Yugoslav Journal of Operations Research
32 (2022), Number 1, 111–134
DOI: https://doi.org/10.2298/YJOR200918042B

ESTIMATING THE OUTCOME OF AN ACTIVE PHASE IN THE MILITARY CONFLICT ON EASTERN UKRAINE BASED ON THE SUGENO FUZZY INTEGRAL

Victor BOCHARNIKOV
Center for Military Strategic Studies, National Defense University of Ukraine, Kyiv, Ukraine
bocharnikovvp@yandex.ua

Sergey SVESHNIKOV
Center for Military Strategic Studies, National Defense University of Ukraine, Kyiv, Ukraine
sveshnikovsv@yandex.ua

Radion TIMOSHENKO
Center for Military Strategic Studies, National Defense University of Ukraine, Kyiv, Ukraine
timishenkori@yandex.ua

Received: September 2020 / Accepted: December 2020

Abstract: A military conflict (especially its active phase) is a time of maximum exertion of all the powers of the state and society, a time that requires quick and correct decisions from state bodies. The quality of these decisions is largely determined by the estimation adequacy of the current situation. As the analysis shows, modern military conflicts start suddenly and develop rapidly. The official informing system turns out to be ineffective, what leads to numerous mistakes in decision-making. In addition, modern military conflicts are of a hybrid nature. The outcome of such military conflicts depends on many factors of a non-military nature, for example, the quality of governance, support from the population, international assistance. These factors are often formulated qualitatively (linguistically), and the conditions of the active phase of a military conflict do not give time to check the adequacy of quantitative data. Therefore, it is necessary that the method for estimating the outcome of the active phase takes into account the data uncertainty and ensures a generalization of the partial characteristics of the current situation. Based on the analysis of known approaches to the description and processing of uncertainty, the authors proposed using the methods of fuzzy integral calculus to describe partial characteristics and calculate a generalized characteristic, which is an estimation of the success of the outcome of an active phase. The authors have solved the following subproblems: identification of structure and parameters of standard for estimating; choice of the observation channel of the characteristics of the current situation; constructing the algorithm for estimations generalization. The authors demonstrated the work of the proposed algorithm by the example of estimating the results of hostilities in eastern Ukraine in July 2014.

Keywords: Military Conflict, Decision-Making, Estimation, Algorithm, Fuzzy-Integral Calculus.

MSC: 28E10, 90B50.

1. INTRODUCTION

Currently, the characteristic features of military conflicts have significantly changed in comparison with military conflicts which have happened in the past. Authoritative researchers name modern military conflicts as hybrid conflicts [19], proxy-conflicts [27] and network conflicts [2]. Obviously, it is possible to agree with all the view-points because modern military conflicts have signs of all these types of conflicts at the same time. Even though, modern conflicts are interstate ones by their nature, they still proceed in the form of internal conflicts. Moreover, the participants of the conflicts, widely employ diplomatic, economic, information and other non-military means to achieve own goals. All this makes it necessary to reconsider approaches to decision making in the conditions of modern military conflicts.

The evolution of military conflicts in the Balkans, Syria and Ukraine has shown that military conflicts evolved cyclically. After a phase of an active armed struggle, participants are trying to achieve intermediate military-political agreements to freeze active actions, regroup forces, replenish resources, etc. However, such agreements are temporary inasmuch as they, as a rule, do not satisfy both participants. Therefore, over time, the unsatisfied participant activates armed struggle. We have observed similar cyclicity during the internal conflict in Croatia. At that time, the Croatian army with support of NATO had routed the Serbian armed formations [15] after the long lull period (about 2.5 years). We can also observe the cyclicity of the military conflict in Syria where the conflict phases consistently replace each other: at first comes the lull, attempts to begin negotiations between the governmental forces and the armed opposition, next the activation of the conflict [1].

The conflict on eastern Ukraine has cyclic character as well. The first active phase of the conflict began in July and ended in September, 2014 when the conflict participants had signed the first Minsk agreements. However, the goals of the participants were not reached. The second active phase of the conflict began in January, 2015 after regrouping and accumulating the forces. The outcome of this phase was also unsuccessful and the participants have signed the second Minsk agreements.

Proceeding from empirical experience, it is possible to suppose, that the evolution of a conflict substantially depends on the outcome of its active phase. Successful offensive will create preconditions for continuation of an active phase of the conflict with the purpose of final rout of the enemy. Ineffective offensive, most likely, will cause lull, and unsuccessful offensive - the need of arrangement for armistice. Therefore, in order to analyze the conflict evolution, it is necessary to estimate a possible outcome of its active phase. Modern management practices require this estimate to be quantitative and calculated using a computerized system, since quantitative estimation provides confidence that all important factors have been taken into account. Moreover, the quantitative estimation can be visualized and interpreted, what significantly improves the support of decision-making by government bodies.

Supporting decision-making was significantly hampered at the beginning of the conflict in eastern Ukraine, when the Ukrainian Armed Forces were forced to launch hostilities without proper preparation. We mean not only combat training, logistics and other issues of support for military operations. Information and psychological training of the population, providing international support and others are important elements of preparation for a military conflict. A system for informing was also in its infancy. Units of the Armed Forces often transmitted data on the results of hostilities in free unformalized form using ordinary cell phones. In these conditions, it was difficult to ensure the completeness and reliability of the data in order to adequately estimate the current situation. Therefore, practice required to solve the following problem. It is necessary to develop an algorithm for calculating a generalized quantitative estimation of the outcome of the active phase of a military conflict, taking into account the uncertainty of the input estimates, which are formed on the basis of data from many open information sources.

However, there are three questions, which are associated with collecting and presenting input data and processing it. These questions can be viewed as requirements for solving the problem.

1. How to obtain input estimates? The expert formulates the input estimates in terms of natural language. It is necessary to convert the linguistically formulated estimate into a number for further calculations. Since the expert analyzes a very intensive flow of informational messages, this transformation should not be complex. The uncertainty of input data is the main difficulty at solving the problem and a natural consequence of the fact that a person is used as meter of characteristics, and not a physical device. An expert is a specialist in the field of military conflicts, not in the field of estimating methods. During the active phase of the conflict, the expert works in real time. Therefore, additional questions to him are not appropriate here.

2. How to obtain the importance of characteristics? The outcome of an active phase depends not only on the success of the armed struggle, but also on factors of a non-military nature, for example, the quality of governance, support from the population, international assistance, and others. The hybridity of mod-

ern military conflicts is manifested in a combination of military and non-military factors. The estimation of the outcome of an active phase should generalize many partial estimations in accordance with their importance, which is specified by the expert. Therefore, the formal presentation of the importance of characteristics should describe the subjective preferences of the expert as accurately as possible.

3. How to generalize multiple estimations? The properties of an operation, which uses for generalization, should also correspond, as much as possible, to the properties of non-additive human logic. Non-additivity means that characteristics do not interact with each other. However, in our case, this condition is not met. For example, the support of hostilities, like the mood of the population, depends on international assistance.

Consider further the known approaches from the view-point of these requirements.

2. KNOWN APPROACHES

The problem of estimating military conflicts outcome is of interest to many researchers and organizations. At the same time, there are few available scientific publications in this area, possibly due to the sensitivity of the problem. We did not find any publication about solving problems that would take into account the above-mentioned questions. Therefore, we will consider scientific approaches that can be used to solve the problem. These approaches can be conditionally divided into non-formalized and formalized.

Non-formalized approaches.

The authors of paper [9] discuss the conceptual questions of measuring effectiveness in irregular warfare. The manuscript [26] is devoted to the measurement of conflicts. The author offers a set of indicators, such as the state of military-political relations, the level of militarization of border areas, and others. However, these indicators cannot be used to estimate the active phase of a conflict when parties are actively fighting. In the study [32], the author examines difficulties associated with estimating the performance of units in an irregular military conflict, and also considers factors that can prevent dysfunctional behavior of servicemen and increase the effectiveness of their actions. The guide [28] contains a description of the methods and procedures for estimating military operations developed by the Air Land Sea Application Center and recommended for the US Army. In particular, the document describes the process of estimating operations, indicators for estimating the success of hostilities, as well as estimation standards (on a five-point scale) for the subject areas: security, governance, socio-economic sphere, regional relations. Using a five-point scale, you can get numerical estimates of partial characteristics and visualize them. However, the guide does not provide data on the procedure for generalizing the partial estimates.

Formalized approaches.

The analysis of publications had shown that, basically, four classes of mathematical models are being used to describe the evolution of military conflicts: descriptive approaches; methods of imitating modeling; optimization methods and decision-making methods. Each of these classes includes the big number of subclasses which differ by mathematical constructions.

Descriptive approaches, as a rule, use the methods of probability theory [21], the statistical theory of decision-making [25], the theory of reliability [3], and queuing theory [14]. The basic obstacle of using these methods to solve the formulated problem is the complexity of taking into account mutual influence of partial characteristics inasmuch as the listed methods require are the characteristics to be independent of each other.

Methods of imitating modeling use the mathematical constructions of Markovian processes, as well as differential equations, finite state machines or multi-agent systems. These methods can be used for realization of imitating games [12]. The method of dynamics for averages on the basis of differential equations of Lanchester [22] has been widely used for the dynamic description of military conflicts [18]. However, methods of imitating modeling do not allow taking into account military-political factors, which greatly influence evolution of modern military conflicts. Neglecting these factors potentially reduces the adequacy of these methods.

Optimization methods use linear and dynamic programming, optimal control, discrete optimization, the graphs theory, the theory of network planning and control [35]. In order to use these methods, the evolution of a conflict should be formulated in kind of optimization task, which is not always possible. With the help of optimization methods, it is difficult to describe initial conditions of conflicts, and also to take into account qualitative characteristics of conflicts.

Decision-making methods are more suitable for solving the problem of estimating the outcome of an active phase of a military conflict, because they allow taking into account uncertainty in the best way.

To date, a large number of scientific studies are devoted to modeling and processing uncertainty. In accordance with the requirements discussed above, our problem can be classified as a multi-criteria decision-making problem under conditions of uncertainty in input data and criteria weights. Methods for modeling numerical or linguistic uncertainty are widely used to solve similar problems.

The study [29] uses interval fuzzy numbers to make decisions based on multiple criteria. The article [20] considers the transformation of linguistic data into fuzzy numbers and issues of building indexes, which are used to estimate the performance of a certain process. In the article [38], the authors propose a new aggregation operator in problems in which the weights of criteria and experts are described by real numbers, and the values of the criteria are described by intuitionistic fuzzy numbers. In the study [7], the author examines the solution of a multicriteria choice problem based on fuzzy arithmetic. Input data and criterion weights are presented in the form of triangular or trapezoidal fuzzy numbers, and the arithmetic operations of addition and multiplication are used to aggregate the estimates. Study [6] considers the use of triangular Z-numbers to represent input data in multicriteria decision-making problems. Z-numbers require additional determination of the reliability of each input estimate.

To describe linguistic uncertainty, researchers use fuzzy sets [39] and fuzzy measures [36], which are given on discrete sets of subjective categories. The article

[10] by D. Dubois is one of the first studies that describes the use of membership functions to represent values and weights of criteria, as well as the use of logical operations (in particular, the minimum) to combine membership functions. Today fuzzy sets are widely used for solving technical problems such as developing fuzzy controllers [31]. Article [32] discusses the use of soft multi-set topology in multicriteria decision-making problems when the input data can be determined as subsets with a certain number of appearances. The study [13] examines the solution of multicriteria decision-making problems using hesitant fuzzy sets, which are a generalization of Zadeh's fuzzy sets, as well as intuitionistic fuzzy sets. Hesitant fuzzy sets are used when an expert has doubts in assigning membership to elements of a set from the domain of definition. In similar cases, fuzzy sets of type 2 are also used [31]. However, in order to determine such a set, it is necessary to ask the expert additional questions, which is not always acceptable in real-time problems.

Fuzzy measures can also be widely used to solve multicriteria decision-making problems in various subject areas. For example, the article [23] presents a fuzzy integral-based model for estimating a supplier. The study [24] also uses a fuzzy integral to generalize several criteria. In the article [16], the author draws attention to the advantages of a fuzzy integral for its use in practical applications of multi-criteria estimation. In particular, he emphasizes that fuzzy integrals can flexibly model interactions between criteria.

Based on the analysis of known approaches to the description and processing of uncertainty, to solve the problem of estimating the outcome of the active phase of a military conflict, we propose an approach based on the use of fuzzy measures and fuzzy integrals. According to this approach, the input estimates should be represented in the form of fuzzy sets, the criterion weights - in the form of fuzzy measures, and the generalization should be performed using a fuzzy integral. A fuzzy measure is a more universal representation of uncertainty [37], since the definition domain of a fuzzy measure is the set of all subsets of the universal set, and therefore a fuzzy measure can be viewed as an extension of the concept of a fuzzy set. In turn, the fuzzy integral can be viewed as an extension of the logical operations of minimum and maximum, which combine the estimates and weights of the criteria. Two versions of the implementation of the fuzzy integral are known: by Sugeno [36], and by Choquet [8]. The Sugeno fuzzy integral uses logical operations of the supremum and minimum, and the Choquet fuzzy integral uses subtraction and multiplication. We propose to use the Sugeno integral, since this integral has a slight advantage. The technique of calculating this integral allows to determine the elements of the universum that influenced the result of integration. Thus, the novelty of our approach lies in the fact that we propose to solve the problem of estimating the active phase of a military conflict using the methods of fuzzy integral calculus.

3. FORMAL DESCRIPTION AND ALGORITHM

The formulated problem can be considered as a static estimating problem [5] and can be described by the following cortege:

$$< \Omega, X, \{B_j\}, J, Y, \gamma > . \tag{1}$$

The evolution of a military conflict can be described as a sequence of situations. Each situation corresponds to one time moment. The situations' set is denoted as Ω. In turn, each situation $\omega \in \Omega$ is described by a characteristics set as X. In common case, each characteristic accepts values from its sets of values $B_J = \{b_{jk}, k = \overline{1, K_j}\}$. The outcome of an active phase of a military conflict can be estimated by means of successfulness criterion of an active phase $J \in [0, 1]$. This criterion reflects the view-point of the participant who is an attacker.

The characteristics $x_j \in X$ of the situation $\omega \in \Omega$ have different importance, which depends on researcher's preferences system $P : X \times X \to J$. The set of the interconnected characteristics X together with the formed preferences system is the implicit estimation standard $M(X, P)$. The standard $M(X, P)$ is intended for estimating the situation $\omega \in \Omega$ in criterion $J \in [0, 1]$. This standard describes the answer to the question: what values should characteristics accept (taking into account the importance of characteristics) so that the criterion $J \in [0, 1]$ becomes maximum? If at least one of the characteristics $x_j \in X$ does not accept the best value, then the criterion $J \in [0, 1]$ is reduced. Calculation is carried out by means of an estimation algorithm $Est : \Omega \to J \in [0, 1]$.

Evolution of the situation $\omega \in \Omega$ can lead to one of the outcomes of an active phase. We denote the outcomes set as $Y = \{y_1, y_2, y_3\}$, where:

y_1 means "Successful offensive" (rout of the enemy, capture of one or several settlements);

y_2 means "Ineffective offensive" (return onto initial positions);

y_3 means "Unsuccessful offensive" (a defeat and retreat).

This specified outcomes $y_m \in Y, m = 1, 2, 3$ are ordered by a successfulness degree of an active phase and have the unique sense. The outcome of an active phase can be estimated by means of interpretation procedure of criterion values $J \in [0, 1]$. This procedure is the mapping $Int : J \to Y$, and an estimation operator of situations is determined by the mappings composition $\gamma = Int \circ Est : \Omega \to Y$.

Taking into account the done designations, the formulated problem can formally be described as: Let the static estimation problem be described in the form of the cortege (1). We should find the mapping (algorithm, procedure) $\gamma : \Omega \to Y$ which calculates the estimation $y_m \in Y$ of the situation $\omega \in \Omega$ according to the chosen estimation standard $M(X, P)$. A problem solving scheme is shown in Figure 1.

The problem contains several sub-problems:

1) Structural identification of the standard $M(X, P)$, i.e., definition of characteristics set X and connections of these characteristics with criterion J.

2) Parametrical identification of the standard, i.e., definition of importance of partial characteristics $x_j \in X$ of the situation $\omega \in \Omega$.

3) Choice of the observation channel for each characteristic $x_j \in X$ of the situation $\omega \in \Omega$.

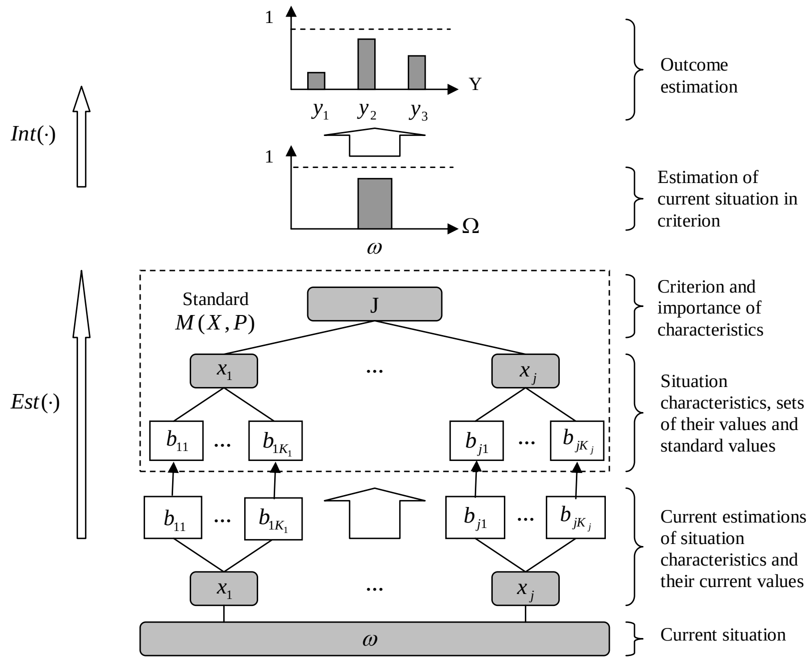

Figure 1: A problem solving scheme

4) Constructing the transformation algorithm $\gamma : \Omega \to Y$ of entrance estimations of characteristics X of the situation $\omega \in \Omega$ into the set Y, i.e., constructing the estimation algorithm $Est : \Omega \to J \in [0, 1]$ and estimation interpretation procedure $Int : J \to Y$.

Our contribution to solving the problem of estimating the outcome of the active phase of a military conflict is determination of: structure of the estimation standard (etalon), its characteristics, the observation channel of these characteristics to obtain input data and the algorithm for their generalization.

Sub-problem 1. Structural identification of the standard.

We pay attention to two aspects. Firstly, a problem of static estimating does not consider the time. We use the standard $M(X, P)$ for estimation of only situation $\omega \in \Omega$, which corresponds to the current time-moment. For the analysis of conflict dynamics, the situation should be estimated daily on the basis of observation over situation characteristics $x_j \in X$. Secondly, different characteristics of the situation can be measured by means of different numerical or linguistic scales. However, estimations of all characteristics $x_j \in X$ should be led to unified basis which is compatible with the standard.

Using empirical experience, we assumed that the structure of the standard $M(X, P)$ should be a two-level hierarchical system, as shown in Figure 1. Characteristics $x_j \in X$ of the situation $\omega \in \Omega$ form the lower level, and the criterion J-upper level of hierarchy. This two-level standard $M(X, P)$ is understandable and not complicated for use in practice.

To solve any estimation problem, it is necessary to fulfill two main requirements for the characteristics of the estimated situation. First, the list of these characteristics should be sufficient to adequately describe the problem. It must also correspond to the data sources, that is, these sources must contain all the necessary data to obtain the values of the characteristics. Second, the formal presentation of the characteristics should best match the nature of the information that is used to obtain the input data. This should minimize errors in the input data and contribute to the adequacy of the solution results. Let us consider further the choice of characteristics of the current situation from the view-point of these requirements.

We consider it a mistake to estimate the outcome of the active phase of a military conflict by analyzing only hostilities. If a military conflict began suddenly and the armed forces did not prepare for it properly, the outcome of hostilities becomes dependent on many non-military factors. Therefore, the list of characteristics that is used to estimate the current situation should be expanded. Above we mentioned the shortage and unreliability of input information to estimate the current situation in conditions of the military conflict in eastern Ukraine. At the same time, local, central and foreign media, social networks contained a large number of messages about events that were in one way or another related to the military conflict and could influence the evolution of hostilities immediately or in the future. In particular, these messages contained data on hostilities, logistics, the attitude of the population to the actions of the conflicting parties, as well as often on the state of combat units. Analyzing the messages of the central media, we could form an opinion about the actions of state bodies, international military assistance, the reaction of civil society to events, the possibility of improving the provision of the armed forces. Analysis of international media messages allowed us to form an opinion about political and economic support, pressure on the enemy, and so on.

This stream of messages had both positive and negative properties. On the one hand, a large number of messages compensated the lack of information and the poor performance of the official system for informing. But on the other hand, this stream contained a lot of fake messages. Despite this, the large volume of the input stream of messages allowed the experienced expert, using their experience and intuition, to identify fake messages and obtain quite adequate input data. In addition, the used formal presentation of these data in terms of "good or bad" not required an expert to make unambiguous numerical estimates, what greatly facilitated his work.

Based on the content analysis of information messages, we grouped them by spheres (for example, the state management, logistics and so on) and used these groups as characteristics $x_j \in X$ of the situation $\omega \in \Omega$. As a result, we have formed the set from seven characteristics $X = \{x_j\}, j = 1, ..., 7$ which are shown in Table 1.

Table 1: The characteristics of the situation

X	Characteristic of the Situation	What is considered during estimating of characteristic	How the characteristic influences criterion
x_1	Quality of the state management	Internal and foreign policy decisions of the state bodies.	If the quality is better, the confidence concerning the success of actions is higher.
x_2	Troops structure in the conflict zone	Troops conditions; Conformity of troops structure to needs of situation; Conditions of mobilization resources.	If conditions of the troops in the conflict are better, the confidence concerning the success of actions is higher.
x_3	Logistics quality	Logistics conditions; The Foreign military-technical help; The Volunteers help.	If the logistics quality is better, the actions of the troops are more effective.
x_4	External influence	Actions of the states which directly show support in military, political or economic sphere.	If the direct support of other states is more active, the confidence concerning the success of actions is higher.
x_5	Strategic background	Actions of the states which are involved in the conflict and cooperate with the participants of the conflict.	The favorable strategic background increases confidence concerning success of actions.
x_6	Resistance of civilians in the conflict zone	Resistance level of civilians; Psychological conditions of public opinion; The Migratory flows.	Strong resistance by civilians reduces confidence in the success of actions.
x_7	Conditions outside of the conflict zone	Auspiciousness of internal conditions in political, economic and social spheres outside of the conflict zone.	If the conditions are better, the confidence concerning the success of actions is higher.

Sub-problem 2. Parametrical identification of the standard.

Above, we have proposed to use methods of fuzzy-integral calculus to solve the formulated problem inasmuch as they better take into account various uncertainties. Therefore, for formalization of importance of characteristics $x_j \in X$, we use the fuzzy measure $g_X(\cdot) : 2^X \to [0, 1]$ (notation $g_X(\cdot)$ means function of set of all subsets of X-set).

For the identification of fuzzy measure $g_X(\cdot)$ we have used an algorithm on the basis of approximation by iterations. According to this algorithm, the fuzzy measure $g_X(\cdot)$ is calculated as a result of the following steps:

the assignment of target value of normalization parameter of the fuzzy measure (a choice of semantics of the fuzzy measure);

the assignment of initial densities of the fuzzy measure with the help of pair comparisons method;

consecutive refinement of densities of the fuzzy measure, until current value of normalization parameter does not equal target value.

In the theory of expert systems, this fuzzy measure can be considered as a knowledge base. Using the broader terminology of artificial intelligence methods, the fuzzy measure is a subsymbolic form of some knowledge about preferences on the set of characteristics X. There are only two sources for machine learning: an expert (several experts) or a big enough set of data. Since the conditions of each military conflict are unique, in each case the importance of the characteristics $g_X(\cdot)$ will be different. For example, in an expeditionary conflict, logistics will be of great importance. In a proxy conflict, the conflict outcome will depend more on external influences (x_4) and strategic background (x_5). Therefore, for specifying the fuzzy measure $g_X(\cdot)$, we refused to use learning on statistical data and used supervised learning.

According to this method, the expert answers questions and formalizes his knowledge using some formal procedure. In our case, we use the paired comparisons procedure [33]. An expert, based on his life experience, consistently answers $j^2 - j$ questions about the preferences strength of each characteristic over the other characteristics. As a result, the procedure calculates the normalized weights of the characteristics, which, using an iterative procedure, we transform into the fuzzy measure $g_X(\cdot)$, taking into account its axiomatics.

Consider a formal description of this algorithm for specifying the fuzzy measure $g_X(\cdot)$ of the importance of characteristics.

The assignment of target value of normalization parameter of the fuzzy measure.

The criterion J should take into account all the characteristics from set $X = \{x_j\}, j = 1, ..., 7$. Therefore, we have chosen the following target value of normalization parameter of the fuzzy measure: $\lambda = 2.3$. Fuzzy measures with this parameter value are belief measures. They attune fuzzy integral for executing logic operation with the semantics which is close to the semantics of logic multiplication "AND".

The assignment of initial densities of the fuzzy measure.

Using widely known method of paired comparisons and a scale [33], we built paired comparisons matrix $A = [a_{kr}], k = 1, ..., 7, r = 1, ..., 7$. With the help of this matrix, we determined values of the normalized weights $\{w_i\}, i = 1, ..., 7$ as follows:

$$w_i = s_i / \max_{k=1,...,7} s_k, s_k = \sqrt[7]{\prod_{r=1}^{7} a_{kr}}, i = 1, ..., 7. \tag{2}$$

Consecutive refinement of densities of the fuzzy measure.

Initial densities of the fuzzy measure are specified as follows.

Step 1. We use additional variable $p \in [0, 1]$ with initial value $p = 0.5$.

Step 2. For all characteristics $x_j \in X$ the algorithm calculates densities of the fuzzy measure as follows: $g_X(x_i) = p \cdot w_i$.

Step 3. For the increasing set of subsets $\{x_1, ..., x_j\} = A_j \subseteq X, j = 1, ..., 7$ the algorithm consistently calculates values of fuzzy measures $g_X(A_j)$ according to the formula:

$$g_X(A_j) = g_X(A_{j-1}) + g_X(x_j) + \lambda \cdot g_X(A_{j-1}) \cdot g_X(x_j). \tag{3}$$

Step 4. If $g_X(A_j) > 1$ for $j < 7$, then the algorithm decreases the value $p \in [0, 1]$ and proceeds to the Step 2.

If $g_X(A_j) < 1$ for $j = 7$, then the algorithm increases the value $p \in [0, 1]$ and proceeds to the Step 2.

In case of $g_X(A_j) = 1$, the algorithm is stopped.

With the help of considered algorithm we have formed the fuzzy measure, shown in Figure 2.

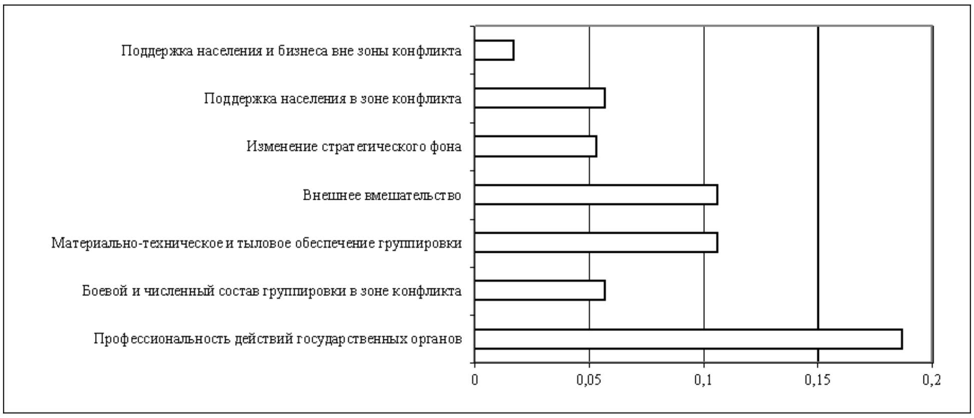

Figure 2: The fuzzy measure of the importance of characteristics $x_j \in X$

<u>Subproblem 3.</u> Choice of the observation channels.

For the assignment of values by various characteristics $x_j \in X$ of the situation $\omega \in \Omega$ we used the single observation channel [4], constructed on the basis of "desirability" curve of Harrington (see Figure 3), as shown in Table 2. In our case, all characteristics of the situation are defined on the same set of linguistic estimations, what greatly simplifies the work for experts. The use of this observation channel is described below.

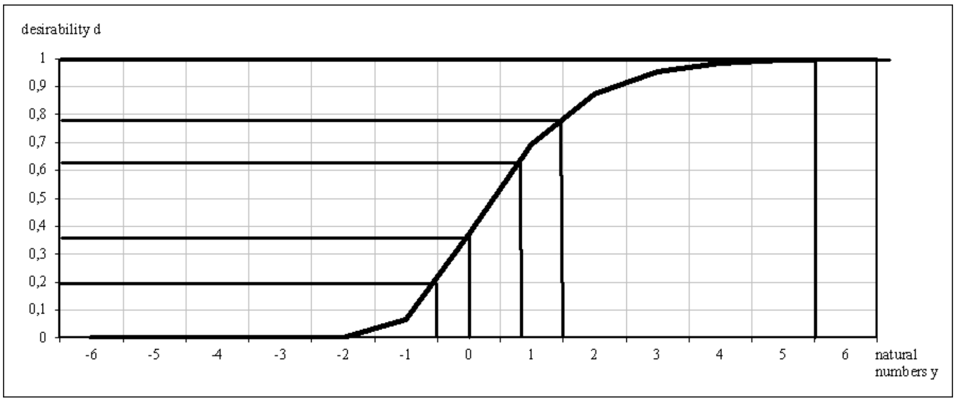

Figure 3: "desirability" curve of Harrington $d = e^{-e^{-y}}$ [17]

Table 2: Channel observation parameters

k	Linguistic estimation of the characteristic value	Value of Harrington's curve	Numeric estimation of the characteristic value $m(b_k)$	Set of estimations $B = \{b_k, k = 1, ..., 5\}$
1	Excellent	$0.8 - 1$	1	b_1
2	Good	$0.63 - 0.79$	0.79	b_2
3	Satisfactory	$0.37 - 0.62$	0.62	b_3
4	Bad	$0.2 - 0.36$	0.36	b_4
5	Very bad	$0 - 0.19$	0.19	b_5

Sub-problem 4. Constructing the transformation algorithm of entranceestimations of situation characteristics into generalized estimation.

With the help of the observation channel, we need to form the distribution of confidence $\mu(x_i|\omega_t) : X \to [0, 1]$ that describes the value of each characteristic $X = \{x_j\}, j = 1, ..., 7$ in the situation $\omega_t \in \Omega$.

An estimation of each characteristic should be executed by several (from 3 to 7) experts N who having approximately equal qualification. Answers of each expert are represented by means of the following function:

$$o(b_k) = \begin{cases} 1, & \text{if choice } b_k, \\ 0, & \text{if choice } b_p, p \neq k \end{cases}. \tag{4}$$

In this case the distribution of confidence $\mu(x_i|\omega_t) : X \to [0, 1]$ is calculated by means of the following formula:

$$\mu(x_i|\omega_t) = \sqrt[N]{\prod_{n=1}^{N} \left\{ \sum_{k=1}^{5} o_n(b_k) \cdot m(b_k) \right\}}. \tag{5}$$

Using this distribution of confidence and the fuzzy measure of importance $g_X(\cdot)$, value of criterion $J(\omega_t) \in [0, 1]$ can be calculated by means of fuzzy integral [36]:

$$J(\omega_t) = (s) \int_X \mu(x_i|\omega_t) \circ g_X(\cdot), \tag{6}$$

where $(s) \int_X$ is a designation of fuzzy Sugeno integral on characteristics set X.

For determination of the outcome $y_m \in Y, m = 1, 2, 3$ of an active phase of the conflict we used procedure $Int : [0, 1] \to Y$ of linguistic interpretation of the criterion value $J \in [0, 1]$. We remind you that the outcomes $y_m \in Y, m = 1, 2, 3$ are ordered by a successfulness degree. For each outcome $y_m \in Y, m = 1, 2, 3$, we determined the area of the criterion values $J_{y_m} \in [0, 1]$ with taking into account the following condition:

$$\forall m, l = 1, 2, 3, m \neq l, J_{y_m} \cap J_{y_l} = \emptyset, \bigcup_{m=1,2,3} J_{y_m} = [0, 1]. \tag{7}$$

These areas of values are determined as follows: $J_{y_1} = [0.5, 1]$, $J_{y_2} = [0.25, 0.5)$, $J_{y_3} = [0, 0.25)$. For each of the outcomes $y_m \in Y, m = 1, 2, 3$, the area of criterion values J_{y_m} is assigned with the help of the following characteristic function:

$$\kappa(y_m|\omega_t) = \begin{cases} 1 & J(\omega_t) \in J_{y_m} \\ 0 & J(\omega_t) \notin J_{y_m} \end{cases}. \tag{8}$$

Then the estimation of the most possible outcome from the set $y_m \in Y, m = 1, 2, 3$ for the situation $\omega_t \in \Omega$ is determined by dependence:

$$y^*(\omega_t) = arg \max_{y_m \in Y} \kappa(y_m|\omega_t). \tag{9}$$

4. USING THE ALGORITHM IN PRACTICE

To use the proposed algorithm in practice, first, it is necessary to determine the estimation period (the periodicity of issuing the estimation results), which depends on the decision-making needs, but the collection of input data must be continuous. We suggest using the algorithm as follows.

1. Experts study the content of information messages from available sources: foreign media, government, local authorities; the media of the opposing states; non-governmental organizations, bloggers and others. The main purpose of the studying is to identify events that may affect the result of the active phase of a military conflict.
2. Experts distribute the identified events according to the characteristics, which are indicated in Table 1.
3. For each characteristic, experts estimate the impact of the entire set of events collected over a specified period. The estimating is carried out using the linguistic scale shown in Table 2.
4. Experts enter estimates of all characteristics into the proposed algorithm.
5. The algorithm converts linguistic estimates into numerical estimates using the Harrington function in accordance with Table 2.
6. The algorithm generalizes opinions of all experts in accordance with formula (5).
7. The algorithm calculates the estimation of the main criterion by generalizing the partial estimations in accordance with formula (6).
8. The algorithm converts the estimation of the main criterion into a linguistic estimation in accordance with the formula (9).

The solution of the proposed algorithm is carried out at the end of each estimation period. To analyze trends in the active phase of the conflict, the set of estimations of the main criterion can be presented in the form of a graph.

5. NUMERICAL EXAMPLE OF USING THE PROPOSED ALGORITHM

We used the proposed algorithm to analyze the first active phase of the military conflict in Ukraine in early July 2014.

In early 2014, on Eastern Ukraine, the militants began to create illegal armed formations (IAF), which were supported by the Russian Federation. They have paralyzed activity of regional authorities in Donetsk and Lugansk regions. In response to it, Ukraine began antiterrorist operation. In this operation on the side of the Ukraine's Government have participated the armed formations of armed forces and law enforcement agencies (governmental armed formations - GAF). At that time, the continuous front line practically did not exist. Armed collisions between IAF and GAF had no system character, even though, were fierce. Confrontational actions in a civil society and on international scene have prevailed among all actions. The analysis of military actions only did not allow investigating this conflict adequately. It also caused complications in using classical methods for the analysis of the current military-political situation. Therefore, we have created a special algorithm which has been described above.

With the help of this algorithm we have been estimating the evolution of the conflict since 1/7/2014 on a daily basis. The beginning of the analysis coincides with the declaration of the new President of Ukraine about necessity of establishing the order by force. Below, we have shown the calculations results for 18 days of observation (up to 18/7/2014). At that moment, our calculations predicted the termination of the first active phase of the conflict.

Estimations of characteristics of the current situation.

Estimations of characteristics of the situation $\mu(x_i|\omega_t) : X \to [0,1]$ are shown in Table 3.

Table 3: Estimations of characteristics of the current situation

Date	Characteristic						
	x_1	x_2	x_3	x_4	x_5	x_6	x_7
01.07.14	0.924	1.000	0.672	0.517	0.729	0.620	0.790
02.07.14	0.924	1.000	0.620	0.517	0.620	0.620	0.924
03.07.14	1.000	1.000	0.729	0.620	0.517	0.729	0.924
04.07.14	1.000	1.000	0.729	0.517	0.432	0.620	0.790
05.07.14	1.000	1.000	1.000	0.517	0.729	0.790	0.729
06.07.14	1.000	1.000	1.000	0.620	0.924	0.924	0.729
07.07.14	1.000	1.000	1.000	0.620	0.924	0.924	0.620
08.07.14	0.924	1.000	0.855	0.620	0.729	0.790	0.790
09.07.14	0.790	0.924	0.790	0.620	0.620	0.729	0.790
10.07.14	0.790	0.924	0.790	0.729	0.620	0.729	0.790
11.07.14	0.729	0.924	0.790	0.729	0.517	0.517	0.924
12.07.14	0.620	0.924	0.729	0.729	0.517	0.517	1.000
13.07.14	0.620	0.790	0.729	0.729	0.517	0.432	1.000
14.07.14	0.517	0.790	0.729	0.729	0.432	0.432	1.000
15.07.14	0.620	0.790	0.790	0.729	0.672	0.432	1.000
16.07.14	0.620	0.729	0.729	0.729	0.432	0.432	0.924
17.07.14	0.620	0.620	0.620	0.790	0.432	0.432	0.924
18.07.14	0.432	0.517	0.517	0.855	0.620	0.360	0.790

Analysis of the successfulness of the first active phase from the view-point of GAF.

During the first active phase of the military conflict, GAF launched an offensive in eastern Ukraine. With the help of fuzzy integral (6) it has been calculated the values of the criterion $J \in [0,1]$, which describes the successfulness of the offensive from the view-point of GAF. These values are shown in Figure 4.

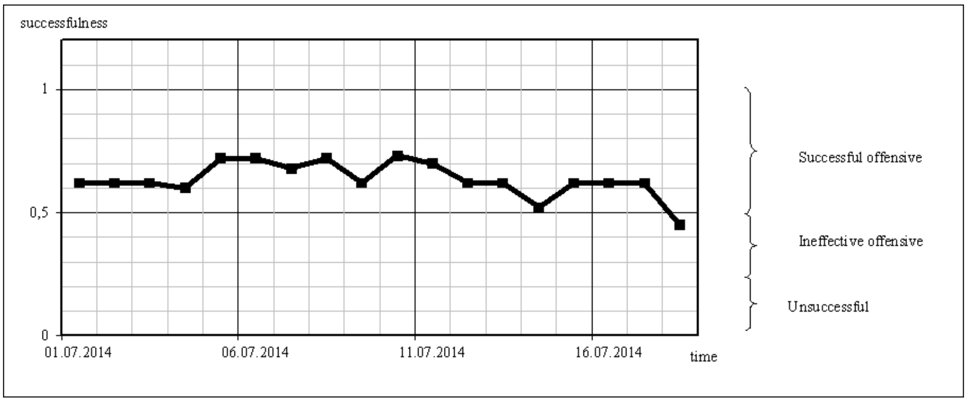

Figure 4: The dynamics of successfulness of the offensive from the view-point of GAF

The analysis of the dynamics of the conflict evolution shown the following. In the early days, the estimation of GAF's actions was in the "Successful offensive" area, although it did not exceed the level of 0.6. On 7/7/2014, this estimate increased, but after a short period of fluctuations, a falling trend arose. On 18/7/2014, the estimation of GAF's actions entered the area "Inefective offensive."

Thus, on 18/7/2014 it was possible to assert that the purpose of GAF (the defeat of IAF) would not be achieved. This conclusion was made despite the fact that at that time GAF was conducting successful operations and restoring the control over many settlements. In particular, on 22/7/2014, GAF restored the control above Popasna, Rubegne, Dzerzhinsk, Lisichansk, Soledar. On 27/7/2014, GAF tried to restore the control over Shahtersk and to finish encirclement of Donetsk. In other words, the conclusion made at that time was not completely obvious. However, further conflict evolution completely confirmed a conclusion about unsuccessfulness of the offensive. On 28/7/2014, IAF repelled the offensive near the Saur-Mogila, Shahtersk, and Gorlovka, and on 10/8/2014, IAF conquered Ilovaisk. The unsuccessful offensive of GAF caused the signing of the first Minsk agreements in early September, 2014.

Taking into account this fact of prediction, we have assumed that the proposed approach is predictive. Moreover, the proposed algorithm also allowed to estimate dynamics of partial characteristics of the situation and to analyze the possible reasons of unsuccessful offensive of GAF.

The analysis of characteristics which describe the military actions.

Figure 5 shows the dynamics of the characteristics x_2 and x_3.

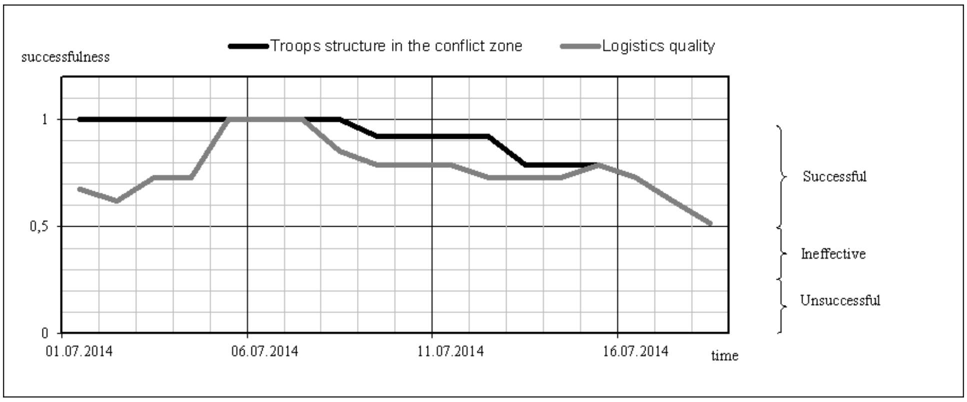

Figure 5: The dynamics of the characteristics estimations which describe the military actions

In early July, 2014 the structure of GAF troops was sufficient to conduct successful offensive operations. The volume of mobilization resources was also sufficient, due to the participation in the conflict of voluntary military formations. However, GAF logistics left much to be desired. By the end of the first ten days of July, a slight improvement of conditions of logistics was observed owing to the improvement of work of support services and considerable help from volunteers. However, by the middle of July, the successfulness of GAF actions tended to deteriorate. It was connected with significant losses in large settlements. Later on, the unsuccessful actions in the offensive and heavy losses of GAF near Shahtersk, Zelenopilja, Ilovaisk worsened the structure of GAF troops and disturbed logistics. It was the main cause of ineffective offensive.

The analysis of characteristics which describe external environment.

Figure 6 shows the dynamics of the characteristics x_4 and x_5.

From the view-point of GAF, in early July, the external strategic background had positive dynamics. In that time strategic partners were maintaining Ukraine not only in political and diplomatic, but also in military-technical sphere. However, after the first failures of GAF, the strategic background began to deteriorate, although partner support remained favorable and even increased.

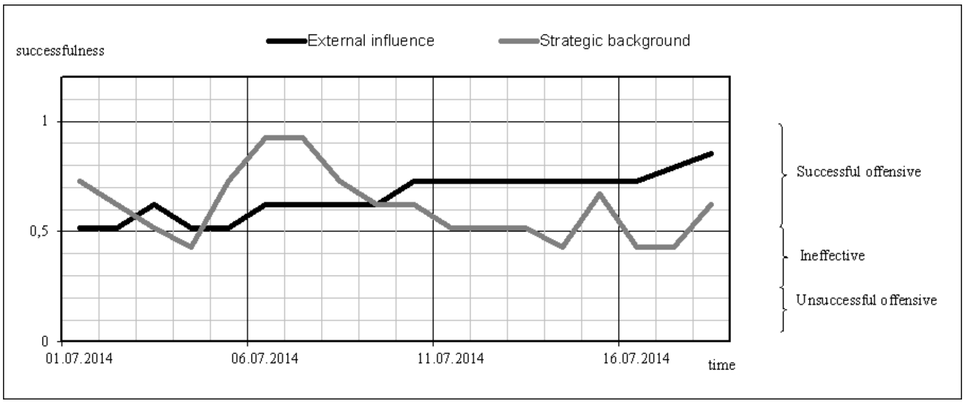

Figure 6: The dynamics of the characteristics which describe external environment

The analysis of characteristics which describe internal environment.

Figure 7 shows the dynamics of the characteristics x_1, x_6 and x_7.

Internal processes in the country also essentially influenced the outcome of the active phase of the conflict.

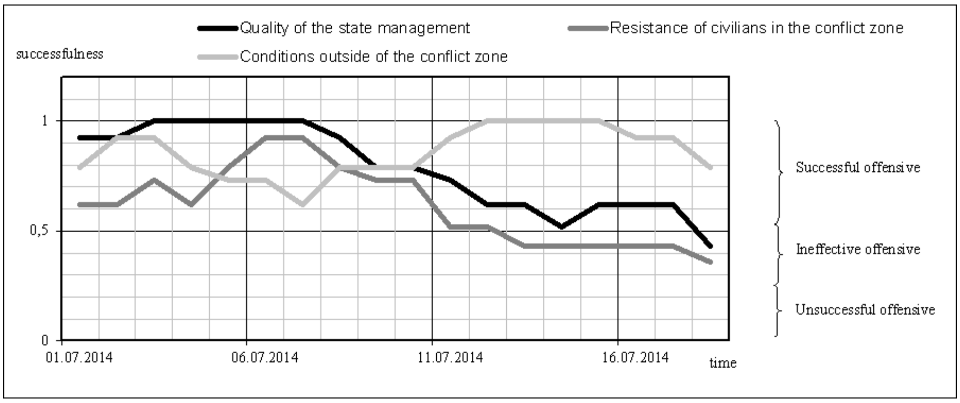

Figure 7: The dynamics of the characteristics which describe internal environment

In the conflict area, the civilians increased their resistance to GAF actions (characteristic x_6). In the capital and big cities, the civilians increased their support to GAF actions due to the patriotic moods (characteristic x_7). However, in general, the level of contradictions within the society started to grow. The actions of the State management (characteristic x_1) were not always professional as well.

6. DISCUSSION OF THE RESULTS

We want to discuss the results of our research from the view-point of the following questions.

Question 1. What are the advantages of the Sugeno fuzzy integral over other generalization methods?

To date, there are a lot of generalization methods, and it is not possible to compare the used approach with all known methods. Therefore, we limited ourselves to comparison with the most common method of maximin composition, having previously presented the measure of the characteristics importance as a measure of possibility in accordance with the work [11]. Figure 8 shows the dynamics of offensive successfulness calculated using the Sugeno fuzzy integral (see figure 4) and maximin composition.

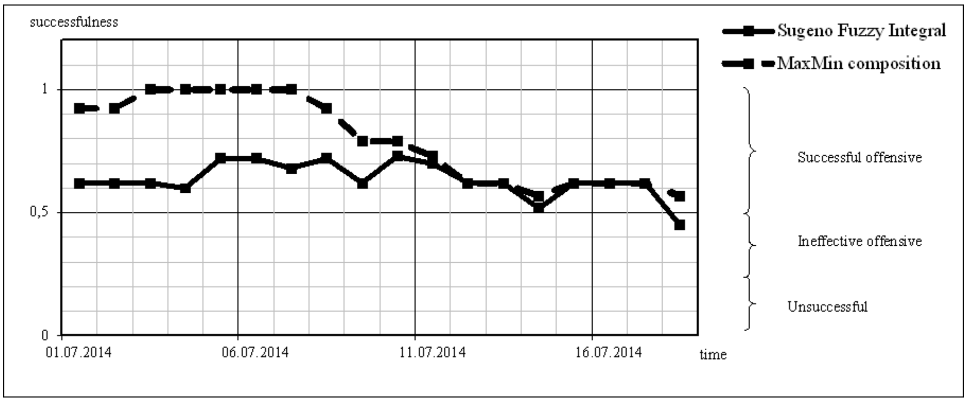

Figure 8: The dynamics of offensive successfulness calculated using the Sugeno fuzzy integral and maximin composition

The analysis shows that the estimate calculated using the maximin composition is less sensitive and does not have the property of prediction. This can be explained from the view-point of modal logic, which interprets possibility as an extreme case of a plausibility function, that is, a case when nothing prevents the occurrence of an event.

Question 2. Is it possible to extend the results to other types of military conflicts?

Modern conflicts arise suddenly, government bodies and the armed forces do not have time to prepare and create reserves. In such conditions, the state is unable to wage a long, large-scale war. Since military resources are insufficiently and government policy is very sensitive to losses, non-military tools will be used to ensure success. Therefore, modern military conflicts are often hybrid. In addition, due to insufficient resources, the possibility of success in such conflicts, as a rule, will be limited by the initial phases. If victory is not achieved after active initial phases, the parties, as a rule, freeze the conflict. In the freezing phase, the conflict acquires a different logic of development - the parties use non-military instruments and transfer the arena of confrontation to the international level. Therefore, the proposed algorithm can be used to estimate the outcome, as a rule, of the initial active phases of hybrid conflicts. For the analysis of conflicts of other types or other phases, the proposed approach requires revision. Since hybrid conflicts are usually geographically limited, local differences will significantly influence the outcome. Therefore, the composition of the set of characteristics must be revised in accordance with the conditions of concrete conflict.

Question 3. Is it difficult in practice to ensure the adequacy of the input data?

The adequacy of the estimating results directly depends on the adequacy of the input data that are prepared by experts. The proposed algorithm assumes the participation of 3 to 7 experts, who must have approximately equal qualifications. This limitation is determined by the properties of arithmetic convolution in formula (5) and is similar to the limitation of the analytic hierarchy process [33]. In our problem, we use 7 characteristics for estimation. These characteristics can be conditionally called the areas of experts competence. If each expert is competent in only one area, then 21 to 49 experts are required to solve the problem. Therefore, it is difficult to strictly fulfill this condition and find such a number of specialists in practice. To mitigate this condition, one can try to develop another procedure for generalizing experts estimations. However, this requires additional research.

Question 4. How can the predictive properties of the proposed approach be explained?

The proposed approach involves the preparation of input data based on the processing of information messages. The flow of information messages has some property that we called predictive potential. At the beginning of the military conflict in eastern Ukraine, quite a few messages contained information about events that influenced the result of hostilities not immediately, but in the future, after some time. Therefore, the estimation of the current situation, obtained taking into account these events, acquired a predictive character. At the same time, this property of the proposed approach requires further study in the framework of a special research.

Question 5. How can we improve the proposed approach?

In the problem, we use a five-point scale to convert linguistic estimations into numerical estimations (table 2). It is possible to increase the number of scale gradations. This will increase the accuracy of estimates, but complicate the work of experts. It is also possible to interpret each linguistic gradation for each of the characteristics of the current situation, that is, to develop guidelines for assigning estimations by experts. If necessary, it is possible to clarify the characteristics of situations by expanding their hierarchy. For example, it is possible to consider the quality of logistics separately in the Army, Air Force, and Naval Forces. It is also possible to organize information support for the algorithm so that experts transfer their formalized estimates of partial characteristics from remote workplaces. Thus, the proposed algorithm can become part of your automated decision-making system.

7. CONCLUSIONS

The analysis of the modern military conflicts has shown that they are accompanied by the elements of hybrid influences. Moreover, often the state does not participate in the conflict directly, and uses the illegal armed formations. Such conflicts arise suddenly and have a high degree of evolutionary uncertainty. A key phase of evolution of the modern conflict is the active phase. This phase determines the conflict outcome. The estimation of the outcome of an active phase of the military conflict should take into account not only the characteristics of the armed struggle, but the multitude of other characteristics as well, inasmuch, the conflict participants widely use non- military resources to achieve own goals. This is the specificity of the modern hybrid military conflict.

The uncertainty of data on characteristics of modern military conflicts significantly limits the use of known methods. Therefore, we have used the methods of fuzzy-integral calculus to present the input data and generalize estimates of characteristics of the current situation. As the main criterion of the estimation, we have chosen the successfulness of actions of the participant that is attacking. We have also defined structure of the estimation standard (etalon) and identified its characteristics, chosen the observation channel of these characteristics and constructed algorithm calculation of the main criterion. Interpretation of the value of this criterion has allowed estimating the outcome of an active phase of the conflict.

With the help of proposed algorithm, we investigated the first active phase of the conflict on Eastern Ukraine during 1/7/2014 - 18/7/2014. We were estimating the current situation during that phase on a daily basis. By 18/7/2014 our analysis shown that the offensive's purpose of the governmental armed formations would not be achieved, though the current situation was not negative and this conclusion was not obvious. However, further evolution of the conflict confirmed our conclusion. We also discussed the limitations regarding the use of the obtained results and the directions for their improvement.

We draw attention to the following advantages of our approach:

quantitative measurement of the outcome of the active phase of a military conflict;

visual representation of the success of the active phase of the conflict improves the conditions for decision-making;

estimation of the outcome of the active phase may have the property of prediction;

full consideration of all quantitative and qualitative factors that affect the outcome of the active phase helps to describe the hybridity of modern conflicts;

simplicity of describing the input data - we do not ask the expert additional questions and do not require additional training;

the formalization of uncertainty allows for the interaction of criteria, since the criteria weights are presented as fuzzy measures.

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