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AN INVENTORY MODEL FOR COORDINATING ORDERING, PRICING AND ADVERTISEMENT POLICY FOR AN ADVANCE SALES SYSTEM

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Abstract: In this paper we study a coordinated stock replenishment, pricing, and advertisement problem for an inventory with advance booking system. A single period planning horizon is considered, consisting of both advance sales and spot sales periods. The discount is offered to the customrs for booking the product in advance when the replenishment arrives. The product demand is price and advertising expenditure sensitive. This paper aims to find the optimal ordering quantity, selling price, and advertising expenditure of the product, which maximizes the total profit. The solution algorithm is suggested for computing the optimal solution, which is illustrated numerically.

Keywords: Joint decision, Inventory-pricing, Inventory-advertisement, Advance Sales, Discount, Advertisement.

MSC: 90B85, 90C26.

1. INTRODUCTION

In today's competitive market scenario, effective inventory planning requires consideration of all the factors which affect demand, and particularly seling as the key factor. By the law of demand, the demand of a product decreases when its selling price increases. Hence, pricing of a product becomes the critical decision which impacts the profitability of a firm. Setting a very high price may reduce demand, leading to a loss in all potential market shares of the firm. On the other hand, a very low price may increase demand, however, revenue generated this way may not be enough to cover the inccured costs.

A common strategy used by firms to boost the demand and enhance revenues is to offer a discount to customers who book the product in advance. During some period, customers are offered to book the product at a discounted price in advance, which they receive on some specified future date when the replenishment arrives. After that period, the product is sold to customers at a non-discounted rate and at purchase time. This way, they are attracted to purchase in advance at a low price, which in turn increases the total generated sales revenue as compared to that of non-discounted spot sales.

Product sales (demand) is also influenced by the advertisement effort [11]. Advertisement efforts help companies to make consumers aware of their products and new launches in the market. Advertisement effort increases sales by influencing consumers with an immediate reason to make the purchase [10]. These efforts may include activities like promotions in various media such as print, radio, television, telemarketing, through search engines and social media. Advertisements as a promotional tool tend to influence the decisions of the consumers by enlightening, educating, and persuading them on their acceptability of the product [2]. Thus, advertisements help firms to generate more sales and to sustain in the competition.

There exista a substantial research in joint inventory control and pricing literature. Whitin [22] was a pioneer in studying the inventory planning problem that considers inventory and pricing simultaneously. He proposed models considering both stationary and uncertain demand as a general function of the unit selling price. Mills [15, 16] explicitly specified selling price as a dependent mean demand function. Further, Chen and Levi[6, 7] considered random price-dependent demand for a single product. Thomas [20] gave the production and pricing decisions with a deterministic demand function for single product. Kunreuther and Richard [12] studied the inventory and pricing problem with deterministic stationary demand curve for non-seasonal items. Chan et al. [5] provide a survey on joint inventory and pricing studies.

There is a growing literature on inventory control problem with advance sales. Tsao [21] gave a cost minimization model, which determines order quantity and the discount offered during advance sales. You [25] considered the case where the firm changes the price during advance sales period. Later, You and Wu [26] extended this model for inventory system to include the scenario of spot sales and order cancellations. Dye and Hsieh [9] extended that model for deteriorating items.

Few researchers have also combined inventory and advertising decisions. Balcer

[3, 4] investigated a joint lot sizing and advertisement decision, which assumed the demand to be advertising expenditure dependent. Sogomonian and Tang [18] considered the combined production and advertising decisions model in which the demand function is adapted from the marketing literature (Little [14]). Further, Cheng and Sethi [8] formulated a joint production and advertising model with demand being stochastic, depending on the level of advertising.

Most of the past studies neglected the combined impact of pricing, advertising, and advance sales discount on the inventory policy. This paper focuses on the problem of ordering, pricing, and advertising policy for an inventory cycle consisting of both advance and spot sales. A discounted price is offered for reserving the product and making payment in advance, which will be available to the customers on a specific future date. This is followed by the spot sales period, in which a non-discounted price is offered, and the demand is immediately met. Here, the demand function is considered to be a multiplicative, which is unit selling price and per unit advertisement expenditure dependent. Similar functions are considered in [13, 17, 19, 23, 27, 24]. Total profit maximization model is formulated to obtain the optimal unit selling price, order quantity, and advertising expenditure.

2. MODEL DEVELOPMENT

2.1. Assumptions

- Instantaneous replenishment rate of the product with zero lead time.
- Single period inventory problem is considered with inventory cycle $[0, T]$, which consists of two periods: advance sales period $[0, T_A]$ and spot sales period $[T_A, T]$.
- Demand generated during $[0, T_A]$ is satisfied immediately as soon as the replenishment is received at a certain specified date.
- Selling price p_A is offered during advance sales period $[0, T_A]$ and p is offered during the spot sales period $[T_A, T]$ such that $p_A = \gamma p; 0 < \gamma < 1$ i.e., $(1-\gamma)$ the percentage of discount is given to customers for booking the product in advance during $[0, T_A]$.
- Demand is assumed to be unit selling price and per unit advertisement expenditure sensitive and is modeled as multiplicative functions i.e.,$D(p, M) = d(p)g(M)$, $d(p)$ being the component due to the influence of unit selling price p, and $g(M)$ the component due to the influence of monetary investment in advertisements, where M is the amount of money spent on advertisements per unit of an item.
- Price dependent demand component is assumed to be linearly affected by unit selling price that is, $d(p) = a - bp; a, b > 0$.
- Advertisement dependent demand function is assumed linearly increasing in per unit advertising expenditure that is, $g(M) = \alpha + \beta M; \alpha, \beta > 0$.

2.2. Notations

T	The total inventory cycle length (decision variable)
T_A	Advance sales period length
$I_A(t)$	Level of inventory at time t during$[0, T_A]$
$I_S(t)$	Level of inventory at time t during$[T_A, T]$
I_{MAX}	Maximum level of inventory
I_{MIN}	The maximum amount of units purchased in advance sales
γ	Discounting factor
p_A	Selling price during advance sales period in($/unit) (decision variable)
p	Selling price during spot sales period ($/unit) (decision variable)
$D(p, M)$	Demand rate
M	Advertising expenditure per unit item ($/unit) (decision variable)
B	Total advertisement budget ($)
c	Purchase cost per unit item ($/unit)
h	Unit inventory holding cost per unit time
A	Fixed ordering cost ($)

2.3. Model formulation

Fig.1 describes the inventory cycle. The level of inventory is zero at the beginning of the cycle, $t = 0$, the and because of the demand generated at rate, $D(p_A, M)$, it changes at rate $-D(p_A, M)$ during $[0, T_A]$. At time T_A, the advance sales demand gets cumulated to I_{MIN}. Immediately after time T_A, replenishment order quantity $Q = I_{MAX} + I_{MIN}$ arrives and the demand generated due to advance sales booking is satisfied, and thus, inventory level becomes I_{MAX}. After time $t = T_A$, because of spot sales demand, the inventory level reduces at the rate $D(p, M)$, and becomes zero at T.

Now, the inventory level for $[0, T_A]$ is given by

$$\frac{dI_A(t)}{dt} = -D(p_A, M); \qquad t \in [0, T_A] \tag{1}$$

Using $I_A(0) = 0$ and solving we get

$$I_A(t) = -D(p_A, M)t; \qquad t \in [0, T_A] \tag{2}$$

The inventory level for $[T_A, T]$ is given by

$$\frac{dI_S(t)}{dt} = -D(p, M); \qquad t \in [T_A, T] \tag{3}$$

Using $I_S(T) = 0$ and solving we get

$$I_S(t) = -D(p, M)(T - t); \qquad t \in [T_A, T] \tag{4}$$

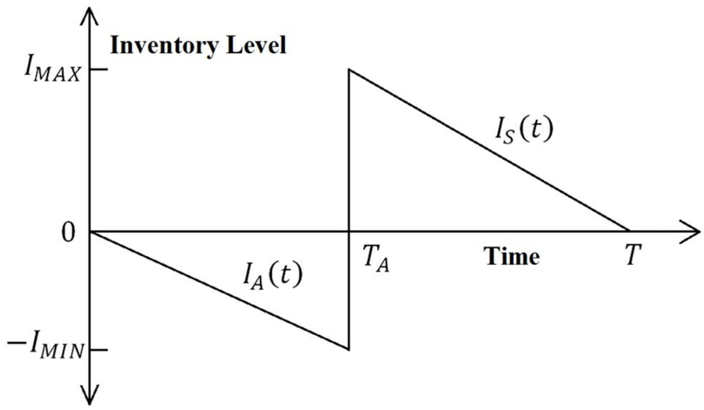

Figure 1: Graphical depiction of Inventory Cycle

Now, the order quantity is given as

$$Q = I_{MAX} + I_{MIN} = -I_A(T_A) + I_S(T_A) \tag{5}$$

where I_{MAX} is the maximum level of inventory and I_{MIN} is the amount of cumulated advance sales. Thus we get,

$$\begin{aligned} Q &= D(p_A, M)T_A + D(p, M)(T - T_A) \\ &= (a - bp)(\alpha + \beta M)T_A + (a - bp)(\alpha + \beta M)(T - T_A) \end{aligned} \tag{6}$$

Total sales during the advance sales period $[0, T_A]$, N_A is given as

$$N_A = D(p_A, M)T_A \tag{7}$$

Total sales during the spot sales period $[T_A, T]$, N_A is given as

$$N_S = D(p, M)(T - T_A) \tag{8}$$

Thus, the total revenue due to advance and spot sales is given by

$$\begin{aligned} TR &= (p_A N_A + pN_S) \\ &= p_A D(p_A, M)T_A + pD(p, M)(T - T_A) \end{aligned} \tag{9}$$

Now,the holding cost, H is,

$$\begin{aligned} H &= \int_{T_A}^{T} hI_S(t)dt \\ &= hD(p, M) \cdot \frac{(T - T_A)^2}{2} \end{aligned} \tag{10}$$

Next, the total profit generated during [0,T] is given as

Total profit = Total Revenue due to total sales − Advertisement expenditure
− Ordering Cost − Purchase cost − Holding Cost

$$\begin{aligned}\Pi &= p_A D(p_A, M)T_A + pD(p, M)(T - T_A) - MT[D(p_A, M) + D(p, M)]\\ &\quad - c[D(p_A, M)T_A + D(p, M)(T - T_A)] - A - hD(p, M) \cdot \frac{(T - T_A)^2}{2}\end{aligned} \tag{11}$$

Using the following substitutions in the above equation

$$\begin{aligned} p_A &= \gamma p; 0 < \gamma < 1\\ D(p_A) &= a - bp_A = a - b\gamma p\end{aligned}$$

$$\begin{aligned}\Pi(p, T, M) &= \gamma p(a - b\gamma p)(\alpha + \beta M)T_A + p(a - bp)(\alpha + \beta M)(T - T_A)\\ &\quad - M(\alpha + \beta M)T[(a - b\gamma p) + (a - bp)]\\ &\quad - c[(a - b\gamma p)(\alpha + \beta M)T_A + (a - bp)(\alpha + \beta M)(T - T_A)] - A\\ &\quad - h(a - bp)(\alpha + \beta M)\frac{(T - T_A)^2}{2}\end{aligned} \tag{12}$$

Now, the following profit maximization model is formulated to obtain the optimal replenishment cycle length, unit selling price, and advertisement expenditure

$$\max_{p,T,M} \quad \Pi(p, T, M)$$

subject to

$$QM \leq B \implies [(a - b\gamma p)T_A + p(a - bp)(T - T_A)](\alpha + \beta M)M \leq B \tag{13}$$

$$T > T_A \geq 0 \tag{14}$$

$$p, M \geq 0 \tag{15}$$

$$(P1)$$

where constraint (13) corresponds to the advertisement budget constraint, and B is the maximum budget of the firm for advertisements. Constraint (14) is to ensure that the advance sales period does not exceed total inventory cycle length, and constraint (15) is the non-negativity constraint.

3. OPTIMALITY AND SOLUTION PROCEDURE

3.1. Optimal Solution

The problem ($P1$) determines the selling price, cycle time, and advertisement expenditure, which maximizes the total profit. This is a non-linear optimization problem in p, T, and M. The procedure similar to Abad [1] is adopted to find the optimal solution. Under this first, p is fixed, then, for the fixed p, total profit $\Pi(T, M|p)$ is maximized, and model reduces to

$$\max_{T,M} \ \Pi(T, M|p)$$

subject to

$$\begin{aligned}[(a-b\gamma p)T_A + p(a-bp)(T-T_A)](\alpha+\beta M)M &\le B\\ T > T_A &\ge 0\\ M &\ge 0 \qquad (P2)\end{aligned}$$

Now, $\Pi(T, M|p)$ is a strictly concave in T, M for given p (proof in Appendix). Since, for the above problem, the objective function is concave and with linear constraints, there exists a unique global maximum of ($P2$).

Next, the existence and uniqueness condition for the optimal unit selling price is established. For any T^*, M^*, the first order necessary condition for maximizing $\Pi(p|T, M)$ is

$$\frac{\partial \Pi(p|T^*, M^*)}{\partial p} = 0 \tag{16}$$

$$\begin{aligned}\Longrightarrow\ & \gamma(a-2b\gamma p)(\alpha+\beta M^*)T_A + (a-2bp)(\alpha+\beta M^*)(T^*-T_A)\\ &+ bM^*(\alpha+\beta M^*)(1+\gamma)T^* + bc(\alpha+\beta M^*)[\gamma T_A + (T^*-T_A)]\\ &+ hb(\alpha+\beta M^*)\frac{(T^*-T_A)^2}{2}\end{aligned} \tag{17}$$

The second order condition is

$$\frac{\partial^2 \Pi(p|T^*, M^*)}{\partial p^2} = -2b\gamma^2(\alpha+\beta M^*)T_A - 2b(\alpha+\beta M^*)(T^*-T_A) < 0 \tag{18}$$

Consequently, $\Pi(p|T^*, M^*)$ is concave in p , given (T^*, M^*), thus p given by equation (17) is a unique solution of

Condition (18) implies that $\exists\, p(p^*)$, which is a unique maximizer of $\Pi(p|T^*, M^*)$, and is given by solving ($P3$). Combining the above findings, the following solution algorithm is proposed.

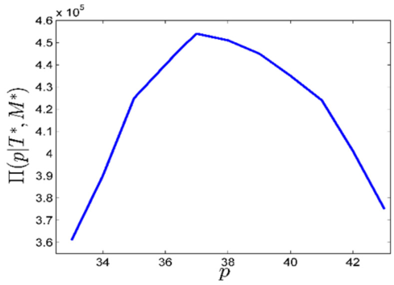

Figure 2: Graph of $\Pi(p|T^*, M^*)$ w.r.t. p

3.2. Solution Procedure

Step 1: Starting at $k = 0$, initiate the trial value of $p_k = p_0$, where $p_0 \in (c, p_{\max})$ is any arbitrary, and $p_{\max}$ is the selling price at which $D(p_{\max}) = 0$.

Step 2: Obtain T^* and M^* for a given p_j from $(P2)$.

Step 3: Using T^* and M^* obtained in Step 2, compute the optimal $p_{(k+1)}$ by solving $(P3)$.

Step 4: If $|p_k - p_{(k+1)}| < \epsilon$, where ϵ is a small number, then $p^* = p_{(k+1)}$, (p^*, T^*, M^*) is the optimal solution, procedure terminates. Else, set k=k+1, and repeat from Step 2.

Step 5: Calculate optimum order quantity Q^* from equation (6) and total profit Π^* from equation (12).

4. NUMERICAL ANALYSIS

Now, the model is illustrated with numerical examples.

Example 1. Consider a firm selling a product for which it needs advertising effort to increase the profit. The maximum advertisement budget is $B = \$80,000$. The firm provides 5% discount to customers for booking the product in advance and advance sale period is 4 weeks.
Let us also consider:
Holding cost is $h = \$0.5$/unit/week;
Unit cost is $c = \$10$/unit.
Ordering cost is $A = \$100$/order
Demand function is $D(p, M) = (750 - 15p)(2.6 + 0.6M)$.
Solution

First, we solve $D(p_{\max}) = a - bp_{\max} = 0$, we obtain $p_{\max} = \$50$. Thus we choose $p_0 \in (3, 50)$ to be \$30 per unit. We used starting value $p_0 = \$30$ and follow the procedure given in Section. After performing nine iterations, we have

k	p_k($/unit)	T(weeks)	M($/unit)	Π($)
1	35.0103	41.4760	0.6082	424146
2	36.4142	49.0573	1.1016	450794
3	36.7499	51.4357	1.1483	453014
4	36.8258	51.9499	1.1616	453311
5	36.8498	52.1208	1.1648	453380
6	36.8557	52.1618	1.1658	453397
7	36.8570	52.1703	1.1660	453401
8	36.8575	52.1725	1.1661	453402
9	36.8577	52.1733	1.1661	453402
10	36.8578	52.1738	1.1661	453402
11	36.8578	52.1738	1.1661	453402

Table 1: Iterative computation of Example 1

$p^* = \$36.8578$ per unit, $T^* = 52.1738$ weeks, $M^* = \$1.1661$ per unit, $Q^* = 34667.3$ units and $\Pi^* = \$453402$.
The computation values are given in Table 1.

After running the solution procedure with distinct starting values of $p_0 \in (3, 50)$, the plot in Fig. 2 has been generated, which shows the strict concavity of the total profit, $\Pi(p|T^*, M^*)$ w.r.t. p.

Example 2. Consider a firm selling a product for which it needs advertising effort to increase the profit. The maximum advertisement budget is $B = \$100,000$. The firm provides 10% discount to customers for booking the product in advance and advance sale period is 4 weeks.
Let us also consider:
Holding cost is $h = \$2.5$/unit/week;
Unit cost is $c = \$250$/unit.
Ordering cost is $A = \$200$/order
Demand function is $D(p, M) = (5000 - 140p)(1.09 + 1.02M)$.

Solution

First, we solve $D(p_{\max}) = a - bp_{\max} = 0$, we obtain $p_{\max} = \$357.143$. Thus we choose $p_0 \in (3, 357.143)$ to be $310 per unit. Similar to the previous example, we choose the starting value $p_0 = \$310$ and follow the procedure given in Section 3. After performing seven iterations, we have
$p^* = \$321.755$ per unit, $T^* = 29.1732$ weeks, $M^* = \$0.4084$ per unit, $Q^* = 244894$ units and $\Pi^* = \$9.56699 \times 10^6$

We now study how the change in the various parameters effects p^*, T^*, M^*, Q^* and Π^*. One parameter is varied at a time, fixing the others for Example-2. Table 2 and plots in Figs.3-6 give the results.

Observations made from the sensitivity analysis shown in Table 2 and Figs. 3-6 are given as follows:

1. With the increase in h, T_A and c, p^* increases. Also, p^* is less sensitive to h and T_A, as compared to that of c. This is quite justified since the increase in

Parameter	Value	p^* (\$/units)	T^* (weeks)	M^* (\$/units)	Q^* (units)	Π^* (million \$)
	0.75	327.967	32.8498	0.3766	265483	6.55141
	0.80	326.180	31.6377	0.3876	257969	7.95976
γ	0.85	342.115	30.4089	0.3982	251106	8.96273
	0.90	321.755	29.1732	0.4084	244894	9.56699
	0.95	319.078	27.9364	0.4179	239280	9.78171
	1.50	321.167	46.8822	0.2850	350854	13.4185
h	2.50	321.755	29.1732	0.4084	244894	9.56699
	3.50	322.435	21.7968	0.5031	198743	7.88961
	4.50	323.116	17.7747	0.5790	172711	6.94943
	5.50	323.762	15.2463	0.6414	155902	6.34785
	3	321.153	27.8814	0.4260	234732	9.09292
	4	321.755	29.1732	0.4084	244894	9.56699
T_A	5	322.315	30.4445	0.3924	254842	10.0394
	6	322.829	31.6933	0.3778	264635	10.5103
	7	323.285	32.9242	0.3644	274426	10.9798
	200	302.933	41.6904	0.2270	440560	25.4008
	225	312.109	35.2748	0.2989	334599	16.2854
c	250	321.755	29.1732	0.4084	244894	9.56699
	275	332.137	23.5912	0.5857	170746	4.82254
	300	343.584	18.6664	0.9074	110198	1.62166

Table 2: Sensitivity analysis results

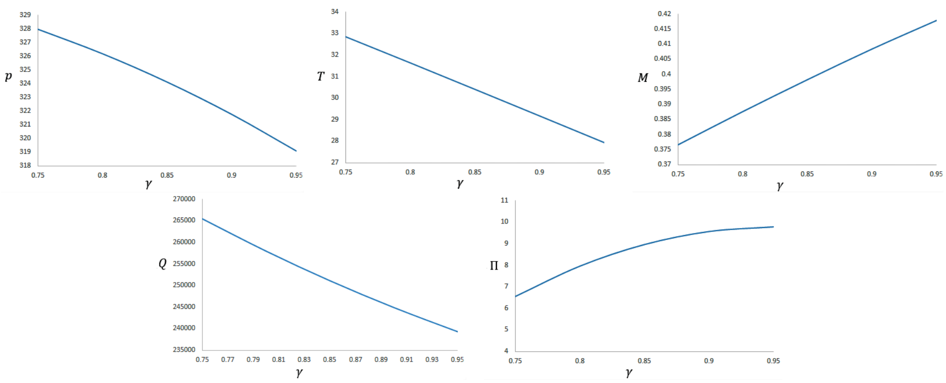

Figure 3: Graphs for sensitivity w.r.t. γ

purchase cost causes increase in the optimal selling price. When γ increases, p^* decreases, that is, when less percentage of discounts is offered to customers for booking the product in advance, the selling price should be kept low.

2. With the increase in parameters γ, h, and c, T^* decreases. That is, the lower the holding cost and cost of purchase, the longer the replenishment period. Also, the lower the advance sales discount the shorter the replenishment period. Moreover, when the advance sales period length T_A increases, the

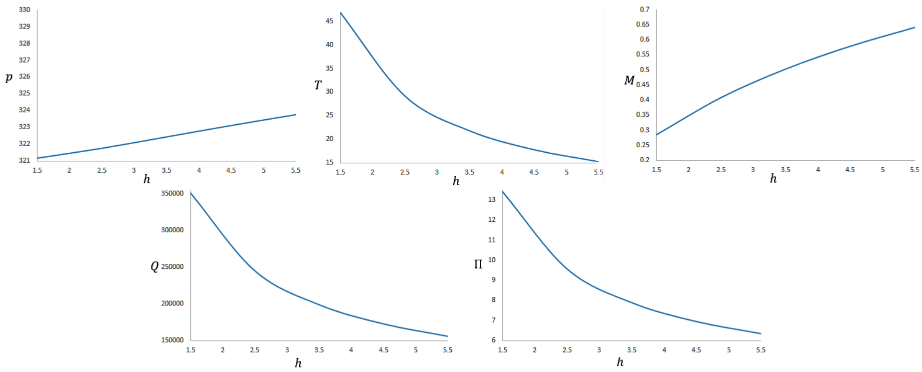

Figure 4: Graphs for sensitivity w.r.t. h

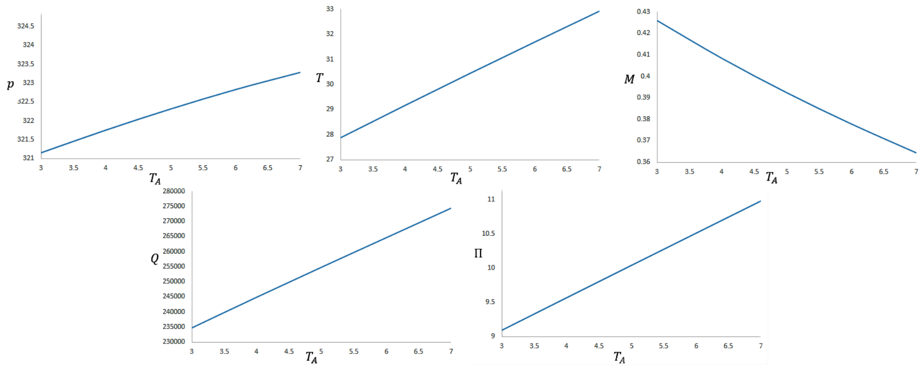

Figure 5: Graphs for sensitivity w.r.t. T_A

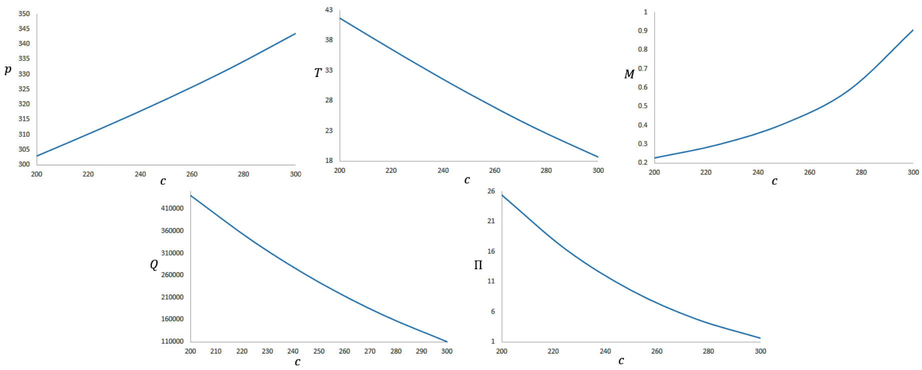

Figure 6: Graphs for sensitivity w.r.t. c

replenishment cycle period also increases.

3. As γ, h and c increases, optimal advertisement expenditure per unit M^*

increases. M^* is less sensitive to γ as compared to h and c. That is when the high discount is offered, firm needs to invest less for advertising the product and purchase cost and holding cost have a high positive influence on the optimal advertisement expenditure. While, when advance sales period increases, advertisement expenditure per unit decreases.

4. When γ, h and c increase, optimum lot size decreases while when T_A increases, optimum lot size increases. That is a high discount for booking the product in advance will induce more sales and thus, firm needs to maintain higher lot size. And when holding cost and purchase cost increase, ordering quantity decreases. Also, since the increase in T_A causes increase in replenishment cycle length, thus ordering quantity increases.
5. With the increase in discount, profit decreases and optimum selling price increases, but Q^* increases. That is a higher discount can be given by the firm in order to increase its sales while compromising with its profit. Also, with an increase in T_A, total profit increases, thus, the firm can earn a higher profit by increasing the length of the period for booking the product in advance. Further, with an increase in h and c, profit decreases, which is reasonable as the higher cost will lead to lower profit.

5. CONCLUSIONS

A profit maximization model is developed for obtaining optimal ordering quantity, selling price, and advertising expenditure for an advance booking system. In this problem, a period of advance booking is considered in which a discounted price is offered to the customers. Advertisement efforts are used to increase the sales of the product. Demand is the unit selling price and per unit advertisement expenditure sensitive. To illustrate the model, numerical analysis is performed. Results suggest that both advance sales period length and discount result in an increased profit of the firm. Thus, it is beneficial for the firm to offer advance booking with discounts as it accelerates the sales as compared to spot sales, and thus helps to fetch more profit by achieving greater sales revenue. Further, when less discount is offered during the advance sales period, the firm should set the lower selling price.

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APPENDIX

Proof concavity of $\Pi(T, M|p)$ with respect to T and M, for given p:

Let (T^*, M^*) be the stationary point $\Pi(T, M|p)$. Then

$$\frac{\partial^2 \Pi(T, M|p)}{\partial T^2}\Big|_{(T^*,M^*)} = -h(a-bp)(\alpha+\beta M^*) < 0$$

$$\frac{\partial^2 \Pi(T, M|p)}{\partial M^2}\Big|_{(T^*,M^*)} = -2\beta[(a-b\gamma p)+(a-bp)] < 0$$

$$\begin{aligned}\frac{\partial^2 \Pi(T, M|p)}{\partial TM}\Big|_{(T^*,M^*)} =& p\beta(a-bp) - (\alpha+2\beta M^*)[(a-b\gamma p)+(a-bp)] - c\beta(a-bp)\\ &- h\beta(a-bp)(T^*-T_A)\end{aligned}$$

Now, the hessian matrix determinant $\det(\mathcal{H})$ at (T^*, M^*) is

$$\det(\mathcal{H}) = \left(\frac{\partial^2 \Pi(T, M|p)}{\partial T^2}\Big|_{(T^*,M^*)}\right)\times\left(\frac{\partial^2 \Pi(T, M|p)}{\partial M^2}\Big|_{(T^*,M^*)}\right)\times\left(\frac{\partial^2 \Pi(T, M|p)}{\partial TM}\Big|_{(T^*,M^*)}\right)^2$$

By assuming $bp_A < a < 2bp_A$, we have

$$\frac{\partial^2 \Pi(T, M|p)}{\partial T^2}\Big|_{(T^*,M^*)} < 0$$

,

$$\left|\frac{\partial^2 \Pi(T, M|p)}{\partial T^2}\Big|_{(T^*,M^*)}\right| > \left|\frac{\partial^2 \Pi(T, M|p)}{\partial TM}\Big|_{(T^*,M^*)}\right|,$$

$$\left|\frac{\partial^2 \Pi(T, M|p)}{\partial M^2}\Big|_{(T^*,M^*)}\right| > \left|\frac{\partial^2 \Pi(T, M|p)}{\partial TM}\Big|_{(T^*,M^*)}\right|$$

Therefore,

$$\det(\mathcal{H}) => 0$$

Hence, Hessian matrix $\mathcal{H}$ at point (T^*, M^*) is negative definite. Consequently, we can conclude that the stationary point (T^*, M^*) is a global maximum.