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EPQ MODEL WITH LEARNING EFFECT FOR IMPERFECT QUALITY ITEMS UNDER TRADE-CREDIT FINANCING

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Abstract: Although high and advanced technologies are used to produce high quality items, some defective items are produced due to an error in technical operation or in maintenance. The defective cost is the expense involving rework, repair and replacement of defective items, and also the cost incurred due to loss of goods quality. The learning function acts as a substantial function for cost diminution. Meanwhile, the impact of learning is an incident which occurs approximately everywhere and enables the workers to carry out new work with better performance after flowing repetition over a course of time. Further, a retailer offers buyers an allowable setback time to arrange the money payable to him and no extra fine if money is paid within the allowable financing time period. On the other hand, if the cash is not paid on trade-credit financing period of time, the retailer will charge on remaining cash provided by the buyer after the allowed period. Keeping these facts,we developed an inventory model for imperfect quality items with a learning effect, in which demand rate is assumed as an exponential function of the trade credit period. The expected total profit function is maximized with respect to trade credit financing period under learning effect. A numerical example is illustrated, and a comprehensive sensitivity analysis is depicted to understand the robustness of the model.

Keywords: Learning Effects, Imperfect Items, Trade-Credit Financing, Defective Cost.

MSC: 90B85, 90C26.

1. INTRODUCTION

Many Corporate firms adopts the trade credit policy to enhance their business dimensions, also to attract more and more customers. The positive aspect behind to implement the trade credit policy is that it helps in the purchase of goods and services without immediate payment. Goyal [8] explored an order quantity formulation for the buyer when the retailer sets an allowable period. Shah [27] assumed a probability inventory formulation when setbacks in payments are allowable. Aggarwal [2] assumed various types of demanding strategy for the decaying items under the condition of the trade credit policy. Wright [32] in the first effort, formulated the numerical method thats links the behavior of learning or gain experience during leading business. After quantitative inspection, the resulting graphs showed a decreasing convex curve with cost. This mathematical formulation named as learning curve, experience curve and progress function. Baloff [5] discussed about the mathematical behavior of the learning. Argote and Epple [4] discussed about the factor by which the rate of learning varies in different situation which is the major factor in the research field. Salameh [23] considered a production inventory model with variable demand rate and learning in time to optimize the cost. Jaber and Salameh [15] discussed about optimal lot sizing with shortage and back-ordered under learning consideration. Jaber and Bonney [9] considered an inventory model with learning and forgetting curve. Author focused on minimizing production time also reduce rework process to find out the optimal production quantity. Salameh and Jaber [25] planned the usual EOQ formulation for the defective items. Jaber and Guiffrida [12] presented on the learning curve for processes generating defects required reworks. Author discussed a modification of Wright's learning curve for processes that generate defects that can be reworked. Jaber and Guiffrida [13] analyzed an order quantity model for defective items with percentage of defective items per batch follow the learning effect. Khan et al. [21] considered an EOQ model with defective quality item in which inspection taken as learning and minimizing the production cost. Jaber and Khan [14]discussed on, how to develop a merger of average dispensation time and process under the shipments and planned the consequence of unreliable learning curve limitations in manufacturing process. Anzanello and Fogliatto [3] explored on literature review of learning and forgetting curves. Konstantaras et al. [22] investigated a mathematical model to maximization of construction when demand is backlogged and an inspection for the defective items taken in the form of learning. Jaggi *e*t al. [17] discussed on construction stock model with shortages under the credit financing strategy with defective items. Givi *e*t al. [7] proposed a mathematical modeling for worker reliability with learning and fatigue. Jaggi et al. [18] used the policy of trade-credit financing in different inventory, ordering strategy for non-instantaneous deteriorating items with two warehouse system. Tiwari *e*t al. [31] proposed a combined store and pricing model for decaying items with partial backlogging under two level trade-credit policies in the provided sequence. Jayaswal *e*t al. [20] discussed the learning phenomenon on seller ordering strategy for defective quality articles with permissible delay in payment. They found out the impact of learning on inventory policies with defective quality and decaying items under the trade financing strategy. In this paper, we have introduced an optimal trade-credit policy for sellers when lot-size contains defective items. in the present article production cost declines and follows the learning curve. We derive the total cost of the inventory system and the optimal trade-credit period by using the Mathematica 9.0 software and plotted the graph for the effective parameters.

Table 1: Contribution of different authors

Author(s)	Learning effects	Inspection	Trade credit financing
Wright (1936)	✓		
Argote and Epple (1990)	✓		
Salameh et al. (1993)	✓	✓	
Jaber and Salameh (1995)	✓	✓	
Jaggi and Aggarwal (1995)		✓	✓
Salameh and Jaber (2000)		✓	
Jaber and Guiffrida(2008)	✓	✓	
Khan et al. (2010)	✓	✓	
Anazanello et al. (2011)	✓		
Jaggi et al. (2013)	✓		✓
Sair et al. (2014)	✓		✓
Khan et al. (2014)	✓		
Sangal et al. (2016)	✓		
Aggarwal et al. (2017)	✓		
This paper	✓	✓	✓

2. NOTATIONS AND ASSUMPTIONS

First of all, the following notations and assumptions are employed throughout this paper so as to develop the proposed models.

(a) **Notations:**

Notations	Units	Description
		Decision Variables
T	year	The buyer trade-credit financing time
y_n	units	Lot/batch size
		Parameter
$D(T)$	unit/year	Demand rate which is the function of trade-credit financing time
A	\$ /order	Ordering cost per order
c_0	\$ / unit	The learning production cost for preparing first unit
h	\$ / unit / year	Holding cost per unit per year
α	–	Defective percentage per lot size y_n with mean μ
u	–	Learning factor
s	per unit in dollar	The selling price per unit with $s > c_0$
r	per year	The buyer per year , CI rate on the opportunity cost
τ	year	Time when the production process started
P	per year	Production rate with $D(T) < P$
S	dollars	Cost incurred by defective items $S < P$
N	–	Number of imperfect items in each lot
$f(N)$	–	Expected number of defective items in each lot
$\phi(T)$	–	Gain function in dollars in year
T^*	year	The buyer maximum trade-credit financing time period
y_n^*	unit	The buyer maximum production batch size in units annually
$\phi(T^*)$	dollars	The buyer's maximum gain in dollar

(b) **Assumptions:**

1. The demand rate for an item is known and rate of replenishment is infinite.
2. Demand is fulfilled and lead time assumed to be zero.
3. The seller's trade-credit period to his/her buyer in years (decision variable).
4. Annual production rate greater than demand rate.
5. It is supposed that each batch contains defective percentage items are α with mean μ which is suggested by Rosenblatt and Lee [24].
6. It can be considered that demand rate is an exponential function of the credit period,T is suggested by Teng et al. [30].

$$D(T) = Ke^{aT} \tag{1}$$

Where $K > 0$ and $a > 0$

7. Wright [32] recommended that the whole unit cost of production declines by the factor from 10% to 50% in each time the accumulative production volume doubles, especially during the introduction phase of a new product and this is equivalent to the assertion that

$$c(t) = c(0)\left(\frac{X(0)}{X(t)}\right)^l \tag{2}$$

Here $c(t)$, is the unit production cost at time t, l is the learning coefficient and $X(t)$ is the accumulated construction volume at time t.

8. It is also considered that the rate of default hazard creating trade-credit financing time T is suggested by Teng et al. [30]

$$R(T) = 1 - e^{-bT} \tag{3}$$

3. MODEL DESCRIPTIONS

As per consideration by Rosenblatt and Lee [24] and above assumptions part that each batch contains not good quality items are α with mean μ and in the batch, the number of imperfect items is $N = 0$ if $\tau \geq t$ and $N = \alpha P(t-\tau)$ if $\tau < t$. Now, the estimated number of imperfect quality items in each batch size is $\int \alpha P(t-\tau)\mu e^{-\mu\tau}dt$ which is equal to $\frac{\alpha\mu t D(T)}{2}$ after simplification. Now, we calculate some costs which are (i) defective cost $(DC) = \frac{S\alpha\mu t y_n}{2}$ (ii)learning production cost $(LPC) = c_0\left(K^u e^{aTu}\right)$ (iii)opportunity cost $(OPC) = sKe^{(a-b-r)T}$(iv)ordering cost $(OC) = \frac{Ke^{aT}}{y_n}A$ (v)holding cost $(HC) = \frac{\left(1-\frac{D(T)}{P}\right)hy_n}{2}$ and the total costs (TC) of this model is

$$TC = \frac{Ke^{aT}}{y_n} + \frac{y_n h}{2}\left(1-\frac{Ke^{aT}}{P}\right) + \frac{S\alpha\mu t y_n}{2} + c_0\left(K^u e^{aTu}\right) + sKe^{(a-b-r)T} \quad (4)$$

As per consideration mentioned above, the inventory production system assumed that the seller must decide his/her trade credit period $T$and production batch length y_n of only items concurrently in order to optimize his gain annually. Considering all assumptions, the annual gain can be expressed as revenue minus total cost,

$$\phi(T) = sKe^{(a-b-r)T} - \frac{Ke^{aT}}{y_n} - \frac{y_n h}{2}\left(1-\frac{Ke^{aT}}{P}\right) - \frac{S\alpha\mu t y_n}{2} - c_0\left(K^u e^{aTu}\right) \quad (5)$$

Then we discuss the $seller's$ optimal solution to production lot size first and then trade credit period next.

3.1. Optimal production lot size

To maximize the annual profit $\phi(T, y_n)$ with respect to y_n is equivalent to minimize the annual total cost of ordering cost, holding cost and defective cost, which is

$$TC = \frac{Ke^{aT}}{y_n} + \frac{y_n h}{2}\left(1-\frac{Ke^{aT}}{P}\right) + \frac{S\alpha\mu t y_n}{2} \quad (6)$$

For the simplicity, we apply an arithmetic-geometric inequality method Teng et al. [30] to obtain the optimal solution of (6). As we now arithmetic mean is always greater or equal to the geometric mean. Suppose that x and y are two real positive numbers than we have

$$\frac{x+y}{2} \geq \sqrt{xy} \quad (7)$$

equation (6) exits if only $x = y$. Here the equations (6) hold if

$$\frac{Ke^{aT}}{y_n} = \frac{y_n h}{2}\left(1 - \frac{Ke^{aT}}{P}\right) + \frac{S\alpha\mu t y_n}{2} > 0 \tag{8}$$

After solving the equation of total cost derivative with respect to lot size which is equal to zero, we will get $seller's$ optimal production lot size is from equation (8)

$$y_n^* = \sqrt{\frac{2e^{aT}AK}{h[1 - \frac{Ke^{aT}}{P}] + \alpha\mu S}} \tag{9}$$

and minimum total cost from the equations (6) and (9), we get

$$TC(y_n^*) = \sqrt{2e^{aT}AK\left[h\left(1 - \frac{Ke^{aT}}{P}\right)\right] + \alpha\mu S} \tag{10}$$

Further, $seller's$ profit, $\phi\,(T)$ will become single decision variable T and which can be represented by

$$\phi\,(T) = sKe^{(a-b-r)T} - c_0\left(K^u e^{aTu}\right) - \sqrt{2e^{aT}AK\left[h\left(1 - \frac{Ke^{aT}}{P}\right)\right] + \alpha\mu S} \tag{11}$$

In order to find the optimal solution T^* of $\phi(T)$we drive the necessary condition for $\phi(T)$ in (9) to be maximized and differentiate with respect to T

$$\begin{aligned}\frac{d\phi(T)}{dT} &= sK(a-b-r)e^{(a-b-r)T} - auc_0\left(K^u e^{aTu}\,)\right)] \\ &- \frac{ae^{aT}AK\left[h\left(1 - \frac{Ke^{aT}}{P}\right)\right]}{\sqrt{2e^{aT}AK\left[h\left(1 - \frac{Ke^{aT}}{P}\right)\right] + \alpha\mu S}} = 0\end{aligned} \tag{12}$$

Sufficient condition can be proved with the help of theorems which are shown below

Theorem 1. *The $seller's$ optimal trade credit period is zero T^* if*

1. *if $a \leq (b + r)$ and $P \geq 2D$.*
2. $[a - (b + r)]sKe^{(a-b-r)T} \leq auc_0\left(K^u e^{aTu}\right)$ *and $P \geq 2D$.*

The proof of theorem-1 is given in the appendix part (A). There is some economical interpretation in the form of condition

1. if $a \leq (b + r)$, then the higher trade credit period, the lower the net revenue after default risk and opportunity cost. In this condition the seller should not offer trade credit financing period to buyer.

2. If the marginal net revenue increase i.e. $[a-(b+r)]sKe^{(a-b-r)T} \leq auc_0\left(K^u e^{aTu}\right)$, then it is no advantages of financing trade credit period from the seller to buyer and it is mentioned that if $D < P < 2D$, then we are unable to prove the theorem-1 is still valid.
3. $Ke^{(a-b-r)T} > auc_0\left(K^u e^{aTu}\right)$. For simplicity, let us define

$$\sum T = auc_0\left(K^u e^{aTu}\right) + \frac{ae^{aT}AK\left[h\left(1-\dfrac{Ke^{aT}}{P}\right)\right]}{\sqrt{2e^{aT}AK\left[h\left(1-\dfrac{Ke^{aT}}{P}\right)\right]+\alpha\mu S}} \tag{13}$$

Then from (12) we get, $\sum T = [a-(b+r)]sKe^{(a-b-r)T}$, which implies that seller's optimal trade credit period is

$$T^* = \frac{1}{a-b-r} In \frac{\sum T}{[a-(b+r)]sK} \tag{14}$$

It can be easily seen that the right hand side of equation (14) is also function of T and not a closed –form solution due to the complexity of the problem. It seems not to be tractable to find a closed-form solution to the seller's optimal trade credit period and to obtain the seller's optimal trade credit period with the help of Mathematica software. Now, for the second-order sufficient condition and taking the derivative of (12) with respect to T and re-arranging terms, we get

$$\begin{aligned}\frac{d^2\phi(T)}{dT^2} &= sK(a-b-r)^2 e^{(a-b-r)T} - (au)^2 c_0\left(K^u e^{aTu}\right) \\ &\quad - \frac{e^{aT}(AaKh)^2\left[\left(\dfrac{2Ke^{aT}}{P}\right)^2 - \dfrac{6Ke^{aT}}{P} + 1\right]}{\left(2e^{aT}AK\left[h\left(1-\dfrac{Ke^{aT}}{P}\right)\right]+\alpha\mu S\right)^{\frac{3}{2}}}\end{aligned} \tag{15}$$

In equation (15) if we take $sK(a-b-r)^2 e^{(a-b-r)T} \leq (au)^2 c_0\left(K^u e^{aTu}\right)$ and $\left[\left(\dfrac{2Ke^{aT}}{P}\right)^2 - \dfrac{6Ke^{aT}}{P} + 1\right] > 0$. Then we know that $\dfrac{d^2\phi(T)}{dT^2} < 0$ in (15) and $\phi(T)$ hence equation in (11) is a strictly concave function of T. We can obtain the following theoretical results from (11) and (15) given below

Theorem 2.

1. *if* $[a-(b+r)]sK - auc_0K^u - \dfrac{aAK\left[h\left(1-\dfrac{K}{P}\right)\right]}{\sqrt{2AK\left[h\left(1-\dfrac{2K}{P}\right)\right]+\alpha\mu S}} > 0$, $sK(a-b-r)^2 e^{(a-b-r)T} \leq (au)^2 c_0\left(K^u e^{aTu}\right)$ *and* $\left[\left(\dfrac{2Ke^{aT}}{P}\right)^2 - \dfrac{6Ke^{aT}}{P} + 1\right] > 0$

then $\phi(T)$ in (11) has a unique optimal solution $T^* > 0$ as in equation in (12).

2. *if* $[a-(b+r)]sK - auc_0K^u - \dfrac{aAK\left[h\left(1-\dfrac{K}{P}\right)\right]}{\sqrt{2AK\left[h\left(1-\dfrac{2K}{P}\right)\right]+\alpha\mu S}} \leq 0$, $sK(a-b-r)^2e^{(a-b-r)T} \leq (au)^2c_0\left(K^ue^{aTu}\right)$ *and* $\left[\left(\dfrac{2Ke^{aT}}{P}\right)^2 - \dfrac{6Ke^{aT}}{P}+1\right] > 0$.*then* $\phi(T)$ *in (11) has a unique optimal solution* $T^* = 0$.
3. *if* $Z = sK(a-b-r)^2e^{(a-b-r)T} - (au)^2c_0\left(K^ue^{aTu}\right) + \dfrac{a^2hKe^{aT}y_n}{2P} \leq 0$,*then there exists a unique optimal solution* (T, y_n) *that maximizes* $\phi(T, y_n)$

The proof of theorem-2 (part (i), (ii) and (iii)) is given in the appendix part (B).

4. NUMERICAL EXAMPLE

Input parameters have been taken from Teng *e*t al. [30] and Rosenblatt and Lee [24]. After putting all the values in the respective equations we got the optimal value of credit period and expected total profit
$b = 0.1, a = 0.2, r = 0.09, u = 0.97$, $c_0 = 8$ dollar for the first production unit, $A = 5$ dollar per order, $h = 1$dollar per unit per year, $K = 1000$ per year, $P = 10000$ units per year, $S = 10$ dollar per unit, $\mu = 0.1$ and $\alpha = 0.05$ per lot. We first check the condition according to theorem-2

$$[a-(b+r)]sK - auc_0K^u - \frac{aAK\left[h\left(1-\frac{K}{P}\right)\right]}{\sqrt{2AK\left[h\left(1-\frac{2K}{P}\right)\right]+\alpha\mu S}} = 18.93 > 0 \quad (16)$$

Then we substitute the values of the parameters into (12) and use Mathematica 9.0 software to obtain a trade credit period $T^* = 0.4642$ year. Now, the check the concavity condition with $T^* = 0.4642$ year, $48.9887 = sK(a-b-r)^2e^{(a-b-r)T} \leq (au)^2c_0\left(K^ue^{aTu}\right) = 149.873$ and $\left[\left(\dfrac{2Ke^{aT}}{P}\right)^2 - \dfrac{6Ke^{aT}}{P}+1\right] = 0.5579 > 0$. According to theorem-2, the unique optimal trade credit period $T^* = 0.4642$ year and substituting $T^* = 0.4642$ year in to (9), get the optimal production lot size $y_n^* = 135.445$ unit. Substituting $T^* = 0.4642$ year and $y_n^* = 135.445$ unit into (5), obtain the optimal annual profit for the seller $\phi(T^*, y_n^*) = 12429$ dollar.

5. SENSITIVITY ANALYSIS

For the sensitivity analysis, we have taken same data which mentioned in above numerical example.The variations of T^*,y_n^* and $\phi(T^*, y_n^*)$ with respect to different parameters are shown below in tables and figures.

Table 2: Impact of learning factor (u) on T^*, y_n^* and $\phi(T^*)$

Learning factor u	Optimal trade-credit financing period T^* (year)	Optimal lot size y_n^* (unit)	Expected profit function $\phi(T^*)$($)
0.97	0.4642	135	12429
0.96	0.8800	146	12983
0.95	1.3110	159	13584
0.94	1.7580	173	14237
0.93	2.2220	190	14948

Table 3: Impact of percentage defective (α) on T^*, y_n^* and $\phi(T^*, y_n^*)$

Percentage of defective α(%)	Optimal trade-credit financing period T^* (year)	Optimal lot size y_n^* (unit)	Expected profit function $\phi(T^*)$($)
0.05	0.4642	135	12429
0.1	0.4602	134	12419
0.15	0.4538	132	12403
0.2	0.4451	130	12380
0.25	0.4344	127	12353

Table 4: Impact of interest rate (r) on T^*, y_n^* and $\phi(T^*, y_n^*)$

Interest rate r	Optimal trade-credit financing period T^* (year)	Optimal lot size y_n^* (unit)	Demand rate $D(T^*)$ unit/year	Expected profit function $\phi(T^*)$($)
0.05	2.626	233	2369	14085
0.06	2.0295	200	1948	13378
0.07	1.4784	176	1625	12901
0.08	0.9602	157	1370	12599
0.09	0.4642	141	1164	12429

Managerial insights:

1. From table 2, we have observed that, when the values of u decreases, the optimal trade-credit period, order quantity and optimal profit for the buyer increase owing to the learning phenomenon.
2. From table 3, we analyze that whenever the number of imperfect items increases, the optimal trade-credit financing period, order quantity and corresponding profit decreases due to the theoretical interpretation from equation (9).

3. From table 4, we analyze that whenever r increases, the optimal trade-credit period T^*, order quantity y_n^*, demand rate $D(T)$ and $\phi(T^*)$ decrease due to dependency on r.
4. From figure 1, it is analyzed that whenever h increases, the optimal trade-credit financing period T^* and $\phi(T^*)$ increase due to the learning effect and trade credit financing policy.
5. From figure 2, we analyze that whenever increases, the optimal trade-credit financing period T^* and $\phi(T^*)$ increase due to $K > 0$ and policy of the proposed model.
6. From figure 3, we analyze that whenever increases, the optimal trade credit financing period T^* and $\phi(T^*)$ increase due to $a > 0$ and policy of the proposed model.

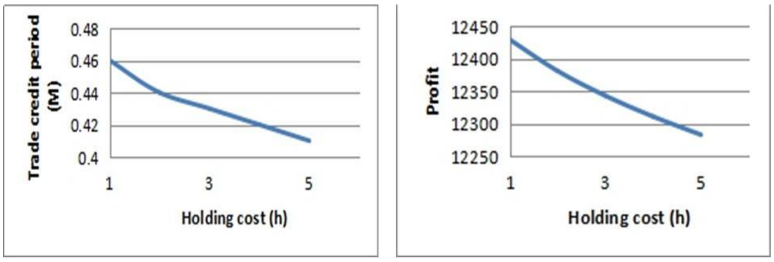

Fig. 1: Impact of holding cost on trade credit period and profit

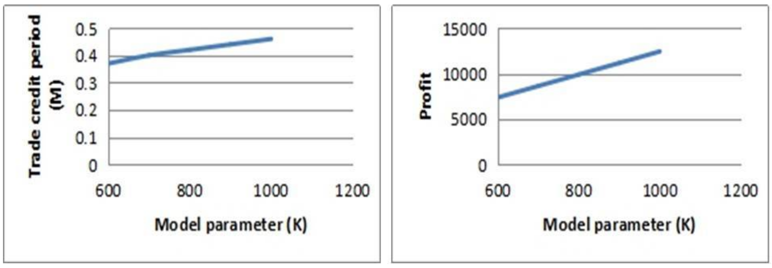

Fig.2: Impact of model parameter (K) on trade credit period and profit

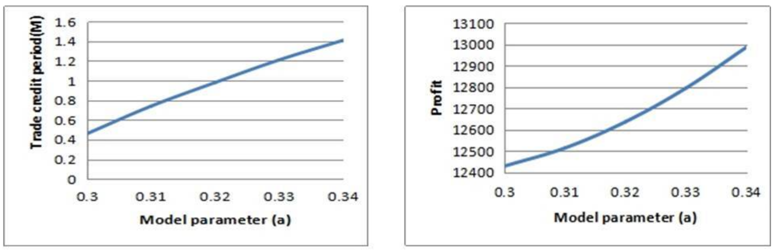

Fig.3: Impact of model parameter (a) on trade credit period and profit

6. CONCLUSION AND FUTURE SCOPE

This paper explained that, how to calculate the optimal credit period for the seller where demand rate is an exponential function of credit period and lots has fixed defective items. Trade credit period have taken as decision variable for seller and maximized it with the help of inventory parameters. Finally, total profit function is maximized with respect to trade credit financing period under learning effect where demand rate is an exponential function of trade credit period. From the sensitive analysis, we examine that higher the percentage of defective items, lower the value of T^*, y_n^* and $\phi(T^*)$. When the value of u decreases, then T^*, y_n^* and $\phi(T^*)$ increase rapidly due to the learning effect. We have given mathematical examples to demonstrate the present model. Hence, we have made managerial insights shown in sensitive analysis for a retailer to establish the best financing period of time and batch size concurrently under imperfect production. This paper can be extended for more realistic situations such as stock dependent demand, shortages etc.

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7. Appendix part (A):

If $a \leq (b+r)$ and $P \leq 2D$, then we know that from (12) that $\frac{d\phi}{dT} \leq 0$. Consequently, the seller's optimal trade credit period is set to be zero. Likewise, we can easily prove that $T^* = 0$ if $sK(a-b-r)e^{(a-b-r)T} - auc_0 \left(K^u e^{aTu}\right)$ and $P \leq 2D$. This completes the proof of theorem.

Appendix part (B):

Now, the part (i) and (ii) can be prove with the help of equation (13)

Proof of part (i)

we know that $\dfrac{d^2\phi(T)}{dT^2} < 0$ if $sK(a-b-r)^2 e^{(a-b-r)T} \leq (au)^2 c_0 \left(K^u e^{aTu}\right)$ and $\left[\left(\dfrac{2Ke^{aT}}{P}\right)^2 - \dfrac{6Ke^{aT}}{P} + 1\right] > 0$. In addition, using the fact that

$\lim_{T\to\infty} \dfrac{sK(a-b-r)e^{(a-b-r)T}}{e^{aT}} = 0$ we get, $\lim_{T\to\infty} \dfrac{d\phi(T)}{dT} = -\infty$.we got $T^* = 0$ and substituting the value of $T^* = 0$ in equation (12) , we obtain

$\dfrac{d\phi(0)}{dT} = [a-(b+r)]sK - auc_0K^u - \dfrac{aAK\left[h\left(1-\dfrac{K}{P}\right)\right]}{\sqrt{2AK\left[h\left(1-\dfrac{2K}{P}\right)\right] + \alpha\mu S}}$. If $\dfrac{d\phi(T)}{dT} > 0$

, then applying the Mean- Value theorem we know that there exists a unique optimal trade credit period $T^* > 0$ such that $\dfrac{\phi(T)}{dT} = 0$. Hence the proved the part (i).

Proof of part (ii)

However if $\dfrac{d\phi(0)}{dT} \leq 0$ then$\dfrac{d\phi(0)}{dT} < 0$ for all T which implies $\phi(T)$ in (11) is a strictly decreasing function of T. Hence, if $\dfrac{d\phi(0)}{dT} < 0$ then $T^* = 0$, is the unique optimal solution of $\phi(T)$ in (11) which is the proof of part (ii). The annual profit $\phi(T, y_n)$ at (T^*, y_n^*) has two decision variables T and y_n. It is needed to prove the Hessian matrix with respect to the annual profit $\phi(T, y_n)$ at (T^*, y_n^*) is negative definite. Hence, we prove the following results.

Appendix part (C):

Taking the second-order partial derivative of $\phi(T, y_n)$ in (5) with respect to T and y_n

$\dfrac{\partial^2\phi(T)}{\partial T^2} = -\dfrac{a^2AKe^{aT}}{y_n} + \dfrac{a^2AKe^{aT}y_n}{2P} < 0$, $\dfrac{\partial^2\phi(T)}{\partial y_n^2} = -\dfrac{2AKe^{aT}}{y_n^3} < 0$,$\dfrac{\partial^2\phi(T)}{\partial y_n \partial T} = \dfrac{AKe^{aT}}{y_n^2} > 0$ and $\left[\dfrac{\partial^2\phi(T)}{\partial T^2}\right]\left[\dfrac{\partial^2\phi(T)}{\partial y_n^2}\right] - \left[\dfrac{\partial^2\phi(T)}{\partial y_n \partial T}\right]^2 > 0$. Hence, the Hessian matrix associated with $\phi(T, y_n)$ is negative definite and applying the part (i) and (ii), we know that the unique solution (T^*, y_n^*) is the global maximum solution. Hence, the theorem is proved.