

INVENTORY AND PRESERVATION INVESTMENT FOR DETERIORATING SYSTEM WITH STOCK-DEPENDENT DEMAND AND PARTIAL BACKLOGGED SHORTAGES

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Abstract: Our model deals with the stock-dependent demand as exhibiting huge volume of commodities leads to more costumers and augment the trading of the goods. As some goods like vegetables, fruits, medicines deteriorate after a period of time, resulting in economical and financial losses, we took this factor into consideration and included a constant deterioration rate, controlled by suitable preservation technologies. Preservation technology investments are made for the valuable business as it helps to decrease the rate of deterioration. Our model allows shortages, and back-ordering is permissible to manage the loss that occurs due to perishable objects and shortages. The objectives are to find the optimal cycle time, preservation technology cost, and positive inventory time. The paper also proves the convexity of total cost through graphs with respect to decision variables. A sensitivity analysis of decision variables with respect to different inventory parameters is carried out.

Keywords: Stock-dependent Demand, Deterioration, Preservation Technology Investment, Shortages, Backlogging, Sensitivity.

MSC: 90B05 .

1. INTRODUCTION

An inventory model deals with the demand rate as to be constant, price-dependent or time-dependent. However, it has been observed that sales is directly proportional to the stockpile, i.e., large product availability in supermarkets attracts more consumers by providing wider selection. By taking care of stock according to the demand, one should also focus on the deterioration rate of the goods, e.g., items such as volatile liquid, fruit, and others. The deterioration cannot be blocked but can be controlled by delaying it. If the company can profitably decrease the deterioration rate by getting better storage services, the total cost can be lowered. Deterioration rate can be controlled up to certain level by using preservation technology. For example, rate of deterioration of seasonal products can be reduced through cold storage. Therefore, causes of deterioration are analysed, and preservation technology investments are made to control the rate of damages, spoilages, and degradation within time. Keeping this in mind, our model considers a stock-dependent demand with constant deterioration rate, and to control the rate of deterioration, preservation technology investments are made. Due to high demand and the rate of deterioration, shortages take place. But in reality, a retailer wants to avoid such kind of situations as in that case, he has to pay some penalty cost. At present, to reduce the loss occurring due to shortages, we suppose partial backordering as being permissible in order to serve consumers according to their demands. The degree of partial backordering depends on consumers' waiting time, where demand increases with longer waiting time. Also, backordering quantity depends on consumers' response during shortages. In our model, shortages are permissible together with partial backordering. Our paper is organised as follows: Section 2 is the literature review. Section 3 brings notations and assumptions that we used in the model. A mathematical model is formulated in section 4; Section 5 includes computational algorithm. Numerical example and sensitivity analysis for different decision variables and objective functions are carried out in section 6; Finally, section 7 provides conclusion and the future scope.

2. LITERATURE REVIEW

Mandal and Maiti [11] introduced a model for deteriorated items where the demand is stock-dependent. Lin *et al.* [10] formulated a model for time-varying demand where items are getting spoiled with time. Shortages are allowed and backordering is permissible. Replenishment number and optimal service level are calculated to minimize the total revenue. Tayal *et al.* [23] developed an integrated model, dealing with perishable items for two-echelon supply chain. They introduced a concept of preservation technology to decrease the deterioration rate, and to make situation more realistic, the time-dependent set up cost and ordering cost are taken into account. Singh and Rathore [18] pointed out a model for deteriorating inventory with preservation technology investment for which the demand rate is a function of time. Partial backlogged shortages are allowed under effect of permissible delay. Bardhan *et al.* [1] generalized a model for non-instantaneous

deterioration and stock-dependent demand. Based on whether stock out takes place before or after deterioration, two distinct inventory scenarios were studied. Diabat [3] presented an economic order quantity model for deteriorating items with partial upstream advance payment and partial downstream delayed payment. This also allows partial backordering, full backordering and shortages. Dye [4] integrated a non-instantaneous deteriorating model and analysis of how preservation technology investment influences inventory decisions. Goyal [5] proposed trended inventory replenishment problem under shortages over finite horizon. Wee [26] presented a lot-size inventory model where demand decreases exponentially. The system considers deteriorating items and shortages are acceptable. Mishra *et al.* [13] developed a model for price sensitive and stock dependent demand with shortages. Deterioration of seasonal products is considered. Khalilpourazari and Pasandideh [7] proposed a multi-constrained economic order quantity model for non-linear holding cost and formulated inventory cycle length to minimize the total cost. Roy and Chaudhari [16] developed a production inventory model for stock-dependent demand, taking Weibull distribution deterioration. Taleizadeh *et al.* [21] introduced a multiproduct inventory problems for dynamic demand and calculated stochastic replenishment intervals. Shah *et al.* [19] evaluated an inventory model for dynamic price, service investment with constant deterioration rate that can be reduced by preservation technology. Wu *et al.* [27] considered a non-instantaneous deteriorating items where the demand is stock dependent and backlogging rate is dependent on waiting time. Cardenas-Barron *et al.* [2] developed an economic order quantity inventory model for both non-linear stock dependent demand and non-linear holding cost. This model offers trade credit for a retailer to obtain optimal solution; theorems are evaluated together with a solution procedure. Khan *et al.* [8] introduced a model for deteriorating products for which the demand is both stock and price dependent. Using this, the paper is formulated for the case of advance payment. Mishra *et al.* [12] presented an inventory model for perishable objects under controllable deterioration rate. The objective is to maximize the total profit for price dependent demand. Tiwari *et al.* [24] developed an economic order quantity model for deteriorating items with permissible delay in payment and allowable shortages. To determine optimal replenishment time and cycle time for stock-out, a theoretical result is presented. Tripathi *et al.* [25] developed an inventory model for stock-dependent demand and time varying holding cost for different trade credits. Krishnamoorthi *et al.* [9] developed a three level production inventory model for deteriorating objects under different production rate and shortages. The optimal solution is derived to minimize the total cost. Kaliraman *et al.* [6] introduced a two warehouse inventory model for deteriorating items with exponential demand rate and permissible payment delay.

3. NOTATIONS AND ASSUMPTIONS

The model is formulated under following notations and assumptions

3.1. Notations

c	Purchase cost per unit (in \$)
h	Holding cost per unit (in \$)
A	Ordering cost per order(in \$)
r	Penalty cost per unit of lost sale that occurs due to shortages and deterioration (in \$)
α	Scale demand
β	Stock-dependent parameter
$R(I(t))$	Stock-dependent demand rate
θ	Constant deterioration rate
a	Reduced rate of deterioration
$m(u)$	Reduced deterioration rate due to the use of preservations technology
$I(t)$	Inventory level during time interval $[0, t_1]$
T	Cycle time(in years)
t_1	Time at which inventory level becomes zero, where $0 < t_1 \leq T$ (in years)
u	Preservation technology cost to decrease the rate of deterioration (in \$)
Q_1	Quantity that is not back-ordered
Q_2	Back-order quantity
$\lambda(\delta)$	Probability that customers agree to wait for a unit of time to purchase the item
TC	Total cost per unit time (in \$/ years)

3.2. Assumptions

- (a) Demand rate is stock-dependent and is given by

$$R(t) = \alpha + \beta I(t)$$

where α is scale demand and $0 \leq \beta < 1$ is stock-dependent parameter. Here, $I(t)$ represents inventory level at time t .

- (b) Deteriorated items are not being repaired or replaced during cycle time.

- (c) Customer's impatience level results into lost sales.

- (d) Initially, backlogged demand is fulfilled when replenishment occurs.

- (e) Cycle time is greater than the shortage time.

- (f) The probability of customers backordered is assumed to be

$$\lambda(\delta) = 1 - \frac{\delta}{T}, 0 \leq \delta < T$$

where δ represent waiting time. It shows that backorder demand depends on customer's patience level that reduces with the increase in waiting time.

4. MATHEMATICAL MODEL

In this section, a model is formulated for stock-dependent demand rate where shortages are permissible and partially backordered when replenishment occurs. Figure 1 represents the model.

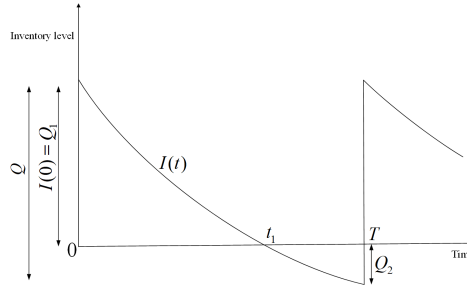


Figure 1

This inventory model deals with the deteriorating items where deterioration rate is assumed to be reduced by $m(u)$, i.e., $\theta(1 - e^{-au})$ because of the retailer's investment of u amount on the preservation technology. Initially, at $t = 0$, the order quantity is Q , out of which Q_2 units are backordered and fulfilled when replenishment occurs. Replenishment occurs after each cycle time T . During time period $[0, t_1]$, the inventory reduces because of the combined effects of demand and deterioration, and at $t = t_1$ the inventory level reaches zero.

The differential equation representing inventory level during time $[0, t_1]$ is

$$\frac{dI(t)}{dt} = (m(u) - \theta)I(t) - R = (\theta(1 - e^{-au}) - \theta)I(t) - (\alpha + \beta I(t)) \tag{1}$$

Using boundary condition $I(t_1) = 0$, the solution is

$$I(t) = \frac{\alpha(e^{(\theta e^{-au} + \beta)(t_1 - t)} - 1)}{\theta e^{-au} + \beta} \tag{2}$$

Here, Q_1 be the order quantity without backordering and is given by

$$Q_1 = I(0) = \frac{\alpha(e^{(\theta e^{-au} + \beta)t_1} - 1)}{\theta e^{-au} + \beta} \tag{3}$$

Whereas, the backordered quantity Q_2 is

$$Q_2 = \int_{t_1}^T R\lambda(T - t)dt = \int_{t_1}^T \frac{(\alpha + \beta I(t))t}{T} dt \tag{4}$$

Therefore, the total order quantity is given by

$$Q = Q_1 + Q_2 = \frac{\alpha(e^{(\theta e^{-au} + \beta)t_1} - 1)}{\theta e^{-au} + \beta} + \int_{t_1}^T \frac{(\alpha + \beta I(t))t}{T} dt \quad (5)$$

Shortages are permissible and are given by

$$S = \int_{t_1}^T R \left(1 - \frac{t}{T}\right) dt \quad (6)$$

List of cost included in objective function is

$$(I) \text{ Purchase cost (PC)} = \left(\frac{\alpha(e^{(\theta e^{-au} + \beta)t_1} - 1)}{\theta e^{-au} + \beta} + \int_{t_1}^T \frac{(\alpha + \beta I(t))t}{T} dt \right) c$$

$$(II) \text{ Lost sale cost (LS)} = Sr = \left(\int_{t_1}^T (\alpha + \beta I(t)) \left(1 - \frac{t}{T}\right) dt \right) r$$

$$(III) \text{ Holding cost (HC)} = h \int_0^{t_1} I(t) dt$$

$$(IV) \text{ Ordering cost (OC)} = A$$

$$(V) \text{ Preservation technology cost (PTC)} = u$$

Hence, the total cost of the integrated inventory model is calculated by

$$\begin{aligned} TC &= \frac{1}{T} (PC + LS + HC + OC + PTC) \\ &= \frac{1}{T} \left(\frac{\alpha(e^{(\theta e^{-au} + \beta)t_1} - 1)}{\theta e^{-au} + \beta} + \int_{t_1}^T \frac{(\alpha + \beta I(t))t}{T} dt \right) c \\ &\quad + \frac{1}{T} \left(\left(\int_{t_1}^T (\alpha + \beta I(t)) \left(1 - \frac{t}{T}\right) dt \right) r + h \int_0^{t_1} I(t) dt + A + u \right) \end{aligned}$$

The total cost per unit time is a continuous function of positive inventory time (t_1), cycle time (T), and preservation investment (u) required to reduce the deterioration rate.

5. COMPUTATIONAL ALGORITHM

The classical optimization method is used to solve the objective function. The aim is to minimize the total cost. The solution procedure is dependent on the following steps.

Step 1: Assign the values to the inventory parameters in suitable units.

Step 2: Evaluate first-order partial derivative of total cost with respect to the decision variables, i.e., t_1, u, T and equate them to zero.

$$\frac{\partial TC}{\partial t_1} = 0, \frac{\partial TC}{\partial u} = 0, \frac{\partial TC}{\partial T} = 0 \tag{8}$$

from (8), values of (t_1, u, T) are obtained and used in equation (7) to find the total cost.

Step 3: Convexity of objective function is verified using graph.

6. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Example 1. Consider $a=0.05, \alpha=100, \beta=0.2, \theta=0.2, h=\$10/\text{unit}/\text{year}, A=\$100/\text{order}, c=\$25/\text{unit}, r = \$45/\text{unit}.$

Using the above computational algorithm, the feasible values of decision variables are $t_1=0.414 \text{ year}, \mu=\$18.213, T=0.798 \text{ year}$ that gives $TC=\$3019.306.$

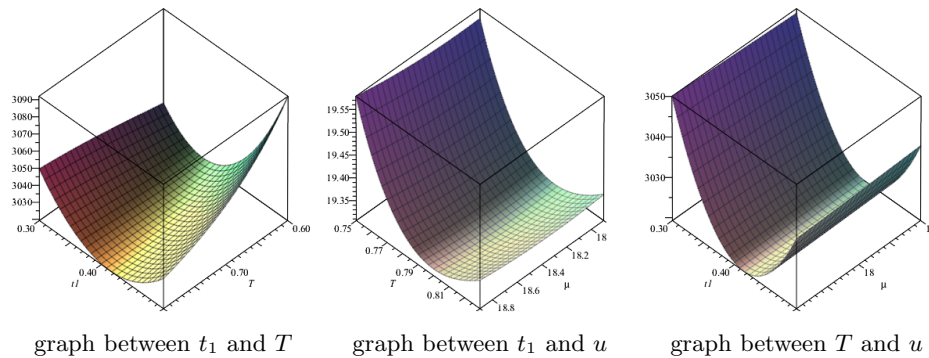


Figure 2: Convexity of objective function

An effect on decision variable together with total cost is examined, changing inventory parameters by -20%, -10%, 10% and 20%. An analysis is performed by changing one inventory parameters at a time and keeping other parameters unchanged.

Table 1: Sensitivity analysis

Parameters	Values of parameters	Preservation technology cost(u) (in \$)	Positive inventory cycle time (t_1) (in years)	Cycle time(T) (in years)	Total cost(TC) (in \$)
a	0.04	-	-	-	-
	0.045	17.75	0.41	0.81	3021.67
	0.050	18.21	0.41	0.80	3019.31
	0.055	18.49	0.42	0.80	3017.12
	0.060	-	-	-	-
α	80	-	-	-	-
	90	-	-	-	-
	100	18.21	0.41	0.80	3019.31
	110	22.842	0.44	0.92	3307.35
	120	25.71	0.45	0.98	3596.14
β	0.16	27.05	0.51	1.14	3021.19
	0.18	23.12	0.46	0.98	3019.28
	0.20	18.21	0.41	0.80	3019.31
	0.22	-	-	-	-
	0.24	-	-	-	-
θ	0.16	15.36	0.43	0.87	3014
	0.18	17.07	0.42	0.84	3016.74
	0.20	18.21	0.41	0.80	3019.31
	0.22	-	-	-	-
	0.24	-	-	-	-
h	8	0.87	22.35	0.46	2995.66
	9	20.27	0.44	0.84	3007.84
	10	18.21	0.41	0.80	3019.31
	11	16.10	0.40	0.76	3030.14
	12	-	-	-	-
A	80	30	0.44	0.99	3002.30
	90	19.65	0.43	0.92	3012.77
	100	16.95	0.41	0.816	3024.20
	110	-	-	-	-
	120	-	-	-	-
c	20	34.77	0.64	1.61	2597.99
	22.50	28.39	0.54	1.25	2803.82
	25	18.21	0.41	0.80	3019.31
	27.50	-	-	-	-
	30.00	-	-	-	-
r	36.00	-	-	-	-
	40.50	-	-	-	-
	45.00	18.21	0.41	0.80	3019.31
	49.50	31.87	0.56	1.29	3082.47
	54.00	40.14	0.67	1.62	3153.96

Based on the sensitivity analysis table, the following results are observed:

- (a) The greater the value of a , the higher is the preservation technology cost as the investment on preservation technology helps to reduce the deterioration rate. Here, the increase is beneficial as it minimizes the total cost of the retailer.
- (b) Increases in scale demand significantly increases the total cost and, at the same time, cycle time also increases. The increase is not advisable as it increases the preservation technology cost that increases the company's total cost.
- (c) Higher the value of the stock-dependent demand rate, i.e., β , lower the total cycle time, preservation technology cost, and shortage time. The increase has a positive impact as reduction in preservation technology cost results in total cost decrease.
- (d) An increase in rate of deterioration decreases the cycle time and increases the preservation technology cost. Larger deterioration rate forces the company to spend more on preservation technology. It has a negative effect on total cost, which is obvious, as it increases the total cost.
- (e) In an inventory model, holding cost is the factor that directly influences the total cost and cycle time. With larger holding cost the cycle time decreases, resulting in higher total cost. Whereas due to the shorter cycle period, the company has to pay out least in preservation technology.
- (f) With the increase in ordering cost and purchase cost, increase in total cost is noticeable. Higher is the costs, lower is the demand and hence, the total cycle time as well as the preservation technology cost reduce because of the smaller order quantity.
- (g) Due to shortages, caused by the high demand rate and deterioration, the partial backordering is permissible. So, a penalty cost is applied on the unit product due to the delay. With an increase in penalty cost, the cycle time increases because of the consumer's patience level, whereas the total cost increases.

7. CONCLUSION

We develop an inventory model for stock-dependent demand with constant deterioration rate because it is believed that increased stock-pile provides consumers with wider selection and better quality of goods, and for the ease of calculation, we take deterioration rate to be constant. The intent of this paper is to determine the optimal cycle time, preservation technology cost, and positive inventory time. The analysis shows that 1) Deterioration has a negative effect on total cost, which forces companies to spend more on preservation technology. 2) The use of preservation technology investments helps to reduce the amount of perishable objects

and, thereby, reduces the unnecessary cost, which minimizes the total cost of the retailer's. 3) The increase in ordering cost and purchase cost increases the total cost. 4) The increase in penalty cost increases the total cost. To relate the model to the real world, shortages and partial backordering are permitted for the precise calculation of the objective function. A sensitivity analysis is done to represent the dependency of decision variables on different inventory parameters. Classical optimization method is adopted to solve the problem. The research work can further be extended for multi-products by taking into account the non-instantaneous deterioration rate. To increase the demand, a researcher can include promotional tools such as advertisement, discounts, credit period, etc. Instead of stock-dependent demand, one can assume the demand to be price or time-dependent.

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