

Research Article

**SINGLE-VALUED NEUTROSOPHIC YAGER POWER
AGGREGATION OPERATORS AND ITS APPLICATION
TO MCDM**

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Abstract: The notion of Single-valued neutrosophic sets (SVNSs) being a generalization of fuzzy sets, intuitionistic fuzzy sets and picture fuzzy sets enhances implications of these methods in various interdisciplinary problems. Aggregation operator is a prevalent tool for unifying the data from various sources and applied to solve the problems of decision-making. In this article, the Yager's operations and power averaging operator are utilized to formulate and investigate some single-valued neutrosophic Yager power operators, i.e., single-valued neutrosophic Yager power weighted averaging (SVNYPWA) operators, single-valued neutrosophic Yager power order weighted averaging (SVNYPOWA) operator, single-valued neutrosophic Yager power weighted geometric averaging (SVNYPWGA) operator, and single-valued neutrosophic Yager power ordered weighted geometric averaging (SVNYPOWGA). Some important properties of the suggested operators are discussed. Furthermore, SVNYPWA and SVNYPWGA operators in the SVN environment are applied to solve the multi-criteria decision-making (MCDM) problem for selecting a suitable road construction company. Also, the proposed method has been verified using the multi-attributive border approximation area comparison (MABAC) method. Finally, a comparative analysis has been done, to establish the advantage of the proposed approach over the existing methods.

Keywords: Neutrosophic set, aggregation operator, power operators, decision-making.

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1. INTRODUCTION

The issue of portraying inaccurate or incomplete information in real-world situations is very crucial to finding viable solutions to the problems. Under these situations, Zadeh [1] first initiated the concept of Fuzzy Sets (FS), as it acquired great attention due to its ability to handle uncertainty due to vagueness. However, the Fuzzy set attains only a membership degree to an element of a generic universe and lacks a non-membership degree, due to which fuzzy sets were expounded by Atanassov [2] and established a new notion called Intuitionistic Fuzzy Sets by adding the degree of non-membership. However, in some cases, FS and IFS are not sufficient to deal with indeterminate and inconsistent information in real-life situations. To overcome this problem, Smarandache [3] introduced Neutrosophic Sets as an extension of FSs and IFSs. To deal with real applications with specific descriptions, a subclass of neutrosophic sets termed Single-valued neutrosophic sets (SVNSs) was introduced by Wang et al. [4]. The power-average (PA) operator and a power-ordered weighted average (POWA) operator, introduced by Yager [5] in which the weighting vectors depend on the input data and allow values to be fused to support and reinforce each other. Xu [6] and Zhou [7] introduced the power geometric and generalized power arithmetic operators. In the existing literature, various researchers provided aggregation operators such as weighted averaging operator in different environments prominent among them are [8], [9] and [10] and applied in solving MCDM (multi-criteria decision-making) problems. The MCDM method is extensively used to rank alternatives under different attributes in real decision-making problems. Recently, aggregation operators emerged as an important tool to deal with MCDM problems and consist of a wide range of research results in the literature [11, 12, 13, 14, 15, 16]. Later, the OWA operator was introduced in intuitionistic fuzzy and interval-valued intuitionistic fuzzy environments. Some of the prominent studies related to aggregation operators based on FS, Bipolar fuzzy set, Picture FS, Interval-valued picture fuzzy set, Pythagorean FS, and IFS are due to [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

Under a neutrosophic linguistic environment, Fan et al. [36] suggested a normalized weighted Bonferroni mean operator and normalized weighted geometric Bonferroni mean and applied in MAGDM problems. Liu et al. [37] introduced Neutrosophic Bonferroni weighted geometric mean operator based on multi-valued functions in a neutrosophic environment. Yager [38] suggested a new aggregation operator namely the "Yager operator" with more flexibility of implications due to the underlying parameter. Akram and Shahzadi [39] introduced the q-rung ortho-pair FS-based Yager aggregation operators to deal with decision-making problems. In complex Pythagorean FS, Akram et al. [40] introduced Yager norm-based aggregation operations. Garg et al. [41] investigated the decision-making problem in testing the COVID -facility using Fermatean FS and Yager norm operator. Khan et al. [42] obtained a novel operational law based on the Yager t-norm and t-conorm under NS environment. The Aczel-Alsina operator under consideration of the interval-valued single-valued neutrosophic value by using t-norm and t-conorm proposed by Liaqat et al. [43]. Zhao et al. [44] combined the Heronian and Power aggregation operator in SVN environment and introduced a novel MADGDM method using the proposed operators. Afterward, Jana and Pal [45] introduced Dombi power aggregation operator and applied it to the MCDM process in the SVNSs

framework. To consider the advantages of power and Yager aggregation to determine uncertain data that arises in a real-world situation, SVNSs have immense capacity, and hence various MADM methods were established by using SVN Yager operations and power averaging functions that made a significant interest in developing our suggested study.

To the best of our knowledge, there is no work in the literature based on aggregation operations that utilized the Yager operator and Power aggregation operation in the SVNS framework. This motivated us to develop the novel aggregation operations based on Yager operations imbibing the Power averaging, and geometric operators.

The main contribution of the paper is as follows.

1. We formulate a single-valued neutrosophic Yager power arithmetic aggregation (SVNYP A) operator, single-valued neutrosophic Yager power geometric averaging (SVNYPGA) operator, and weighted form of this operator namely SVNYPWA and SVNYPWGA.
2. We Propose an MCDM method based on SVNYPWA and SVNYPWGA operators in the SVN environment.
3. The sensitivity and verification of the proposed aggregating operators have also been studied.
4. A comparative study to check the advantage and effectiveness of the proposed methods is also done.

Subsequent content of this paper is organized as follows: In section 2, a background of relevant notions is set to define Yager type aggregation operations in the SVNS framework, and novel single-valued neutrosophic Yager power arithmetic and geometric averaging operators proposed along with some of their properties. Section 3 includes the application of the proposed aggregation operators in the MADM problem. In section 4, the sensitivity analysis of the proposed method has been presented. Section 5, investigates the consistency of the proposed approach with MABAC method. Section 6 contrasts the proposed aggregation operators with some existing aggregating operators. Finally, conclusion is presented in Section 7.

2. PRELIMINARIES

In this section, first we set the appropriate background to define the new aggregation operators.

2.1. Background

Definition 1. [4] Let X be a universe of discourse. A single-valued neutrosophic set A in a universal set X is described by a triplet that constitutes with truth membership value, indeterminacy-membership value, and the falsity-membership value s.t $\alpha_A: X \rightarrow [0, 1]$, $\beta_A: X \rightarrow [0, 1]$ and $\gamma_A: X \rightarrow [0, 1]$ with condition

$$0 \leq \alpha_A(x_i) + \beta_A(x_i) + \gamma_A(x_i) \leq 3. \quad (1)$$

Here, $\alpha_A(x_i)$, $\beta_A(x_i)$, and $\gamma_A(x_i)$ denote the membership value, indeterminacy value, and non-membership value.

Definition 2. [8] Let $A = (\alpha_A, \beta_A, \gamma_A)$ be a single-valued neutrosophic number of $x_i \in X$. Then a score function $SF(A)$ is defined as follows

$$SF(A) = \frac{2 + \alpha_A - \beta_A - \gamma_A}{3}, SF(A) \in [0, 1]. \tag{2}$$

and, accuracy function can be defined as:

$$AF(A) = \alpha_A - \gamma_A, AF(A) \in [-1, 1]. \tag{3}$$

For any two SVN Ss A_1 and A_2 , Deli and Subas [12] following essential requirements:

- (i) If $SF(A_1) < SF(A_2)$, imply $A_1 < A_2$.
- (ii) If $SF(A_1) > SF(A_2)$, imply $A_1 > A_2$.
- (iii) If $SF(A_1) = SF(A_2)$, then
 - a. If $AF(A_1) < AF(A_2)$, imply $A_1 < A_2$.
 - b. If $AF(A_1) > AF(A_2)$, imply $A_1 > A_2$.
 - c. If $AF(A_1) = AF(A_2)$, imply $A_1 \sim A_2$.

Definition 3. [38] Let us suppose that A and B are two real numbers. Then Yager TN (t -norm) and Yager TCN (t -conorm) can be defined as follows:

$$Yager^{TN}(A, B) = 1 - \min\left(1, ((1 - A)^\rho + (1 - B)^\rho)^{\frac{1}{\rho}}\right) \tag{4}$$

and

$$Yager^{TCN}(A, B) = \min\left(1, (A^\rho - B^\rho)^{\frac{1}{\rho}}\right) \tag{5}$$

where $\rho \geq 1$ and $(A, B) \in [0, 1] \times [0, 1]$.

Operations based on Yager TN and Yager TCN [42]:

Using Yager TN and Yager TCN defined in Eq. (4) and Eq. (5), Khan et al. [42] defined the follow sum and product operations.

Let A_1 and A_2 be two SVN Ss and $\rho > 0$, then Yager sum and Yager product operations $A_1 = (\alpha_1, \beta_1, \gamma_1)$ and $A_2 = (\alpha_2, \beta_2, \gamma_2)$ are defined as below:

Y01. $A_1 \oplus A_2 =$
 $\left(\min\left(1, (\alpha_1^\rho - \alpha_2^\rho)^{\frac{1}{\rho}}\right); \min\left(1, (\alpha_1^\rho - \alpha_2^\rho)^{\frac{1}{\rho}}\right); 1 - \min\left(1, ((1 - \gamma_1)^\rho + (1 - \gamma_2)^\rho)^{\frac{1}{\rho}}\right)\right);$

Y02. $A_1 \otimes A_2 =$
 $\left(1 - \min\left(1, ((1 - \alpha_1)^\rho + (1 - \alpha_2)^\rho)^{\frac{1}{\rho}}\right); \min\left(1, (\beta_1^\rho - \beta_2^\rho)^{\frac{1}{\rho}}\right); \min\left(1, (\gamma_1^\rho - \gamma_2^\rho)^{\frac{1}{\rho}}\right)\right);$

Y03. $\lambda A_1 =$
 $\left(\min\left(1, (\lambda \alpha_1^\rho)^{\frac{1}{\rho}}\right); 1 - \min\left(1, (\lambda(1 - \beta_1)^\rho)^{\frac{1}{\rho}}\right); 1 - \min\left(1, (\lambda(1 - \gamma_1)^\rho)^{\frac{1}{\rho}}\right)\right);$

Y04. $A_1^\lambda =$
 $\left(1 - \min\left(1, ((1 - \alpha_1)^\rho)^{\frac{1}{\rho}}\right); \min\left(1, (\beta_1^\rho)^{\frac{1}{\rho}}\right); \min\left(1, (\gamma_1^\rho)^{\frac{1}{\rho}}\right)\right).$

Definition 4. [45] The Power averaging (PA) operator is defined as follows

$$PA(a_1, a_2, \dots, a_m) = \frac{\sum_{z=1}^m (1+T(a_z))a_z}{\sum_{z=1}^m (1+T(a_z))} \tag{6}$$

Where $T(a_z) = \sum_{z=1}^m \sum_{y \neq z} \text{Supp.}(a_z, a_y)$, and $\text{Supp.}(a, b)$ is the support of b for a , follows some properties:

- (i) $\text{Supp.}(a, b) \in [0, 1]$.
- (ii) $\text{Supp.}(a, b) = \text{Supp.}(b, a)$.
- (iii) $\text{Supp.}(a, b) \geq \text{Supp.}(r, s)$ if $|a - b| < |r - s|$.

Where Supp. defines the similarity index when two values are more similar, then two values are closer and then support between them is stronger.

Definition 5. [45] The Power geometric (PG) operator is defined as follows

$$PG(a_1, a_2, \dots, a_m) = \prod_{z=1}^m (a_z)^{\frac{(1+T(a_z))}{\sum_{z=1}^m (1+T(a_z))}} \tag{7}$$

With the fusion of Yager and power operation on SVN Ss, a single-valued neutrosophic Yager power weighted arithmetic (SVNYPWA) operator, and single-valued neutrosophic Yager power ordered weighted arithmetic (SVNYPOWA) operator, Yager power weighted geometric (SVNYPWG) operator, and single-valued neutrosophic Yager power ordered weighted geometric (SVNYPOWG) operator are proposed in the following.

2.2. Single-valued neutrosophic Yager power arithmetic aggregation operator

Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVN Ss for $z = 1, 2, \dots, m$. Then, the single-valued neutrosophic Yager power arithmetic (SVNYP A) operator is a function defined as follows:

$$SVNYP A(A_1, A_2, \dots, A_m) = \frac{\oplus_{z=1}^m (1+T(A_z))A_z}{\sum_{z=1}^m (1+T(A_z))} \tag{8}$$

Where $T(A_z) = \sum_{z=1}^m \sum_{y \neq z} \text{Supp.}(A_z, A_y)$, and $\text{Supp.}(A_z, A_y)$ defines the support of A_y and A_z , holds some important properties:

- (i) $\text{Supp.}(A_z, A_y) \in [0, 1]$.
- (ii) $\text{Supp.}(A_z, A_y) = \text{Supp.}(A_y, A_z)$.
- (iii) $\text{Supp.}(A_z, A_y) \geq \text{Supp.}(A_r, A_s)$ if $d(A_z, A_y) < d(A_r, A_s)$ where d denotes the distance measure.

The aggregating operator in Eq. (8) has been obtained by using Yager’s PA operator and Yager sum operation YO1. By applying Yager operations on SVN Ss, we get the following theorem.

Theorem 1. Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVN Ss for $z = 1, 2, \dots, m$. Then by applying SVNYP A on SVN Ss, the aggregated value obtained is also an SVN S. Therefore,

$$SVNYP A(A_1, A_2, \dots, A_m) = \frac{\oplus_{z=1}^m (1+T(A_z))A_z}{\sum_{z=1}^m (1+T(A_z))} = \left(\begin{array}{l} \sum_{z=1}^m \frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))} \min.(1, \alpha_z^\rho); \\ 1 - \sum_{z=1}^m \frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))} \min.(1, (1 - \beta_z)^\rho); \\ 1 - \sum_{z=1}^m \frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))} \min.(1, (1 - \gamma_z)^\rho) \end{array} \right)^{\frac{1}{\rho}} \tag{9}$$

where $T(A_z) = \sum_{z=1}^m \sum_{y \neq z} \text{Supp.}(A_z, A_y)$. (10)

In the SVNYP A and SVNYPGA operators, we only take into account the power weight vector and interrelationship among SVNNS but not the weight of every SVNNS. However, in many realistic decision-making, the weights of attributes are also an important parameter. Thus, we propose the single-valued neutrosophic numbers Yager power weighted arithmetic (SVNYPWA) and geometric (SVNYPWGA) operator as given below.

Theorem 2. Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVNNSs for $z = (1, 2, \dots, m)$. Then by applying SVNYPWA on SVNNSs, the aggregated value obtained is also a SVNNSs, $\varphi = (\varphi_1, \varphi_2, \dots, \varphi_m)^T$ is the weight vector of A_z where $\varphi_z \in [0, 1]^T$ s.t $\sum_{z=1}^m \varphi_z = 1$. Therefore, SVNYPWA: $\psi^m \rightarrow \psi$ accepts the following

$$\begin{aligned} SVNYPWA_\varphi(A_1, A_2, \dots, A_m) &= \frac{\bigoplus_{z=1}^m \varphi_z(1+T(A_z))A_z}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \\ &= \left(\begin{array}{l} \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min.(1, (\alpha_z)^\rho); \\ 1 - \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min.(1, (1 - \beta_z)^\rho); \\ 1 - \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min.(1, (1 - \gamma_z)^\rho) \end{array} \right)^{\frac{1}{\rho}} \end{aligned} \quad (11)$$

$$\text{where } T(A_z) = \sum_{z=1}^m \varphi_z \text{Supp.}(A_z, A_y). \quad (12)$$

By mathematical induction, we can prove theorem 2 as follows:

Proof. Let $m = 2$, by Yager operations on SVNNSs and by power aggregation operator, we obtain the results $A_1 \oplus A_2 = (\alpha_1, \beta_1, \gamma_1) \oplus (\alpha_2, \beta_2, \gamma_2)$ and from the right side of (9), we have

$$\begin{aligned} SVNYPWA_\varphi(A_1, A_2) &= \\ &= \left(\begin{array}{l} \frac{\varphi_1(1+T(A_1))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (\alpha_1)^\rho) + \frac{\varphi_2(1+T(A_2))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (\alpha_2)^\rho); \\ 1 - \frac{\varphi_1(1+T(A_1))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (1 - \beta_1)^\rho) + \frac{\varphi_2(1+T(A_2))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (1 - \beta_2)^\rho); \\ 1 - \frac{\varphi_1(1+T(A_1))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (1 - \gamma_1)^\rho) + \frac{\varphi_2(1+T(A_2))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (1 - \gamma_2)^\rho) \end{array} \right)^{\frac{1}{\rho}} \\ &= \left(\begin{array}{l} \sum_{z=1}^2 \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (\alpha_z)^\rho); \\ 1 - \sum_{z=1}^2 \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (1 - \beta_z)^\rho); \\ 1 - \sum_{z=1}^2 \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^2 \varphi_z(1+T(A_z))} \min.(1, (1 - \gamma_z)^\rho) \end{array} \right)^{\frac{1}{\rho}} \end{aligned} \quad (13)$$

Thus, Eq. (11) true for $m = 2$. Suppose Eq. (11) holds for $m = q$ where $q \in N$, then based on the Eq. (11), we have

$$SVNYPWA_\varphi(A_1, A_2, \dots, A_q) = \bigoplus_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m (\varphi_z(1+T(A_z)))} A_z$$

$$= \left(\begin{array}{l} \sum_{z=1}^q \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^q \varphi_z(1+T(A_z))} \min. (1, (\alpha_z)^\rho); \\ 1 - \sum_{z=1}^q \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^q \varphi_z(1+T(A_z))} \min. (1, (1 - \beta_z)^\rho); \\ 1 - \sum_{z=1}^q \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^q \varphi_z(1+T(A_z))} \min. (1, (1 - \gamma_z)^\rho) \end{array} \right)^{\frac{1}{\rho}}$$

Now for $m = q + 1$, then

$$\begin{aligned} SVNYPWA_\varphi(A_1, A_2, \dots, A_q, A_{q+1}) &= \oplus_{z=1}^q \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^q \varphi_z(1+T(A_z))} A_z \oplus \frac{(\varphi_{q+1}(1+T(A_{q+1}))^{A_{q+1}})}{(\varphi_{q+1}(1+T(A_{q+1})))} A_{q+1} \\ &= \left(\begin{array}{l} \sum_{z=1}^q \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^q \varphi_z(1+T(A_z))} \min. (1, (\alpha_z)^\rho); \\ 1 - \sum_{z=1}^q \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^q \varphi_z(1+T(A_z))} \min. (1, (1 - \beta_z)^\rho); \\ 1 - \sum_{z=1}^q \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^q \varphi_z(1+T(A_z))} \min. (1, (1 - \gamma_z)^\rho) \end{array} \right) \oplus \\ &\quad \left(\begin{array}{l} \frac{\varphi_{q+1}(1+T(A_{q+1}))}{\varphi_{q+1}(1+T(A_{q+1}))} \min. (1, (\alpha_{q+1})^\rho) \\ 1 - \frac{\varphi_{q+1}(1+T(A_{q+1}))}{\varphi_{q+1}(1+T(A_{q+1}))} \min. (1, ((1 - \beta_{q+1})^\rho)) \\ 1 - \frac{\varphi_{q+1}(1+T(A_{q+1}))}{\varphi_{q+1}(1+T(A_{q+1}))} \min. (1, (1 - (1 - \gamma_{q+1})^\rho)) \end{array} \right)^{\frac{1}{\rho}} \\ &= \left(\begin{array}{l} \sum_{z=1}^{q+1} \frac{\varphi_{z+1}(1+T(A_z))}{\sum_{z=1}^{q+1} \varphi_{z+1}(1+T(A_z))} \min. (1, (\alpha_z)^\rho); \\ 1 - \sum_{z=1}^{q+1} \frac{\varphi_{z+1}(1+T(A_z))}{\sum_{z=1}^{q+1} \varphi_{z+1}(1+T(A_z))} \min. (1, ((1 - \beta_z)^\rho)); \\ 1 - \sum_{z=1}^{q+1} \frac{\varphi_{z+1}(1+T(A_z))}{\sum_{z=1}^{q+1} \varphi_{z+1}(1+T(A_z))} \min. (1, (1 - \gamma_z)^\rho) \end{array} \right)^{\frac{1}{\rho}} \end{aligned}$$

Thus Eq. (11) is true for $m = q + 1$. Hence, it shows that Eq. (11) holds for all $m \in N$.

2.3. Single-valued neutrosophic Yager power ordered weighted averaging operator

Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVN Ss for $z = 1, 2, \dots, m$. Then $SVNOWA$ operator of dimension m is a function $SVNOWA: \Omega^m \rightarrow \Omega$ with weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ such that $\omega_z \in [0, 1]$, and $\sum_{z=1}^m \omega_z = 1$. Thus,

$$SVNOWA_{\omega, \varphi}(A_1, A_2, \dots, A_m) = \frac{\oplus_{z=1}^m \omega_z (1+T(A_{\lambda(z)}))^{A_{\lambda(z)}}}{\sum_{z=1}^m \omega_z (1+T(A_{\lambda(z)}))}$$

$$= \left(\begin{array}{l} \sum_{z=1}^m \frac{\omega_z(1+T(A_{\lambda(z)}))}{\sum_{z=1}^m \omega_z(1+T(A_{\lambda(z)}))} \min. (1, \alpha_{\lambda(z)}^\rho); \\ 1 - \sum_{z=1}^m \frac{\omega_z(1+T(A_{\lambda(z)}))}{\sum_{z=1}^m \omega_z(1+T(A_{\lambda(z)}))} \min. (1, (1 - \beta_{\lambda(z)})^\rho); \\ \sum_{z=1}^m \frac{\omega_z(1+T(A_{\lambda(z)}))}{\sum_{z=1}^m \omega_z(1+T(A_{\lambda(z)}))} \min. (1, (1 - \gamma_{\lambda(z)})^\rho) \end{array} \right)^{\frac{1}{\rho}} \tag{14}$$

Where $(\lambda(1), \lambda(2), \dots, \lambda(m))$ are the permutations of $\lambda(z)$ for which $A_{\lambda(z-1)} \geq A_{\lambda(z)}$ and $T(A_{\lambda(z)})$ denotes the support of the largest SVN Ss $A_{\lambda(z)}$ from the other SVN Ss. Therefore, $T(A_{\lambda(z)}) = \sum_{z=1}^m \sum_{y \neq z} \text{Supp.}(A_{\lambda(z)}, A_{\lambda(y)})$.

In the next subsection, we introduced a single-valued neutrosophic Yager power geometric operator, and a single-valued neutrosophic Yager power ordered weighted geometric aggregation operator (SVNYP OWGA) for SVN Ss.

2.4. Single-valued neutrosophic Yager power geometric operator

Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVN Ss for $z = 1, 2, \dots, m$. Then, the single-valued neutrosophic Yager power geometric (SVNYPG) operator is a function defined as follows:

$$SVNYPG(A_1, A_2, \dots, A_m) = \otimes_{z=1}^m A_z^{\frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))}} \tag{15}$$

Where $T(A_z) = \sum_{z=1}^m \sum_{y \neq z} \text{Supp.}(A_z, A_y)$, and $\text{Supp.}(A_z, A_y)$ defines the support of A_y and A_z satisfying the following properties:

- (i) $\text{Supp.}(A_z, A_y) \in [0,1]$.
- (ii) $\text{Supp.}(A_z, A_y) = \text{Supp.}(A_y, A_z)$.
- (iii) $\text{Supp.}(A_z, A_y) \geq \text{Supp.}(A_r, A_s)$ if $d(A_z, A_y) < d(A_r, A_s)$ where d denotes the distance measure.

The aggregating operator in Eq. (15) has been obtained by using Yager’s PA operator and Yager product operation YO2. By applying Yager operations on SVN Ss, we get the following theorem.

Theorem 3. Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVN Ss for $z = 1, 2, \dots, m$. Then by applying SVNYPG on SVN Ss, the aggregated value obtained is also an SVN S. Therefore,

$$SVNYPG(A_1, A_2, \dots, A_m) = \otimes_{z=1}^m A_z^{\frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))}} = \left(\begin{array}{l} \sum_{z=1}^m \frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))} \min. (1, (1 - \alpha_z)^\rho); \\ 1 - \sum_{z=1}^m \frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))} \min. (1, \beta_z^\rho); \\ \sum_{z=1}^m \frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))} \min. (1, \gamma_z^\rho) \end{array} \right)^{\frac{1}{\rho}} \tag{16}$$

where $T(A_z) = \sum_{z=1}^m \sum_{y \neq z} \text{Supp.}(A_z, A_y)$. (17)

Theorem 4. Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVNNSs for $z = 1, 2, \dots, m$. Then by applying SVNYPWGA on SVNNSs, the aggregated value obtained is also a SVNNSs, $\varphi = (\varphi_1, \varphi_2, \dots, \varphi_m)^T$ is the weight vector of A_z where $\varphi_z \in [0, 1]^T$ s.t $\sum_{z=1}^m \varphi_z = 1$. Therefore, SVNYPWG: $\psi^m \rightarrow \psi$ accepts the following

$$\begin{aligned}
 SVNYPG_\varphi(A_1, A_2, \dots, A_m) &= \otimes_{z=1}^m (A_z) \frac{(1+T(A_z))}{\sum_{z=1}^m (1+T(A_z))}. \\
 &= \left(\begin{array}{l} 1 - \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, (1 - \alpha_z)^\rho); \\ \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, \beta_z^\rho); \\ \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, \gamma_z^\rho) \end{array} \right)^{\frac{1}{\rho}} \tag{18}
 \end{aligned}$$

Where $T(A_z) = \sum_{z=1}^m \varphi_{y \neq z} \text{Supp.}(A_z, A_y)$. (19)

The proof of the given theorem is similar to theorem 2.

2.5. Single-valued neutrosophic Yager power ordered the weighted geometric operator

Let $A_z = (\alpha_z, \beta_z, \gamma_z)$ be SVNNSs for $z = 1, 2, \dots, m$. Then SVNYPWGA operator is a function $SVNYPWGA: \Omega^m \rightarrow \Omega$ with weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ s.t $\omega_z \in [0, 1]$, and $\sum_{z=1}^m \omega_z = 1$. Thus,

$$\begin{aligned}
 SVNYPWGA_\omega(A_1, A_2, \dots, A_m) &= \otimes_{z=1}^m A_{\lambda(z)} \frac{(1+T(A_{\lambda(z)}))}{\sum_{z=1}^m (1+T(A_{\lambda(z)}))} \\
 &= \left(\begin{array}{l} 1 - \sum_{z=1}^m \frac{\omega_z(1+T(A_{\lambda(z)}))}{\sum_{z=1}^m \omega_z(1+T(A_{\lambda(z)}))} \min. (1, (1 - \alpha_{\lambda(z)})^\rho); \\ \sum_{z=1}^m \frac{\omega_z(1+T(A_{\lambda(z)}))}{\sum_{z=1}^m \omega_z(1+T(A_{\lambda(z)}))} \min. (1, \beta_{\lambda(z)}^\rho); \\ \sum_{z=1}^m \frac{\omega_z(1+T(A_{\lambda(z)}))}{\sum_{z=1}^m \omega_z(1+T(A_{\lambda(z)}))} \min. (1, \gamma_{\lambda(z)}^\rho) \end{array} \right)^{\frac{1}{\rho}} \tag{20}
 \end{aligned}$$

where $(\lambda(1), \lambda(2), \dots, \lambda(m))$ are the permutations of $\lambda(z)$ for which $A_{\lambda(z-1)} \geq A_{\lambda(z)}$ and $T(A_{\lambda(z)})$ denotes the support of the z th the largest SVNNSs $A_{\lambda(z)}$ from the other SVNNSs. Hence, $T(A_{\lambda(z)}) = \sum_{z=1}^m \varphi_{y \neq z} \text{Supp.}(A_{\lambda(z)}, A_{\lambda(y)})$.

In the next section, we develop the application of proposed aggregation operators in the MADM problem.

3. APPLICATION OF AGGREGATION OPERATORS IN MADM PROBLEM

In this section, we propose a MADM method using SVNYP aggregation operators under SVN environment. Here MADM problem is studied to investigate the advantage of evaluating the emerging software systems selection under the SVN environment. Let $M = \{M_1, M_2, \dots, M_m\}$ be a finite set of alternatives, and $L = \{L_1, L_2, \dots, L_n\}$ be a set of

attributes having weight vector $\varphi = \{\varphi_1, \varphi_2, \dots, \varphi_l\}$ such that $\varphi_z \in [0,1], \sum_{z=1}^m \varphi_z = 1$. Consider $A_z = (\alpha_{yz}, \beta_{yz}, \gamma_{yz})_{l \times m}$ is the SVN decision matrix, where α_{yz} denotes the degree of truth membership function, β_{yz} denotes the indeterminacy membership function, and γ_{yz} denotes the falsity membership function such that $0 \leq \alpha_{yz} + \beta_{yz} + \gamma_{yz} \leq 3$.

To interpret and solve the MADM problem, we follow an algorithm using SVNYPWA and SVNYPWGA operators as shown in the flowchart (Figure 1).

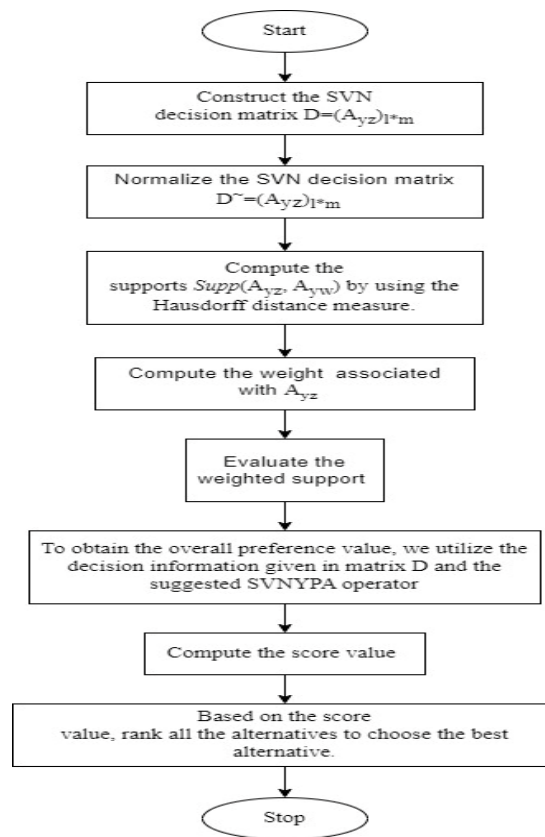


Figure 1: The flowchart of the proposed MADM algorithm.

Algorithm:

Step 1. Select the alternatives and attributes as given in set M and L, respectively. We obtain the single-valued neutrosophic decision matrix $D = (A_{yz})_{l \times m}; y = 1, 2, \dots, l; z = 1, 2, \dots, m$.

Step 2. Compute the normalized SVN decision matrix $\tilde{D} = (A_{yz})_{l \times m}$, where

$$A_{yz} = \begin{cases} (\alpha_{ij}, \beta_{ij}, \gamma_{ij}) & \text{for benefit criteria} \\ (\gamma_{ij}, \beta_{ij}, \alpha_{ij}) & \text{for cost criteria} \end{cases} \tag{21}$$

Step 3. Compute support by using the Hausdorff distance measure given by [46] and is given as below:

$$Supp (A_{yz}, A_{yw}) = 1 - d (A_{yz}, A_{yw}) \tag{22}$$

Where $d (A_{yz}, A_{yw}) = \frac{1}{3} max. (|\alpha_{yz} - \alpha_{yw}|, |\beta_{yz} - \beta_{yw}|, |\gamma_{yz} - \gamma_{yw}|)$.

Step 4. Apply the weight of the attribute φ_z to obtain the weighted support $T (A_{yz})$ between A_{yz} and A_{yw} SVN Ss such that

$$T (A_{yz}) = \sum_{w=1, w \neq z}^m \varphi_z Supp (A_{yz}, A_{yw}) \tag{23}$$

and compute the weight Δ_{yz} associated with A_{yz} ; $y = 1, 2, \dots, l$; $z = 1, 2, \dots, m$

$$\Delta_{yz} = \frac{\varphi_z (1+T(A_{yz}))}{\sum_{z=1}^m \varphi_z (1+T(A_{yz}))} \tag{24}$$

Where $\Delta_{yz} \geq 0$, and $\sum_{z=1}^m \Delta_{yz} = 1$.

Step 5. Using the information in the decision matrix and by SVNYPWA operator.

$$n_y = SVNYPWA_{\varphi}(A_1, A_2, \dots, A_m) = \frac{\oplus_{z=1}^m (\varphi_z(1+T(A_z))A_z)}{\sum_{z=1}^m (\varphi_z(1+T(A_z)))} = \left(\begin{array}{l} \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, \alpha_z^{\rho}); \\ 1 - \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, (1 - \beta_z)^{\rho}); \\ 1 - \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, (1 - \gamma_z)^{\rho}) \end{array} \right)^{\frac{1}{\rho}} \tag{25}$$

We can also utilize, $\tilde{n}_y = SVNYPG_{\varphi}(A_1, A_2, \dots, A_m) = \otimes_{z=1}^m A_z \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m (\varphi_z(1+T(A_z)))}$

$$= \left(\begin{array}{l} 1 - \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, (1 - \alpha_z)^{\rho}); \\ \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, (\beta_z)^{\rho}); \\ \sum_{z=1}^m \frac{\varphi_z(1+T(A_z))}{\sum_{z=1}^m \varphi_z(1+T(A_z))} \min. (1, (\gamma_z)^{\rho}) \end{array} \right)^{\frac{1}{\rho}} \tag{26}$$

For obtaining the preference values n_y ; $y = 1, 2, \dots, l$ of the alternative M_y .

Step 6. Compute the score value $\tau (n_{yz})$ using the definition of the score function to determine the ranking of the alternatives M_y , to select the most suitable choice M_y . We proceed to accuracy value $\sigma (n_y)$ and $\sigma (n_z)$ if τ_y and τ_z are the same.

Step 7. Rank the alternative M_y according to the score value to choose the best alternative. To implement this algorithm, we adapt an example [Ye] that illustrates the effectiveness of the proposed aggregation operators in the field of MADM problems.

Example 1. Consider an investment company, which wants to invest a sum amount of money in the best company. There are four possible alternatives to invest the money: (1) M_1 is a car company; (2) M_2 is a food company; (3) M_3 is a computer company; (4) M_4 is an arms company. The investment company must take under consideration the following

three criteria while selecting the best option: (1) L_1 is the risk; (2) L_2 is the growth; (3) L_3 is the environmental impact with weight vector of the criteria $W = (0.35, 0.25, 0.4)$. The evaluation done by domain an expert for the given criteria is shown in Table 1.

Table 1: Evaluation of expertise for four alternatives under three attributes

	M_1	M_2	M_3	M_4
L_1	(0.4, 0.2, 0.3)	(0.6, 0.1, 0.2)	(0.3, 0.2, 0.3)	(0.7, 0, 0.1)
L_2	(0.4, 0.2, 0.3)	(0.6, 0.1, 0.2)	(0.5, 0.2, 0.3)	(0.6, 0.1, 0.2)
L_3	(0.2, 0.2, 0.5)	(0.5, 0.2, 0.2)	(0.5, 0.3, 0.2)	(0.4, 0.3, 0.2)

For a selection of the most suitable and effective alternative, we use the SVNYPWA and SVNYPWGA operators and follow the algorithm given above:

Step 1. By using the equation from (20) to (22) in the data presented in Table 1, we compute the values of $\Delta_{yz}; y = 1, 2, \dots, l; z = 1, 2, \dots, m$ which is shown below.

$$\Delta = \begin{bmatrix} 0.3371 & 0.1685 & 0.4942 \\ 0.3352 & 0.1676 & 0.4971 \\ 0.3307 & 0.1673 & 0.5019 \\ 0.3346 & 0.1692 & 0.4961 \end{bmatrix}$$

Step 2. With the values of Δ and data provided by decision-makers in Table 1, we compute the aggregated SVN information, by using SVNYPWAS and SVNYPWGA operator to calculate the overall SVNNs for the alternative M_y as shown in Table 2.

Table 2: Aggregated values of the alternatives using SVNYPWA and SVNYPWGA

Alternative	SVNYPWA	SVNYPWGA
M_1	(0.3010, 0.2001, 0.3989)	(0.3012, 0.1999, 0.3987)
M_2	(0.5502, 0.1498, 0.2000)	(0.5503, 0.1497, 0.1999)
M_3	(0.4338, 0.2502, 0.2498)	(0.4339, 0.2501, 0.2497)
M_4	(0.5341, 0.1658, 0.1666)	(0.5342, 0.1657, 0.1665)

Step 3. Obtain the score value corresponding to each alternative which is given in Table 3 by using aggregated values.

Table 3: Score values of the alternatives

Alternative	SVNYPWA	SVNYPWGA
M_1	0.5673	0.5675
M_2	0.7334	0.7335
M_3	0.6445	0.6446
M_4	0.7339	0.7340

Step 4. Rank the alternatives according to the score values shown in Table 4.

Table 4: Ranking of the alternatives

Aggregation operator	Ranking
SVNYPWA	$M_4 > M_2 > M_3 > M_1$
SVNYPWGA	$M_4 > M_2 > M_3 > M_1$

From Table 4, we conclude that alternative M_4 is the best company for the investment.

4. SENSITIVITY ANALYSIS

The different values of the parameter ρ in SVNYP A operator may give different results in a decision-making problem. In this section, we analyze the sensitivity of the parameter ρ . We give variation in the values of parameters ρ . The score value and the corresponding ranking order of the alternatives for different parameters (range from $0 \leq \rho \leq 10$) based on SVNYPWA and SVNYPWGA operators are shown in Table 5 and Table 6.

Table 5: Influence of parameter by SVNYPWA operator in ranking of alternatives

ρ	$\tau(n_1)$	$\tau(n_2)$	$\tau(n_3)$	$\tau(n_4)$	$\tau(n_5)$	Ranking
1	0.1523	0.1628	0.1565	0.0967	0.1026	$M_4 > M_2 > M_3 > M_1$
2	0.1541	0.1636	0.1267	0.0986	0.1028	$M_2 > M_4 > M_3 > M_1$
3	0.1541	0.1637	0.1191	0.0986	0.1028	$M_2 > M_4 > M_3 > M_1$
4	0.1541	0.1636	0.1163	0.0986	0.1028	$M_4 > M_2 > M_3 > M_1$
5	0.1541	0.1636	0.1151	0.0986	0.1028	$M_4 > M_2 > M_3 > M_1$
6	0.1536	0.1638	0.1147	0.0989	0.1031	$M_4 > M_2 > M_3 > M_1$
7	0.1544	0.1639	0.1145	0.0990	0.1032	$M_4 > M_2 > M_3 > M_1$
8	0.1541	0.1636	0.1141	0.0984	0.1028	$M_4 > M_2 > M_3 > M_1$
9	0.1541	0.1637	0.1141	0.0986	0.1028	$M_4 > M_2 > M_3 > M_1$
10	0.1541	0.1636	0.1140	0.0986	0.1028	$M_4 > M_2 > M_3 > M_1$

Table 6: Influence of parameter by SVNYPWGA operator in ranking of alternatives

ρ	$\tau(n_1)$	$\tau(n_2)$	$\tau(n_3)$	$\tau(n_4)$	$\tau(n_5)$	Ranking
1	0.9249	0.9415	0.9148	0.8731	0.8749	$M_4 > M_2 > M_3 > M_1$
2	0.9249	0.9414	0.9371	0.8731	0.8749	$M_2 > M_3 > M_1 > M_4$
3	0.9241	0.9407	0.9406	0.8724	0.8742	$M_2 > M_3 > M_1 > M_4$
4	0.9249	0.9414	0.9423	0.8731	0.8749	$M_2 > M_3 > M_1 > M_4$
5	0.9249	0.9414	0.9426	0.8731	0.8749	$M_2 > M_3 > M_1 > M_4$
6	0.9246	0.9411	0.9425	0.8728	0.8746	$M_2 > M_3 > M_1 > M_4$
7	0.9244	0.9410	0.9424	0.8727	0.8745	$M_2 > M_3 > M_1 > M_4$
8	0.9249	0.9414	0.9428	0.8731	0.8749	$M_2 > M_3 > M_1 > M_4$
9	0.9248	0.9414	0.9427	0.8730	0.8748	$M_2 > M_3 > M_4 > M_1$
10	0.9249	0.9414	0.9428	0.8731	0.8749	$M_2 > M_3 > M_1 > M_4$

From Table 5, it is evident that by using the SVNYPWA operator, the ranking orders for $\rho = 1$ and $\rho > 3$ are identical i.e., $M_4 > M_2 > M_3 > M_1$. But when $\rho = 2, 3$ the ranking order is $M_2 > M_4 > M_3 > M_1$ and M_2 is selected as the best alternative. On the other hand, from Table 6, the ranking order for $\rho = 1$ is $M_4 > M_2 > M_3 > M_1$; for $2 \leq \rho \leq 8$; $\rho = 10$, the ranking order is similar i.e., $M_2 > M_3 > M_1 > M_4$ and for $\rho = 9$, the ranking order is $M_2 > M_3 > M_4 > M_1$. Hence, from both, we conclude that the corresponding ranking order of the alternatives for the SVNYPWA operator can be changed with different values of the parameter. Thus, the algorithm is sensible towards

the parameters ρ . The suitable choice of the parameter ρ as per the conditions of a system is expected to make the decision result more reliable.

5. CONSISTENCY OF PROPOSED METHOD

We utilize the MABAC method on the information given in Table 1. For this, the systematic procedure is as follows:

Step 1. Normalize the decision matrix (Table 1). As all the attributes are of the same type, so there is no need to normalize them.

Step 2. Calculate the weighted normalized decision matrix as shown below in Table 7 by using Eq. (23).

Step 3. By using the normalized matrix $n_{yz} = (\alpha_{yz}, \beta_{yz}, \gamma_{yz})$ and attribute's support weight φ_z , we compute the normalized fuzzy weighting matrix $\varphi F_{yz} = (\alpha'_{yz}, \beta'_{yz}, \gamma'_{yz})$; $y = 1, 2, \dots, l$; $z = 1, 2, \dots, m$ by applying the following formula, $\varphi n_{yz} = \varphi_l \oplus n_{yz}$; $y = 1, 2, \dots, l$; $z = 1, 2, \dots, m$

$$= (1 - (1 - \alpha_{yz})^{\Delta_z}, (\beta_{yz})^{\Delta_z}, (\gamma_{yz})^{\Delta_z}) \tag{27}$$

where $\Delta_{yz} = \frac{\varphi_z (1+T(A_{yz}))}{\sum_{z=1}^m \varphi_z (1+T(A_{yz}))}$.

Table 7: Normalized neutrosophic decision-matrix

	M_1	M_2	M_3	M_4
L_1	(0.1581, 0.5812, 0.6664)	(0.2644, 0.4621, 0.5830)	(0.1112, 0.5872, 0.6937)	(0.3315, 0, 0.4628)
L_2	(0.0824, 0.7624, 0.8163)	(0.1423, 0.6798, 0.7635)	(0.1094, 0.7639, 0.8175)	(0.1436, 0.6773, 0.7616)
L_3	(0.1044, 0.4514, 0.7099)	(0.2914, 0.4493, 0.4493)	(0.2938, 0.5464, 0.4458)	(0.2238, 0.5503, 0.4500)

$$\varphi F_{yz} = [n_{yz}]_{l \times m} = \begin{matrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{matrix} \begin{bmatrix} (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) & (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) & \cdot & \cdot & \cdot & (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) \\ (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) & (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) & \cdot & \cdot & \cdot & (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) & (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) & \cdot & \cdot & \cdot & (\alpha_{yz}, \beta_{yz}, \gamma_{yz}) \end{bmatrix} \tag{28}$$

Step 4. Compute the border approximation areas (BBA) values and for BBA matrix $T = [g_z]_{l \times m}$ can be evaluated as

$$g_z = (\prod_{y=1}^l \varphi n_{yz})^{\frac{1}{l}} = \left\{ (\prod_{y=1}^l \alpha_{yz})^{\frac{1}{l}}, 1 - (\prod_{y=1}^l \beta_{yz})^{\frac{1}{l}}, 1 - (\prod_{y=1}^l \gamma_{yz})^{\frac{1}{l}} \right\} \tag{29}$$

where l is the no. of alternatives.

The BBA matrix is calculated using Eq. (29) as follows: $g_1 = \langle 0.1982, 0.4478, 0.6110 \rangle$, $g_2 = \langle 0.1165, 0.7241, 0.7915 \rangle$, $g_3 = \langle 0.2115, 0.5017, 0.5302 \rangle$.

Step 5. Evaluate the distance between each alternative and BBA matrix by the following equation:

$$d_{yz} = \begin{cases} d(\varphi n_{yz}, g_z) & \text{if } \varphi n_{yz} > g_z \\ 0 & \text{if } \varphi n_{yz} = g_z \\ -d(\varphi n_{yz}, g_z) & \text{if } \varphi n_{yz} < g_z \end{cases} \quad (30)$$

Where $d(\varphi n_{yz}, g_z)$ denotes the distance measure from φn_{yz} to g_z and we calculate the distance measure by using Hamming distance [45] between each alternative and BBA matrix given in Table 8.

Table 8: Distance between alternatives and BBA matrix

	L_1	L_2	L_3
M_1	-0.0762	-0.0324	-0.1123
M_2	0.0266	0.0326	0.0711
M_3	-0.1030	-0.0243	0.0704
M_4	0.2431	0.0345	0.0470

Step 6. Calculate the sum of the distance $S_y = \sum_{y=1}^l S_y$ for each alternative by using Table 8 and the result is as follows: $S_1 = -0.2209, S_2 = 0.1303, S_3 = -0.0569, S_4 = 0.3246$. By verifying the result by the MABAC method for choosing the best alternative. We obtain the following ranking order $M_4 > M_2 > M_3 > M_1$ and hence, it shows that M_4 is the best alternative. Thus, by using SVNYP operators and the SVN- MABAC method, we obtained M_4 as the most suitable and desirable road construction company although there is a slight change in ranking order. Thus, the proposed operators are consistent with MABAC method.

6. COMPARATIVE ANALYSIS

To justify the advantage of the proposed methods, we consider the same decision-making problem given in section 5. The comparative analysis between the proposed method SVNYPWA and SVNYPWGA with some existing methods such as SVNYPWA [10], WAA and WGA operators for simplified neutrosophic sets offered in Ye [34], SVNDPA and SVNDPGA operators established in Jana and Pal [45], and SVNPHA and SVNGPHA operators suggested by Zhao et al. [44] to get the aggregated SVN values, score value and ranking of the alternatives. The aggregated SVN values, score value and ranking of the alternatives are presented in Table 9, 10, and 11, respectively.

Table 9: Aggregate values of the existing and proposed methods

	SVNPWA	WAA	WGA	SVNDPWA	SVNDPWGA
M_1	(0.3464, 0.2000, 0.3492)	(0.3268, 0.2000, 0.3881)	(0.3031, 0.2000, 0.3680)	(0.3335, 0.2782, 0.4422)	(0.2861, 0.1395, 0.3133)
M_2	(0.5707, 0.1246, 0.2000)	(0.5627, 0.1414, 0.2000)	(0.5578, 0.1320, 0.2000)	(0.5654, 0.1460, 0.2057)	(0.5559, 0.0672, 0.1927)
M_3	(0.4423, 0.2281, 0.2630)	(0.4375, 0.2416, 0.2616)	(0.4181, 0.2352, 0.2551)	(0.4444, 0.2775, 0.2695)	(0.4051, 0.1398, 0.2141)
M_4	(0.5872, 0.0000, 0.1599)	(0.5476, 0.1555, 0.1663)	(0.5385, 0.0000, 0.1569)	(0.5936, ND, ND)	(ND, 0.0673, 0.1487)

Table 9: (Continued)

	SVNPHA	SVNGPHA	Prop. (SVNYPWA)	Prop. (SVNYPWGA)
M_1	(0.2885, 0.1565, 0.3200)	(0.3032, 0.1011, 0.3867)	(0.3186, 0.5419, 0.3813)	(0.3187, 0.1145, 0.3812)
M_2	(0.5675, 0.0576, 0.2015)	(0.5567, 0.1256, 0.1985)	(0.5597, 0.5796, 0.6232)	(0.5598, 0.0505, 0.0941)
M_3	(0.4670, 0.2341, 0.5786)	(0.4327, 0.6798, 0.2236)	(0.4295, 0.6239, 0.6644)	(0.4296, 0.0983, 0.1388)
M_4	(0.5534, 0.1236, 0.2390)	(0.6754, 0.2145, 0.8897)	(0.5534, 0.5487, 0.5888)	(0.5536, 0.0188, 0.0589)

Table 10: Score values of the existing and proposed methods

Alternatives	SVNPWA	WAA	WGA	SVNDPWA	SVNDPWGA
M_1	0.6524	0.5922	0.5863	0.5377	0.6110
M_2	0.9243	0.9169	0.9188	0.7378	0.7652
M_3	0.7858	0.7756	0.7696	0.6323	0.6837
M_4	0.9649	0.9297	0.9601	ND	ND

Table 10: (Continued)

	SVNPHA	SVNGPHA	Prop. (SVNYPWA)	Prop. (SVNYPWGA)
M_1	0.5678	0.6237	0.4651	0.6076
M_2	0.7376	0.7865	0.4522	0.8050
M_3	0.6541	0.6654	0.3803	0.7308
M_4	0.9867	0.9843	0.4719	0.8253

Table 11: Ranking order of the alternatives

Operators	Ranking
SVNPWA	$M_4 > M_2 > M_3 > M_1$
WAA	$M_4 > M_2 > M_3 > M_1$
WGA	$M_4 > M_2 > M_3 > M_1$
SVNDPWA	ND
SVNDPWGA	ND
SVNPHA	$M_4 > M_2 > M_3 > M_1$
SVNGPHA	$M_4 > M_1 > M_2 > M_3$
SVNYPWA	$M_4 > M_1 > M_2 > M_3$
SVNYPWGA	$M_4 > M_2 > M_3 > M_1$

ND: Divisible by zero problem.

From the numerical example, it is evident that M_4 is the most suitable and desirable company. Although the ranking orders of the five methods (SVNPWA, WAA, WGA, SVNPHA, SVNGPHA) and the proposed methods are slightly different, but still, appears that the alternative M_4 is a desirable and suitable company.

Example 2. [45] Consider the problem of selecting the best road construction company among the five road construction companies (M_1) Jaihind Road Builders private (Pvt.) limited (Ltd.), (M_2) J.K. Construction, (M_3) Tata Infrastructure Ltd, (M_4) Birla Pvt. Ltd., and (M_5) Relcon Infra projects Ltd which are alternatives among the five possible alternatives $M_r (r = 1, 2, \dots, 5)$ under four criteria: L_1 : Contractor background experience, L_2 : Technical Capability, L_3 : Tender price, and L_4 : Completion time.

From the comparative analysis between the proposed method SVNYPWA and SVNYPWGA with SVNDPA and SVNDPGA operator as well as SVNPHA and SVNGPHA established by Jana and Pal [45] and Zhao et al. [44] respectively to get the aggregated SVN values, score value and ranking of the alternatives are given in Tables 12, 13 and 14.

From Table 14, we observed that the ranking produced by [44-45] and the proposed method are slightly different, but still, it provides that similar best alternative i.e., M_3 . This justifies that the proposed methods proposed by us are more advanced and effective.

Likewise, to determine the advantages of the proposed operator, we compare the existing operator proposed by [44] and [45] with the suggested operator. To justify the advantages, we reconsider a numerical Example of Jana and Pal [45].

Table 12: Aggregated values of the existing and proposed operators

	SVNDPWA	SVNDPWGA	SVNPHA	SVNGPHA	Proposed (SVNYPWA)	Proposed (SVNYPWGA)
M_1	(0.5810, 0.4377, 0.2214)	(0.5636, 0.5934, 0.2856)	(0.5871, 0.4382, 0.2132)	(0.5567, 0.5876, 0.2376)	(0.5777, 0.5353, 0.2558)	(0.5778, 0.5352, 0.2557)
M_2	(0.6000, 0.3916, 0.2220)	(0.4934, 0.5444, 0.1524)	(0.6234, 0.4352, 0.3456)	(0.5055, 0.5555, 0.1524)	(0.5431, 0.4775, 0.1550)	(0.6245, 0.4773, 0.1548)
M_3	(0.7159, 0.4005, 0.1820)	(0.7217, 0.4853, 0.2105)	(0.7349, 0.4238, 0.2210)	(0.7100, 0.4865, 0.2564)	(0.7004, 0.4542, 0.2119)	(0.7006, 0.4540, 0.2117)
M_4	(0.6673, 0.4241, 0.2302)	(0.6443, 0.5375, 0.2425)	(0.6578, 0.4376, 0.2576)	(0.6534, 0.5467, 0.2569)	(0.6559, 0.5011, 0.2330)	(0.6561, 0.5009, 0.2328)
M_5	(0.7002, 0.5315, 0.1333)	(0.6089, 0.5763, 0.1528)	(0.8543, 0.6548, 0.1234)	(0.7120, 0.6200, 0.1527)	(0.6225, 0.5507, 0.1555)	(0.6227, 0.5548, 0.1553)

Table 13: Score values of the existing and proposed methods

Alternatives	SVNDPWA	SVNDPWGA	SVNPHA	SVNGPHA	Proposed (SVNYPWA)	Proposed (SVNYPWGA)
M_1	0.6406	0.5615	0.6210	0.5534	0.5955	0.5956
M_2	0.6621	0.5989	0.6532	0.6210	0.6368	0.6641
M_3	0.7111	0.6753	0.7233	0.6845	0.6780	0.6782
M_4	0.6710	0.6214	0.6875	0.6332	0.6405	0.6407
M_5	0.6785	0.6266	0.6834	0.6256	0.6373	0.6375

Table 14: Ranking order of the alternatives

	SVNDPWA	SVNDPWGA	SVNPHA	SVNGPHA	Proposed (SVNYPWA)	Proposed (SVNYPWGA)
Ranking	$M_3 > M_5 > M_4 > M_2 > M_1$	$M_3 > M_5 > M_4 > M_2 > M_1$	$M_3 > M_5 > M_4 > M_2 > M_1$	$M_3 > M_5 > M_4 > M_2 > M_1$	$M_3 > M_4 > M_5 > M_2 > M_1$	$M_3 > M_2 > M_4 > M_5 > M_1$

Example 3. In this example, we only change a little data from the example. We can find that there are slight changes in the truth-membership value of $L_1, L_2,$ and L_4 attributes for alternative M_3 . The old value is $(0.7, 0.3, 0.4), (0.7, 0.5, 0.2), (0.6, 0.7, 0.1),$ and $(0.8, 0.3, 0.2)$ and now the new value is $(0.6, 0.3, 0.4), (0.6, 0.5, 0.2), (0.6, 0.7, 0.1), (0.6, 0.3, 0.2),$ then we find the changes in the ranking results for the method proposed by Jana and Pal [45], and Zhao et al. [44] and the suggested method with SVNYPWA operator. The ranking results are given in Table 15.

From Table 15, we observe that the ranking results of the proposed method with the SVNYPWA operator and the existing method are different. The ranking result by the proposed method with the SVNYPWA operator remains the same while it is changed by existing method. For existing method, the best alternative in both examples given in section 6 are different, but on the other hand for similar examples given in section 6, M_3 is the best alternative by our suggested method. This justifies the advantages of the proposed method, which can relieve the influence of too big or too small data. This concludes that the result of the proposed method is more reasonable than the existing method [44] and [45].

Table 15: Ranking order of the alternatives

Method	Score function with WAA operator	Ranking
The method proposed by Jana and Pal [45]	SF (M_1) = 0.6406, SF (M_2) = 0.662, SF (M_3) = 0.6724, SF (M_4) = 0.6710, SF (M_5) = 0.6785	$M_5 > M_3 > M_4 > M_2 > M_1$
The method proposed by Zhao et al. [44]	SF (M_1) = 0.6400, SF (M_2) = 0.656, SF (M_3) = 0.6432, SF (M_4) = 0.6700, SF (M_5) = 0.6785,	$M_5 > M_4 > M_2 > M_3 > M_1$
The proposed method with the SVNYPWA operator	SF (M_1) = 0.5955, SF (M_2) = 0.6368, SF (M_3) = 0.6445, SF (M_4) = 0.6405, SF (M_5) = 0.6373	$M_3 > M_4 > M_5 > M_2 > M_1$

7. CONCLUSION

In this article, the fusion of Yager operators and power operators resulted in the development of the SVNYPWA operator, SVNYPWA operator, SVNYPGA operator, and SVNYPWA operator. However, the newly developed aggregation operators were found suitable for handling SVN information in MCDM problems in certain situations. The proposed operators have also shown sensitivity towards the hyperparameter ρ . From the analysis, it is shown that the corresponding ranking order of the alternatives for the SVNYPWA operator can be changed with different values of the parameter and thus, the algorithm is sensible towards the parameters ρ . The MABAC method was applied for the verification to show the consistency of the proposed SVNYPWA aggregation operator. Theoretically, our study guides the fusion of information using aggregation operators in different situations. Practically, it has implications in the decision making, where we need to aggregate the different information from the sources. The application discussed in this article utilized artificial data. However, to investigate the proposed method's real-time implications, we need to create SVN data in a real-time situation, as it is not available in any repository. So, in the future, we will try to create SVN data to investigate the proposed work's real-time implications. We shall also expand our models to single-valued neutrosophic soft set environment.

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