Yugoslav Journal of Operations Research
31 (2021), Number 3, 363–372
DOI: https://doi.org/10.2298/YJOR200618032P

THE APPLICATION DOMAIN OF DIFFERENCE TYPE MATRIX $D\,(r, 0, s, 0, t)$ ON SOME SEQUENCE SPACES

Avinoy PAUL
Department of Mathematics, Cachar College, Silchar−788001, Assam. India
avinoypaul@gmail.com and avinoypaul@rediffmail.com

Binod Chandra TRIPATHY
Department of Mathematics, Tripura University, Suryamaninagar, Agartala − 799022, Tripura, India
tripathybc@yahoo.com and tripathybc@rediffmail.com

Received: June 2020 / Accepted: October 2020

Abstract: In this paper we introduce new sequence spaces with the help of domain of matrix $D(r, 0, s, 0, t)$, and study some of their topological properties. Further, we determine β and γ duals of the new sequence spaces and finally, we establish the necessary and sufficient conditions for characterization of the matrix mappings.

Keywords: β and γ Duals, Matrix Transformation, Schauder Basis.

MSC: 40A05; 40A25; 40C05; 40H05; 46A35; 47A10.

1. INTRODUCTION

Throughout the paper we denote w, ℓ_∞, c, c_0, and ℓ_p be the space of all, bounded, convergent, null and p-absolutely summable sequences, respectively.

Let X and Y be two sequence spaces and $B = (b_{nk})$ an infinite matrix of real or complex numbers b_{nk}, where $n, k \in \mathbb{N} = \{1, 2, \ldots\}$. Then, we say that B defines a matrix mapping from X into Y, denoted by $B : X \to Y$, if for every sequence $x = (x_n) \in X$, the sequence $Bx = \{(Bx)_n\}$ is in Y, where

$$(Bx)_n = \sum_{k=1}^{\infty} b_{nk}\, x_k \qquad (1)$$

provided the right hand side converges for every $n \in \mathbb{N}$ and $x \in X$.

If μ is a normed sequence space, we can write $D_\mu(B)$ for $x \in w$, for which the sum in Eqn. 1 converges in the norm of μ. We write $(\lambda : \mu) = \{B : \lambda \subseteq D_\mu(B)\}$ for the space of those matrices B transform the all sequences in λ into μ in this sense.

The sequence space $\lambda_B = \{x = (x_k) \in w : Bx \in \lambda\}$ is called the domain of an infinite matrix B in a sequence space λ. One can easily verify that the sequence spaces λ_B and λ are linearly isomorphic when B is a triangle. The continuous dual space of the space λ_B is defined by $\lambda_B^* = \{f : f = goB, g \in \lambda^*\}$.

The idea for constructing a new sequence space by means of the matrix domain of a particular limitation method has recently been employed by Altay and Basar [1, 2], Malkowsky and Savas [11], Basar et al. [3], Kirisci and Basar [8], Ng and Lee [12], Sönmez [14] and many more. In summability theory, different classes of matrices have been investigated. Characterization of matrix classes is found in Rath and Tripathy [13], Tripathy and Sen [19] and many others.Recently, Khan et al. [6, 7] have studied the concept of I-convergence of the sequence where I is an ideal.

Let r, s, t be non-zero real numbers, and define as in [16] matrix $D = D(r, 0, s, 0, t) = \{d_{nk}(r, s, t)\}$ as follows

$$d_{nk}(r,s,t) = \begin{cases} r, & (n = k) \\ s, & (n = k+2) \\ t, & (n = k+4) \\ 0, & \text{otherwise.} \end{cases}$$

2. SOME NEW SEQUENCE SPACES AND THEIR TOPOLOGICAL PROPERTIES

Now, we introduce the new sequence spaces, derived by the matrix D as follows

$$\begin{aligned}
(\ell_\infty)_D &= \{x = (x_k) \in w : Dx \in \ell_\infty\} = \{x = (x_k) : \sup_{k \in \mathbb{N}} |rx_k + sx_{k-2} + tx_{k-4}| < \infty\}, \\
(c_0)_D &= \{x = (x_k) \in w : Dx \in c_0\} = \{x = (x_k) \in w : \lim_{k\to\infty} |rx_k + sx_{k-2} + tx_{k-4}| = 0\}, \\
(c)_D &= \{x = (x_k) \in w : Dx \in c\} \\
&= \{x = (x_k) \in w : \exists\, l \in \mathbb{C}, \lim_{k\to\infty} |rx_k + sx_{k-2} + tx_{k-4} - l| = 0\}, \\
(\ell_p)_D &= \{x = (x_k) \in w : Dx \in \ell_p\} = \{x = (x_k) \in w : \sum |rx_k + sx_{k-2} + tx_{k-4}|^p < \infty\}.
\end{aligned}$$

We quate the following results, useful for our study from Stieglitz and Tietz [15]

$$\sup_{n \in \mathbb{N}} \sum_k |a_{nk}|^q < \infty, \tag{2}$$

$$\sup_{k,n \in \mathbb{N}} |a_{nk}| < \infty, \tag{3}$$

$$\lim_{n\to\infty} a_{nk} = \alpha_k, \quad (k \in \mathbb{N}), \tag{4}$$

$$\lim_{n\to\infty} \sum_k |a_{nk}| = \sum_k |\alpha_k|, \tag{5}$$

$$\lim_{n\to\infty} \sum_k a_{nk} = \alpha. \tag{6}$$

Lemma 1. *The necessary and sufficient conditions for $A \in (\lambda : \mu)$, where $\lambda \in \{\ell_\infty, c, c_0, \ell_p, \ell_1\}$ and $\mu \in \{\ell_\infty, c\}$ can be read from Table 1:*

(i)	(2) with $q = 1$
(ii)	(2)
(iii)	(3)
(iv)	(4) and (5)
(v)	(2) with $q = 1$, (4) and (6)
(vi)	(2) with $q = 1$ and (4)
(vii)	(2) and (4)
(viii)	(3) and (4)

Table 1: The characterization of the class $(\lambda : \mu)$, with $\lambda \in \{\ell_\infty, c, c_0, \ell_p, \ell_1\}$ and $\mu \in \{\ell_\infty, c\}$.

From⟶ To↓	ℓ_∞	c	c_0	ℓ_p	ℓ_1
ℓ_∞	(i)	(i)	(i)	(ii)	(iii)
c	(iv)	(v)	(vi)	(vii)	(viii)

Lemma 2. *We give the following results from Tripathy and Paul [15, 16]*

(i) *Let $\lambda \in \{\ell_\infty, c, c_0, \ell_p\}$, s be a complex number such that $\sqrt{s)} = -s$, and define the set $S = \left\{\alpha \in \mathbb{C} : \left|\frac{2(r-\alpha)}{-s+\sqrt{s^2-4t(r-\alpha)}}\right| \leq 1\right\}$. Then $\sigma(D(r,0,s,0,t),\lambda) = S$.*

(ii)$\sqrt{s^2} = s$ *and S be defined as above, we obtain, $\sigma(D(r,0,s,0,t),\lambda) = S$.*

(iii)*Let $S_1 = \left\{\alpha \in \mathbb{C} : \left|\frac{2(r-\alpha)}{-s+\sqrt{s^2-4t(r-\alpha)}}\right| < 1\right\}$, then $\sigma_p(D(r,0,s,0,t)^*, \lambda^*) = S_1$.*

(iv) *Let* S_1 *be defined as in above and* $S_2 = \left\{\alpha \in \mathbb{C} : \left|\frac{2(r-\alpha)}{-s+\sqrt{s^2-4t(r-\alpha)}}\right| = 1\right\}$, *then (a)* $\sigma_r\left(D\left(r,0,s,0,t\right),\lambda\right) = S_1$ *and (b)* $\sigma_c\left(D\left(r,0,s,0,t\right),\lambda\right) = S_2$.

Theorem 3. *Let* $\lambda \in \{\ell_\infty, c, c_0, \ell_p\}$ *and* $D = D(r,0,s,0,t)$ *then*

(i)$\lambda = \lambda_D$ *if* $|r| > \dfrac{|-s+\sqrt{s^2-4tr}|}{2}$.

(ii) $\lambda \subset \lambda_D$ *is strictly if* $|r| \leq \dfrac{|-s+\sqrt{s^2-4tr}|}{2}$.

Proof. Let, $\lambda \in \{\ell_\infty, c, c_0, \ell_p\}$ and $D = D(r,0,s,0,t)$. Since the matrix D satisfies the conditions

$\sup_{n\in\mathbb{N}} \sum_k |d_{nk}| = |r| + |s| + |t|$, $\lim\limits_{n\to\infty} d_{nk} = 0$, $\lim\limits_{n\to\infty} \sum_k d_{nk} = r+s+t$,

and $\sup_{k\in\mathbb{N}} \sum_n |d_{nk}| = |r| + |s| + |t|$ and using Lemma 1, $D \in (\lambda : \lambda)$.

For any sequence $x, Dx \in \lambda$; hence $x \in \lambda_D$. This shows that $\lambda \subset \lambda_D$.

Let, $|r| > \dfrac{|-s+\sqrt{s^2-4tr}|}{2}$.

Since the inverse matrix $D^{-1} = A = (a_{nk}) = \begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & \cdots \\ a_2 & a_1 & 0 & 0 & 0 & \cdots \\ a_3 & a_2 & a_1 & 0 & 0 & \cdots \\ a_4 & a_3 & a_2 & a_1 & 0 & \cdots \\ a_5 & a_4 & a_3 & a_2 & a_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \end{pmatrix}$

of the matrix D also satisfies the conditions

$\sup_{k\in\mathbb{N}} \sum_n |a_{nk}| < \infty$, $\lim\limits_{n\to\infty} a_{nk} = 0$, $\lim_{n\to\infty} \sum_k a_{nk}$ exits,

and $\sup_{n\in\mathbb{N}} \sum_k |a_{nk}| < \infty$, $D^{-1} \in (\lambda : \lambda)$ [see 15, 16], where

$$a_{2n+1} = \frac{1}{\sqrt{s^2-4tr}}\left\{\left[\frac{-s+\sqrt{s^2-4tr}}{2r}\right]^{n+1} - \left[\frac{-s-\sqrt{s^2-4tr}}{2r}\right]^{n+1}\right\}, \quad \text{for } n \in \mathbb{Z}^+$$

$$a_{2n} = 0, \quad \text{for } n \in \mathbb{N}.$$

Therefore, if $x \in \lambda_D$, then $y = Dx \in \lambda$ and $x = D^{-1}y \in \lambda$. Then, $\lambda_D \subset \lambda$. Hence, $\lambda = \lambda_D$.

Let, $|r| < \dfrac{|-s+\sqrt{s^2-4tr}|}{2}$. Consider the sequence $X = (x_n)$, where

$$x_{2n+1} = \frac{1}{\sqrt{s^2-4tr}}\left\{\left[\frac{-s+\sqrt{s^2-4tr}}{2r}\right]^{n+1} - \left[\frac{-s-\sqrt{s^2-4tr}}{2r}\right]^{n+1}\right\} \quad \text{for } n \in \mathbb{Z}^+$$

$$x_{2n} = 0, \quad \text{for } n \in \mathbb{N}$$

$$\text{and } Y = \left(\frac{n}{r}\right). \quad \text{Then, } Dx = e^{(0)} = (1,0,0,\cdots) \in \lambda.$$

Thus, we have $X \in \lambda_D$. But, if $r \neq 0$, and $s^2 \neq 4tr$, from [15, Theorem 5] we have $|u_1| > |u_2|$, since $|u_1| > 1$, we obtain

$$\begin{aligned} x_{2n+1} &= \frac{1}{\sqrt{s^2-4tr}}(u_1^{n+1} - u_2^{n+1}), \quad \text{for } n \in \\ &= \frac{1}{\sqrt{s^2-4tr}}\{1 - (\frac{u_2}{u_1})^{n+1}\}u_1^{n+1} \end{aligned}$$

where $u_1 = \frac{-s+\sqrt{s^2-4tr}}{2r}$ and $u_2 = \frac{-s-\sqrt{s^2-4tr}}{2r}$. Thus, the sequence is unbounded, and $X \in \lambda_D \setminus \lambda$. If $r \neq 0$ and $s^2 = 4tr$, then $u_1 = u_2 = \dfrac{-s}{2r}$. Hence, we have $x_{2n+1} = \dfrac{2(n+1)}{-s}\left(\dfrac{-s}{2r}\right)^{n+1}$ for $n \in \mathbb{Z}^+$ and $x_{2n} = 0$ for $n \in \mathbb{N}$. Since, $|\dfrac{-s}{2r}| > 1$, the sequence X is unbounded and then $X \in \lambda_D \setminus \lambda$.
Next, suppose that $|r| = \frac{|-s+\sqrt{s^2-4tr}|}{2}$

(a) Let $\lambda = c_0, \ell_p$, then, $X \in \lambda_D \setminus \lambda$.

(b) Let $\lambda = c, \ell_\infty$, then the following hold. If $r + s + t = 0$, then

$$DY = \{1, 2, 3 + \frac{s}{r}, \frac{2(r-t)}{r}, \frac{2(r-t)}{r}, \frac{2(r-t)}{r}, \cdots\}$$

and hence $DY \in \lambda$, thus $Y \in \lambda_D \setminus \lambda$. Therefore, we conclude that $\lambda \subset \lambda_D$ is strict.

□

The idea of dual sequence space was introduced by Köthe and Toeplitz [9]. Then, Maddox [10] generalized this notion to $X-$ valued sequence classes where X is a Banach space. Further, Chandra and Tripathy [4] studied on generalized Köthe-Toeplitz duals of some sequence spaces.
The set $S(\lambda, \mu)$ defined by

$$S(\lambda, \mu) = \{z = (z_k) \in w : x\,z \in \mu, \forall x = (x_k) \in \lambda\} \tag{7}$$

is called the multiplier space of the spaces λ and μ. One can easily observe for a sequence space γ with $\lambda \supset \gamma \supset \mu$ that the inclusions $S(\lambda, \mu) \subset S(\gamma, \mu)$ and $S(\lambda, \mu) \subset S(\lambda, \gamma)$ hold. With the notation (7), the β and γ duals of a sequence space λ, which are respectively denoted by λ^β and λ^γ, are defined by $\lambda^\beta = S(\lambda, cs)$ and $\lambda^\gamma = S(\lambda, bs)$.

Lemma 4. *(Kamthan and Gupta*[5, p.52, Exercise 2.5 (i)]) *Let* λ, μ *be the sequence spaces and* $\zeta \in \{\beta, \gamma\}$. *If* $\lambda \subset \mu$, *then* $\mu^{\zeta} \subset \lambda^{\zeta}$.

Lemma 5. ([1, Theorem 3.1]) *Let* $C = (c_{nk})$ *be defined via sequence* $a = (a_k) \in w$ *and the inverse matrix* $V = (v_{nk})$ *of the triangle matrix* $U = (u_{nk})$ *by*

$$c_{nk} = \begin{cases} \sum_{j=k}^{n} a_j v_{jk} & (0 \le k \le n) \\ 0 \quad (k > n) \end{cases}$$

for all $k, n \in \mathbb{N}$. *Then*

$$\begin{aligned} &\{\lambda_U\}^{\gamma} = \{a = (a_k) \in w : C \in (\lambda : \ell_\infty)\} \\ \textit{and } &\{\lambda_U\}^{\beta} = \{a = (a_k) \in w : C \in (\lambda : c)\}. \end{aligned}$$

Combining Lemma 4 and Lemma 5, we have

Corollary 6. *Define the sets* $L_1(r,s,t), L_2(r,s,t), L_3(r,s,t), L_4(r,s,t)$ *and* $L_5(r,s,t)$ *by*

$$L_1(r,s,t) = \{b = (b_k) \in w : \sup_{n \in \mathbb{N}} \sum_{k=0}^{n} |\sum_{j=k}^{n} a_{jk} b_j|^q < \infty\},$$

$$L_2(r,s,t) = \{b = (b_k) \in w : \lim_{n\to\infty} \sum_{j=k}^{n} a_{jk} b_j \ \ \textit{exist}\},$$

$$L_3(r,s,t) = \{b = (b_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} |\sum_{j=k}^{n} a_{jk} b_j| = \sum_{k=0}^{\infty} |\lim_{n\to\infty} \sum_{j=k}^{n} a_{jk} b_j|\},$$

$$L_4(r,s,t) = \{b = (b_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} \sum_{j=0}^{k} a_{jk} b_k \ \ \textit{exist}\},$$

$$L_5(r,s,t) = \{b = (b_k) \in w : \sup_{n,k \in \mathbb{N}} |\sum_{j=k}^{n} a_{jk} b_j| < \infty\},$$

$$L_6(r,s,t) = \{b = (b_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} |\sum_{j=k}^{n} a_{jk} b_j| = 0\}.$$

Then,

(i) $\{(\ell_\infty)_D\}^{\gamma} = \{(c)_D\}^{\gamma} = \{(c_0)_D\}^{\gamma} = L_1(r,s,t)$ *with* $q = 1$,

(ii)$\{(\ell_p)_D\}^{\gamma} = L_1(r,s,t)$,

(iii) $\{(\ell_1)_D\}^{\gamma} = L_5(r,s,t)$,

(iv) $\{(\ell_\infty)_D\}^\beta = L_2(r,s,t) \cap L_3(r,s,t)$,

(v) $\{(c_0)_D\}^\beta = L_1(r,s,t) \cap L_2(r,s,t)$, *with* $q = 1$,

(vi)$\{(\ell_p)_D\}^\beta = L_1(r,s,t) \cap L_2(r,s,t)$,

(vii) $\{(\ell_1)_D\}^\beta = L_2(r,s,t) \cap L_5(r,s,t)$,

(viii) $\{(c)_D\}^\beta = L_1(r,s,t) \cap L_2(r,s,t) \cap L_4(r,s,t)$ *with* $q = 1$.

3. MATRIX MAPPING

In this section, we list the characterizations of some classes of infinite matrices related to the classes of sequences introduced in this article. The results can be established using standard techniques.

Lemma 7. ([5, Theorem 4.1]) *Let λ be an FK-space, U be a triangle, V be its inverse and μ be arbitrary subset of w. Then, we have $F = (f_{nk}) \in (\lambda_U : \mu)$ if and only if $C^{(n)} = (c^{(n)}_{mk}) \in (\lambda : c)$ for all $n \in \mathbb{N}$ and $C = (c_{mk}) \in (\lambda : \mu)$, where*

$$c^{(n)}_{mk} = \begin{cases} \sum_{j=k}^{m} f_{nj} v_{jk}, & (0 \le k \le m) \\ 0 \quad k > m \end{cases}$$

for all $k, m, n \in \mathbb{N}$

$$\text{and } \; c_{nk} = \sum_{j=k}^{\infty} f_{nj} v_{jk}.$$

We list the following conditions:

$$\sup_{m\in\mathbb{N}} \sum_{k=0}^{m} |\sum_{j=k}^{m} a_{jk} f_{nj}|^q < \infty, \tag{8}$$

$$\lim_{m\to\infty} \frac{1}{r} \sum_{j=k}^{m} a_{jk} f_{nj} = c_{nk}, \tag{9}$$

$$\lim_{m\to\infty} \sum_{k=0}^{m} |\frac{1}{r} \sum_{j=k}^{m} a_{jk} f_{nj}| = \sum_{k} c_{nk} \;\text{ for each} n \in \mathbb{N}, \tag{10}$$

$$\lim_{m\to\infty} \sum_{k=0}^{m} \sum_{j=0}^{k} a_{jk} f_{nk} = \alpha_n \;\text{ for all }\; n \in \mathbb{N}, \tag{11}$$

$$\underset{m,\,k\,\in\,\mathbb{N}}{sup} |\frac{1}{r} \sum_{j=k}^{m} a_{jk} f_{nj}| < \infty, \tag{12}$$

$$\underset{n\,\in\,\mathbb{N}}{sup} \sum_{k} |c_{nk}|^q < \infty, \tag{13}$$

$$\lim_{n\to\infty} c_{nk} = \beta_k, \tag{14}$$

$$\lim_{n\to\infty} \sum_k |c_{nk}| = \sum_k |\beta_k|, \tag{15}$$

$$\lim_{n\to\infty} \sum_k c_{nk} = \beta, \tag{16}$$

$$\sup_{n,k\in\mathbb{N}} |c_{nk}| < \infty, \tag{17}$$

$$\sup_{k\in\mathbb{N}} \sum_n |c_{nk}| < \infty, \tag{18}$$

$$\lim_{n\to\infty} \sum_k c_{nk} = 0, \tag{19}$$

$$\sup_{\mathbb{N},K\in\wp} |\sum_{n\in\mathbb{N}} \sum_{k\in K} c_{nk}| < \infty, \tag{20}$$

$$\sup_{\mathbb{N}\in\wp} \sum_k |\sum_{n\in\mathbb{N}} c_{nk}|^q < \infty, \tag{21}$$

where $\wp$ denotes the collection of all finite subsets of $\mathbb{N}$.

Table 2: The characterization of the class (λ_D, μ) with $\lambda \in \{\ell_\infty, c_0, \ell_p, \ell_1\}$ and $\mu \in \{\ell_\infty, c, c_0, \ell_1\}$
.

From→ To↓	$(\ell_\infty)_D$	$(c)_D$	$(c_0)_D$	$(\ell_p)_D$	$(\ell_1)_D$
ℓ_∞	A1.	A2.	A3.	A4.	A5.
c	A6.	A7.	A8.	A9.	A10.
c_0	A11.	A12.	A13.	A14.	A15.
ℓ_1	A16.	A17.	A18.	A19.	A20.

We have the following Corollary from Lemma 7:

Corollary 8. *The necessary and sufficient conditions for* $A \in (\lambda : \mu)$ *when* $\lambda \in \{(\ell_\infty)_D, (c_0)_D, (c)_D, (\ell_p)_D\}$ *and* $\mu \in \{\ell_\infty, c_0, c, \ell_1\}$ *can be read from the Table 2, where*

A1.	(9), (10) and (13) with $q = 1$.
A2.	(9), (11) and (8), (13) with $q = 1$.
A3.	(9) and (8), (13) with $q = 1$.
A4.	(8), (9) and (13).
A5.	(9), (12) and (17).
A6.	(9), (10), (14) and (15).
A7.	(9), (11), (14), (16) and (8), (13) with $q = 1$
A8.	(9), (14) and (8), (13) with $q = 1$.
A9.	(8), (9), (13) and (14).
A10.	(9), (12), (14) and (17).
A11.	(9), (10) and (19).
A12.	(9), (11), (14) with $\beta_k = 0$ and (16) with $\beta = 0$ and (8), (13) with $q = 1$.
A13.	(9), (14) with $\beta_k = 0$ and (8), (13) with $q = 1$.
A14.	(8), (9), (13) and (14) with $\beta_k = 0$.
A15.	(9), (12), (14) with $\beta_k = 0$ and (17).
A16.	(9), (10) and (20).
A17.	(8) with $q = 1$, (9), (11) and (20).
A18.	(8) with $q = 1$, (9) and (20).
A19.	(8), (9) and (21).
A20.	(9), (12) and (18).

4. CONCLUSION

Lot of research work has been conducted on almost each convergent sequence space, but a few on their structure, algebraic and topological.To overcome this gap,we investigated the problem of the almost convergence domain of difference of matrix $D(r, 0, s, 0, t)$ and obtained β and γ duals of the new sequence spaces. Moreover, we developed criterion for characterization of the matrix mappings in the almost convergence domain.

REFERENCES

[1] Altay, B., and Basar, F., "Certain topological properties and duals of the matrix domain of a triangle matrix in a sequence space", *Journal of Mathematical Analysis and Applications*, 336 (1) (2007) 632–645.

[2] Altay, B., and Basar, F., "The matrix domain and the fine spectrum of the difference operator on the sequence space $\ell_p, (0 < p < 1)$", *Communications in Mathematical Analysis*, 2(2) (2007) 1–11.

[3] Basar, F., Altay, B., and Mursaleen, M., "Some generalizations of the space bv_p of $p-$bounded variation sequences", *Nonlinear Analysis*, 68(2) (2008) 273–287.

[4] Chandra, P., and Tripathy, B.C., "On generalized Köthe-Toeplitz duals of some sequence spaces", *Indian Journal of Pure and Applied Mathematics*, 33(8) (2002) 1301-1306.

[5] Kamthan, P., and Gupta, P.K., *Sequence Spaces and Series*, Marcel Dekker Inc., New York, Basel, 1981.

[6] Khan, V.A., Ahmad, M., Hasan, SN, and R., Ahmad, R., "I-Convergent Difference Sequence Spaces", *Journal of Mathematical Analysis*, 10(2) (2019) 58–68.
[7] Khan, V.A., Rababah, R.K.A., Ahmad, M., Esi, A., and Idrisi, M.I., "I-Convergent Difference Sequence Spaces Defined by Compact Operator and Sequence of Moduli" *ICIC Express Letters*, 13(10) (2019), 907–912.
[8] Kirisci, M., and Basar, F., "Some new sequence spaces derived by the domain of generalized difference matrix", *Computers & Mathematics with Applications*, 60 (2010) 1299–1309.
[9] Köthe, G., and Toeplitz, O., "Lineare räume mit unendlich vielen koordinaten und reigne unendlicher matrizen", *Journal für die reine und Angewandte Mathematik*, 171 (1934), 193–226.
[10] Maddox, I.J., *Infinite Matrices of Operators*, Springer, Heidelberg, Germany, 1980.
[11] Malkowsky, E., and Savas, E., "Matrix transformations between sequence spaces of generalized weighted means", *Applied Mathematics and Computation,*, 147 (2004) 333–345.
[12] Ng, P.N., and Lee, P.Y., "Cesaro sequence spaces of non-absolute type", *Comment. Math.Prace Mat.*, 20(2) (1978) 429–433.
[13] Rath, D., and Tripathy, B.C., "Matrix maps on sequence spaces associated with sets of integers". *Indian Journal of Pure and Applied Mathematics*, 27 (2), (1996) 197–206.
[14] SÖnmez, A., "Some new sequence spaces derived by the domain of the triple band matrix", *Computers & Mathematics with Applications*, 62 (2011) 641–650.
[15] Stieglitz, M., and Tietz, H., "Matrix transformationen von folgenräumen eine ergebnisübersicht", *Mathematische Zeitschrift*, 154 (1977) 1–16.
[16] Tripathy, B.C., and Paul, A., "The spectrum of the operator $D(r, 0, 0, s)$ over the sequence spaces c_0 and c", *Kyungpook Mathematical Journal*, 53 (2013) 247–256.
[17] Tripathy, B.C. and Paul, A., "The spectrum of the operator $D(r, 0, s, 0, t)$ over the sequence spaces c_0 and c", *Journal of Mathematics*, Volume 2013, Article ID 430965, 7 pages.
[18] Tripathy, B.C., and Paul, A., "The spectrum of the operator $D(r, 0, s, 0, t)$ over the sequence spaces ℓ_p and bv_p", *Afrika Matematika*, 26 (2015) 1137-1151.
[19] Tripathy, B.C., and Sen, M., "Characterization of some matrix classes involving paranormed sequence spaces", *Tamkang Journal of Mathematics*, 37(2) (2006) 155–162.