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Research Article

PARAMETERIZATION OF MULTI-DECISIVE HYPERSOFT SET UNDER FUZZY ENVIRONMENT WITH APPLICATION IN MULTI-ATTRIBUTE DECISION MAKING

Muhammad IHSAN
University of Management and Technology Lahore 54000, Pakistan
mihkhb@gmail.com, ORCID: 0000-0001-6107-5493

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Abstract: Fuzzy parameterized hypersoft expert set (FPHSE-set) is the generalisation of fuzzy parameterized soft expert set (FPSE-set). The FPHSE-set overcomes the shortcomings of FPSE-set for the examination of multi-argument approximate function. The FPHSE-set takes on the real-world situation where each attribute is actually supposed to be further categorised. Each category contains non matching sub-attribute valued disjoint set with the use of multi-argument approximate function. The FPHSE-set is more adaptable and dependable in the decision-support system with the comprehensive analysis of attributes. As a result, the primary goal of this paper is to describe how to characterise the FPHSE-set using set-theoretic, axiomatic, and algorithmic approaches. Two algorithms are suggested to investigate the proposed model's function in decision-making while coping with a real-world event of computer system selection for a bank office in order to validate it. A comparative comparison between the suggested approach and a few relevant, current methods is used to assess the advantages of the proposed strategy.

Keywords: Soft set, soft expert set, hypersoft set, hypersoft expert set, decision-making, optimization.

MSC: 03B52; 03E72; 52A01.

1. INTRODUCTION

In order to address the shortcomings of F-sets with regard to the availability of a parameterization tool, Maji et al. [1] created the notion of fuzzy soft set (FS-set) as a generalisation of fuzzy set [2] (F-set) and soft set [22] (S-set). In addition to validating F-sets, FS-set satisfies S-set requirements. Instead of using a power set, it essentially employs the collection of F-subsets. This collection can be taken as a set of single-argument approximate functions. These functions are used throughout the domain of discourse. Hussain and Ullah [3] offered a cutting-edge method for managing ambiguity and imprecision in a substantial body of human opinion in a decision support system. Asif et al. [4] introduced different types of operators by using the structure of the Pythagorean fuzzy set for the purpose of decision-making methods. Ghazi et al.[5] looked for the most crucial factor in demonstrating the role of women in sports. A review of research, and specialists are conducted. While the direct communication with those connected to the sports industry are used to narrow down the pertinent criteria. Li and Yang [6] looked at the issue of disturbing fuzzy multi-attribute decision-making (MADM), where the disturbing fuzzy numbers represent the attribute weight,and information. Yue et al. [7] developed a unique algorithm to determine the weight of the group consistency characteristic. They also suggested a multi-attribute personnel-position matching group decision-making process. Based on the expertise of specialists, Ferreira et al. [8] built multi-attribute models that handle both quantitative and qualitative data simultaneously. In some situations when attribute weights are uncertain and attribute values fluctuate within the range of the Pythagorean fuzzy set. To handle this situation, Rasool et al. [9] proposed a gray relational analysis projection. For the innovative multi-criteria decision-making (MCDM) technique, Uluçay et al. [10] used a fuzzy method based on the Q-single-valued neutrosophic sets.

The suggested strategy incorporates a key mathematical concept as an alternative approach to solving the MCDM problem. Andalib Ardakani et al. [11] created a model for classifying and detecting the variables affecting green supply chain management in Iran's large tile and ceramic sector. Donyavi Rad et al. [12] developed a mathematical model of crisis logistics planning that takes into account the issue of main and secondary crises in disaster relief. Mehmood et al. [13] developed two essential methods for taking measurements. Next, they provided vague soft sets pertaining to distinct space points. Jaikumar et al. [14] introduced the novel cardinality and integrity of the picture fuzzy soft graph (PFSG), giving an idea of the vulnerability parameters of the PFSG. Additionally, an algorithm for choosing the best site to build a city diagnosis center has been suggested utilizing this as part of a decision-making process for the PFSG. Uluçay and Şahin [15] first described the idea of an intuitionistic fuzzy soft expert graph and its specific characteristics before applying a MCDM technique. Li et al. [16] developed an innovative structure for rating methods. The antifragility analysis algorithm, which is a cutting-edge future-based situation strategy that can maximize choice outcomes, was introduced in this work. Chusi et al. [17] looked at how Africa could get the most out of trading carbon credits in order to turn conservation efforts into financial gains. Sezgin and Yavaz [18] proved that the soft binary piecewise plus operation is a coherent semigroup under certain conditions and a right-left basis. Vellapandi and Gunasekaran [19] introduced a new method of decision-making based on a multi-soft set. Smarandache [20] made changes in the soft set by giving the concept of a hypersoft set. In the midst of the COVID-19 epidemic, Pethaperumal et al. [21] made a significant contribution to the field of healthcare supply chain management by introducing a strong MADM approach for supplier evaluation. Theoretical studies on the theory of soft sets and some algebraic operations on soft sets were provided by Maji et al. [23]. The S-sets are increasingly being used in decision-making issues, texture categorization, and other domains. Some new operations on S-sets were discovered based on Maji [23] work, and some De Morgan's laws with regard to these new operations were demonstrated by Ali et al. [24]. The idea of a S-set's complement has also been developed. The idea of S-sets offers a wide range of possible applications. The S-set relations were created by Babitha and Sunil [25] as a sub S-set of the S-set's Cartesian product, and many related ideas are also covered, including identical S-set relations, division, composition, and function. In order to make Molodtsov's S-sets more useful for enhancing a number of new findings, Çağman and Enginoĝlu [26] revised their operations. Additionally, Çagman and Enginoĝlu defined S-set products and the uni-int decision function. Çağman and Enginoĝlu next created a uni-int decision-making procedure that picks a set of ideal components from the alternatives using these new definitions. Herawan and Deris [27] provided an alternate method for S-set theory based mining of maximum and regular association rules from transactional data sets. Using this concept of FS-sets, some academics have expanded many novel applications of S-set theory. On the basis of the notion of S-sets, some [28] define soft relation and fuzzy soft relation. The S-sets and the related ideas of fuzzy sets and rough sets were contrasted by Aktaçs and Çağman. [29]. The S-set and FS-set were introduced into the incomplete environment by Zou and Xiao [30]. The S-set models place emphasis on a single expert's judgement within a single model. However, there are some circumstances where different viewpoints in various models are required. A soft expert set (SE-set) was developed by Alkhazaleh et al. [31] to address the shortcomings of the S-set with regard to the opinions of various experts. Ihsan et al. [32, 33] made an extension in Alkhazaleh's work by adding the concept of convexity with its characteristics. Alkhazaleh et al. [34] additionally expanded on their previous work on the topic (DMPs). Smarandache [35] replaced the single argument estimate function with multiple argument approximate functions in 1998 to generalise the S-set to the hypersoft set (HS-set). Abbas et al. [36] made an extension in the structure of HS-set by introducing its different operations. Musa and Asaad [37] introduced some new structures of topological space on the HS-set. Kamacï [38] introduced the hybrids of HS-set and rough sets. Ajay and Charisma [39] used multi-criteria decision making with the help of alpha open HS-set. Dalklç [40] calculated the membership degree for HS-set using the range of $(0,1)$.

Fuzzy parameterized soft set (FPS-set) was conceptualised by Çağman et al. [41] and applied to parameters to a significant degree. They provided an application for the best product choice and recommended a solution to the DMPs. The S-set-like structures deal with a single expert's viewpoint within a model. However, there are some circumstances where we want various expert viewpoints in a single model. Alkhazaleh and Salleh [42] introduced the idea of SE-set in order to resolve this issue without the need of any additional operations. Tella et al. [43] gave application of multi-decision making using the structure of FPS-set. Bashir et al. [44] merged the structures of fuzzy parameterized with SE-set in order to produce hybrids of fuzzy parameterized soft expert set (FPSE-set) with application in DMPs. They compared the outcomes and talked about the implementation of Alkhazaleh and Salleh's generalised method. By converting the single set of attributes into many discontinuous attribute-valued sets in 2021, Rahman et al. [45]

extended the work of the FPS-set to FP-hypersoft set and examined the applications in DMPs. With applications in DMPs. By providing an application of DMPs, Ihsan et al. [46] expanded the work of the HS-set into the hypersoft expert set. Motivated by the work on parametrization done by various researchers above, With an application related to DMPs, a new structure of FPHSE-set is developed. The suggested study makes the following important contributions: The literature looks at the definitions of the hypersoft set, hypersoft expert set, and fuzzy hypersoft expert set. Fuzzy parameterized theory, the concept of the hypersoft expert set, which includes its axiomatic features, set-theoretic operations, and laws, is supported by numerical illustrative examples. Two algorithms are put forth, and they are then proven to work by using them to solve real-world decision-making problems. Its comprehensive structural comparison with some pertinent existing structures is discussed, and its generalization is discussed by describing some of its particular cases, in order to demonstrate the dependability and flexibility of the proposed model i.e., FPHSE-set. To assess the benefits of the proposed study, it is compared to similar models that are already in use. To encourage readers to request more extensions, the paper is summarised with a description of its goals and future objectives. The pattern of rest of the paper is ordered as section 2 is about the description of some necessary notions of soft set and hypersoft sets. Section 3 has been constructed for the description of purely new concept of FPHSE-set along with its important operations. While the purpose of section 4 is the use of purposed structure in decision making problems by discussing an example related to daily life situation. Another important section 5 has been discussed by taking the concept of weighted fuzzy parameterization. Comparison analysis and discussion have been done in sections 6 and 7. The last section is devoted to conclusions and future directions for research.

2. PRELIMINARIES

The basic concepts of hypersoft expert set theory are reviewed in this section of the article. The $\mathscr{I}$ and are the specialists and parameters, respectively, in this article. The $\mathscr{P}$ is the product of $\times\mathscr{I}\times\mathscr{U}$, where $\mathscr{U}$ subsets of $\mathscr{P}$. Although $\mathscr{U}$ is a collection of judgments, i.e. $\mathscr{U}$ = $\{0, 1\}$, $\hat{\Delta}$ is used for the universe of discourse.

Definition 1. *[22] The pair $(\Upsilon_s, \mathscr{A}))$ is referred to as the soft set on the $\hat{\Delta}$. The function $\Upsilon_s : \mathscr{A} \rightarrow P(\hat{\Delta})$ is its approximate function and $\mathscr{A}$ are utilised as a subset of , respectively.*

Definition 2. *[35] Let $\c{C}_1, \c{C}_2, \c{C}_3, ..., \c{C}_l$, for $l > 1$ be distinct attributes, while the $\infty_1, \infty_2, \infty_3, ..., \infty_l$ are non-overlapping attributed-valued sets corresponding these attributes. Then the pair $(\eta, \mathfrak{G})$, where $\mathfrak{G} = \infty_1 \times \infty_2 \times \infty_3 \times ... \times \infty_l$ and $\eta : \mathfrak{G} \rightarrow P(\hat{\triangle})$ is named as a hypersoft set over $\hat{\triangle}$.*

Definition 3. *[46] A hypersoft expert set $\bowtie$ is defined by as $\bowtie : \propto \rightarrow P(\hat{\triangle})$ where $\propto \subseteq \breve{A} = \times\mathscr{I}\times\mathscr{U}$ and $=_1 \times_2 \times_3 \times ... \times_n$, while $_{1,2,3}, ..., _n$ are different parametric valued sets corresponding to n different parameters $q_1, q_2, q_3, ..., q_n$.*

Definition 4. *[45] Let the non-overlapping sub-parametric valued sets for different parameters a_i; i = 1,2,...,n are shown by $\mathfrak{U} = \{\mathfrak{U}_1, \mathfrak{U}_2, \mathfrak{U}_3, ...\mathfrak{U}_n\}$ respectively. A fuzzy parameterized hypersoft set $\Psi_{\mathfrak{Z}}$ over $\hat{\triangle}$ is defined as*

$\Psi_{\mathfrak{I}} = \{(\hat{g}/\zeta_{\mathfrak{I}}(\hat{g}), \psi_{\mathfrak{I}}(\hat{g})) : \hat{g} \in, \psi_{\mathfrak{I}}(\hat{g}) \in P(\hat{\triangle}), \zeta_{\mathfrak{I}}(\hat{g}) \in \mathbb{I} = [0,1]\}$ *and also* $= \mathfrak{U}_1 \times \mathfrak{U}_2 \times \mathfrak{U}_3 \times \ldots \times \mathfrak{U}_n$ *and* $\mathfrak{I}$ *is a fuzzy set over and membership function of FPHS-set is* $\zeta_{\mathfrak{I}} :\rightarrow \mathbb{I}$. *While the approximate function of FPHS-Set is* $\psi_{\mathfrak{I}} :\rightarrow P(\hat{\triangle})$.

Definition 5. *[45] Let* $\eta_{\mathscr{F}_1}$ *and* $\eta_{\mathscr{F}_2}$ *be the two FPHS-sets over* $\hat{\triangle}$, *then* $\eta_{\mathscr{F}_1} \subseteq \eta_{\mathscr{F}_2}$ *if* $\zeta_{\mathfrak{I}_1}(\hat{g}) \leq \zeta_{\mathfrak{I}_2}(\hat{g})$ *and* $\psi_{\mathfrak{I}_1}(\hat{g}) \subseteq \psi_{\mathfrak{I}_2}(\hat{g})$.

Definition 6. *[45] Let* $\eta_{\mathscr{F}_1}$ *and* $\eta_{\mathscr{F}_2}$ *be the two FPHS-sets over* $\hat{\triangle}$, *then* $\eta_{\mathscr{F}_1} = \eta_{\mathscr{F}_2}$ *if* $\zeta_{\mathfrak{I}_1}(\hat{g}) = \zeta_{\mathfrak{I}_2}(\hat{g})$ *and* $\psi_{\mathfrak{I}_1}(\hat{g}) = \psi_{\mathfrak{I}_2}(\hat{g})$.

Definition 7. *[45] Let* $\eta_{\mathscr{F}_1}$ *and* $\eta_{\mathscr{F}_2}$ *be the two FPHS-sets over* $\hat{\triangle}$, *then their union is defined by* $\eta_{\mathscr{F}_1} \cup \eta_{\mathscr{F}_2}$, *for all* $\hat{g} \in$ *and* $\psi_{\mathfrak{I}_1}(\hat{g}) \cup \psi_{\mathfrak{I}_2}(\hat{g}) = \psi_{\mathfrak{I}_3}(\hat{g})$ *with*

1. $\zeta_{\psi_{\mathfrak{I}_1} \cup \psi_{\mathfrak{I}_2}}(\hat{g}) = max\{\zeta_{\psi_{\mathfrak{I}_1}(\hat{g})}, \zeta_{\psi_{\mathfrak{I}_2}(\hat{g})}\}$
2. $\psi_{\mathfrak{I}_3}(\hat{g}) = \begin{cases} \psi_{\mathfrak{I}_1}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_1} \setminus \psi_{\mathfrak{I}_2} \\ \psi_{\mathfrak{I}_2}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_2} \setminus \psi_{\mathfrak{I}_1} \\ \psi_{\mathfrak{I}_1}(\hat{g}) \cup \psi_{\mathfrak{I}_2}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_1} \cap \psi_{\mathfrak{I}_2}. \end{cases}$

Definition 8. *[45] Let* $\eta_{\mathscr{F}_1}$ *and* $\eta_{\mathscr{F}_2}$ *be the two FPHS-sets over* $\hat{\triangle}$, *then their intersection is* $\eta_{\mathscr{F}_1} \cap \eta_{\mathscr{F}_2}$, *defined by for all* $\hat{g} \in$ *and* $\psi_{\mathfrak{I}_1}(\hat{g}) \cap \psi_{\mathfrak{I}_2}(\hat{g}) = \psi_{\mathfrak{I}_3}(\hat{g})$

1. $\zeta_{\psi_{\mathfrak{I}_1} \cap \psi_{\mathfrak{I}_2}}(\hat{g}) = min\{\zeta_{\psi_{\mathfrak{I}_1}(\hat{g})}, \zeta_{\psi_{\mathfrak{I}_2}(\hat{g})}\}$
2. $\psi_{\mathfrak{I}_3}(\hat{g}) = \begin{cases} \psi_{\mathfrak{I}_1}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_1} \setminus \psi_{\mathfrak{I}_2} \\ \psi_{\mathfrak{I}_2}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_2} \setminus \psi_{\mathfrak{I}_1} \\ \psi_{\mathfrak{I}_1}(\hat{g}) \cap \psi_{\mathfrak{I}_2}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_1} \cap \psi_{\mathfrak{I}_2}. \end{cases}$

3. PROMINENT PROPERTIES OF THE METHODOLOGY

This section contain different parts of the methodology that contain valuable techniques. It has been shown in Figure 1.

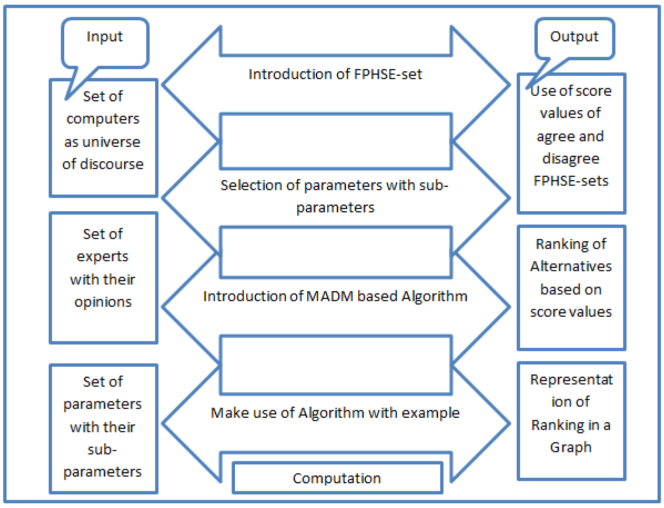

Figure 1: An adopted methodology of the paper

3.1. Fuzzy Parameterized Hypersoft Expert Sets (FPHSE-set)

In this section, a completely new structure of FPHSE-set is established with the help of already existing concept of FPHSE-set and some of its major characteristics like union, intersection, compliment and laws are discussed with the help of examples. To clearly articulate the rationale for selecting specific criteria for the model, consider breaking down and explaining the following key points:

- The FPHSE-set was selected as it addresses the shortcomings of the FPSE-set, particularly with regard to the analysis of multi-argument approximate functions. This indicates that it is capable of handling more complicated scenarios than FPSE-sets.
- An extension of the FPSE-set is the FPHSE-set. It was chosen because it enables the categorization of each feature into the relevant sub-attributes, making it appropriate for examining the attributes and sub-attributes of real-world situations.
- Because the FPHSE-set allows for the thorough examination of numerous variables and their interconnections, it provides decision-support systems with increased flexibility and dependability. Because of this, it can be a useful tool to support complex processes of decision-making.

The body of existing literature is insufficient to offer a mathematical model that addresses each of the aforementioned scenarios separately. This deficiency serves as the study's inspiration. All of the scenarios listed above can be handled together by the suggested model, the FPHSE-set, as one framework.

Definition 9. *Let non-overlapping sub-parametric valued sets for different parameters a_i; i = 1,2,...,n are shown by $\mathfrak{U} = \{\mathfrak{U}_1, \mathfrak{U}_2, \mathfrak{U}_3, ...\mathfrak{U}_n\}$ respectively. A fuzzy parameterized hypersoft expert set $\Psi_{\mathscr{F}}$ over $\hat{\triangle}$ is defined as*
$\Psi_{\mathscr{F}} = \left\{(\hat{g}/\zeta_{\mathfrak{I}}(\hat{g}), \psi_{\mathfrak{I}}(\hat{g})) : \hat{g} \in \mathscr{P}, \psi_{\mathfrak{I}}(\hat{g}) \in P(\hat{\triangle}), \zeta_{\mathfrak{I}}(\hat{g}) \in \mathbb{I}\right\}$ *and* $\mathscr{P} = \times \mathscr{I} \times \mathscr{U}$.
Also $= \mathfrak{U}_1 \times \mathfrak{U}_2 \times \mathfrak{U}_3 \times ... \times \mathfrak{U}_n$ *and $\mathfrak{I}$ is a fuzzy set over $\mathscr{P}$ and membership function of FPHSE-set is $\zeta_{\mathfrak{I}} : \mathscr{P} \to \mathscr{I}$. While the approximate function of FPHSE-set is $\psi_{\mathfrak{I}} : \mathscr{P} \to P(\hat{\triangle})$.*

Example 10. *Imagine that a chain of colleges is looking for a construction company to modernise the campus to keep up with globalisation and requires the advice of some specialists. Let $\hat{\triangle} = \{\tilde{œ}_1, \tilde{œ}_2, \tilde{œ}_3, \tilde{œ}_4, \tilde{œ}_5, \tilde{œ}_6\}$ be a set of construction company, let*
$\{\vartheta_1/0.2, \vartheta_2/0.4, \vartheta_3/0.5, \vartheta_4/0.8, \vartheta_5/0.7, \vartheta_6/0.1, \vartheta_7/0.3, \vartheta_8/0.9\}$ *be the fuzzy subset of I(a set of fuzzy subsets of) and $\check{\mathscr{J}}_1 = \{p_{11}, p_{12}\}$, $\check{\mathscr{J}}_2 = \{p_{21}, p_{22}\}$, $\check{\mathscr{J}}_3 = \{p_{31}, p_{32}\}$, be disjoint attributive sets for distinct attributes p_1= quality characteristics, p_2= cheap, p_3= quality of design. Now $\mathscr{G} = \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2 \times \check{\mathscr{J}}_3$*

$$= \left\{ \begin{array}{l} \vartheta_1/0.2 = (p_{11}, p_{21}, p_{31}), \vartheta_2/0.4 = (p_{11}, p_{21}, p_{32}), \vartheta_3/0.5 = (p_{11}, p_{22}, p_{31}), \\ \vartheta_4/0.8 = (p_{11}, p_{22}, p_{32}), \vartheta_5/0.7 = (p_{12}, p_{21}, p_{31}), \vartheta_6/0.1 = (p_{12}, p_{21}, p_{32}), \\ \vartheta_7/0.3 = (p_{12}, p_{22}, p_{31}), \vartheta_8/0.9 = (p_{12}, p_{22}, p_{32}) \end{array} \right\}.$$

Now $\mathscr{H} = \mathscr{G} \times \mathscr{I} \times \mathscr{U}$

$$\mathscr{H} = \left\{\begin{array}{l} (\vartheta_1/0.2,s,0),(\vartheta_1/0.2,s,1),(\vartheta_1/0.2,t,0),(\vartheta_1/0.2,t,1),(\vartheta_1/0.2,u,0),\\ (\vartheta_1/0.2,u,1),(\vartheta_2/0.4,s,0),(\vartheta_2/0.4,s,1),(\vartheta_2/0.4,t,0),(\vartheta_2/0.4,t,1),\\ (\vartheta_2/0.4,u,0),(\vartheta_2/0.4,u,1),(\vartheta_3/0.5,s,0),(\vartheta_3/0.5,s,1),(\vartheta_3/0.5,t,0),\\ (\vartheta_3/0.5,t,1),(\vartheta_3/0.5,u,0),(\vartheta_3/0.5,u,1),(\vartheta_4/0.8,s,0),(\vartheta_4/0.8,s,1),\\ (\vartheta_4/0.8,t,0),(\vartheta_4/0.8,t,1),(\vartheta_4/0.8,u,0),(\vartheta_4/0.8,u,1),(\vartheta_5/0.7,s,0),\\ (\vartheta_5/0.7,s,1),(\vartheta_5/0.7,t,0),(\vartheta_5/0.7,t,1),(\vartheta_5/0.7,u,0),(\vartheta_5/0.7,u,1),\\ (\vartheta_6/0.1,s,0),(\vartheta_6/0.1,s,1),(\vartheta_6/0.1,t,0),(\vartheta_6/0.1,t,1),(\vartheta_6/0.1,u,0),\\ (\vartheta_6/0.1,u,1),(\vartheta_7/0.3,s,0),(\vartheta_7/0.3,s,1),(\vartheta_7/0.3,t,0),(\vartheta_7/0.3,t,1),\\ (\vartheta_7/0.3,u,0),(\vartheta_7/0.3,u,1),(\vartheta_8/0.9,s,0),(\vartheta_8/0.9,s,1),(\vartheta_8/0.9,t,0),\\ (\vartheta_8/0.9,t,1),(\vartheta_8/0.9,u,0),(\vartheta_8/0.9,u,1) \end{array}\right\}$$

let

$$\mathscr{S} = \left\{\begin{array}{l} (\vartheta_1/0.2,s,0),(\vartheta_1/0.2,s,1),(\vartheta_1/0.2,t,0),(\vartheta_1/0.2,t,1),(\vartheta_1/0.2,u,0),\\ (\vartheta_1/0.2,u,1),(\vartheta_2/0.4,s,0),(\vartheta_2/0.4,s,1),(\vartheta_2/0.4,t,0),(\vartheta_2/0.4,t,1),\\ (\vartheta_2/0.4,u,0),(\vartheta_2/0.4,u,1),(\vartheta_3/0.5,s,0),(\vartheta_3/0.5,s,1),(\vartheta_3/0.5,t,0),\\ (\vartheta_3/0.5,t,1),(\vartheta_3/0.5,u,0),(\vartheta_3/0.5,u,1), \end{array}\right\}$$

be a subset of $\mathscr{H}$ *and* $\mathscr{I} = \{s,t,u,\}$ *be a set of experts. The choices of three experts are shown in the survey below:*

$$\begin{gathered} \mho_1 = \mho(\vartheta_1/0.2,s,1) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\},\mho_2 = \mho(\vartheta_1/0.2,t,1) = \{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_5\},\\ \mho_3 = \mho(\vartheta_1/0.2,u,1) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_4\},\mho_4 = \mho(\vartheta_2/0.4,s,1) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3\},\\ \mho_5 = \mho(\vartheta_2/0.4,t,1) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_6\},\mho_6 = \mho(\vartheta_2/0.4,u,1) = \{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\},\\ \mho_7 = \mho(\vartheta_3/0.5,s,1) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_5\},\mho_8 = \mho(\vartheta_3/0.5,t,1) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\},\\ \mho_9 = \mho(\vartheta_3/0.5,u,1) = \{\tilde{\oe}_1\ \tilde{\oe}_3,\ \tilde{\oe}_6\},\mho_{10} = \mho(\vartheta_1/0.2,s,0) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\},\\ \mho_{11} = \mho(\vartheta_1/0.2,t,0) = \{\tilde{\oe}_1,\ \tilde{\oe}_4,\ \tilde{\oe}_5\},\mho_{12} = \mho(\vartheta_1/0.2,u,0) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\},\\ \mho_{13} = \mho(\vartheta_2/0.4,s,0) = \{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\},\mho_{14} = \mho(\vartheta_2/0.4,t,0) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_6,\ \tilde{\oe}_4\},\\ \mho_{15} = \mho(\vartheta_2/0.4,u,0) = \{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_4\},\mho_{16} = \mho(\vartheta_3/0.5,s,0) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_6,\ \tilde{\oe}_4\},\\ \mho_{17} = \mho(\vartheta_3/0.5,t,0) = \{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3\},\mho_{18} = \mho(\vartheta_3/0.5,u,0) = \{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_5,\ \tilde{\oe}_4\}. \end{gathered}$$

The fuzzy parameterized hypersoft expert set can be described as

$$(\mho,\mathscr{S}) = \left\{\begin{array}{l} ((\vartheta_1/0.2,s,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),((\vartheta_1/0.2,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_5\}),\\ ((\vartheta_1/0.2,u,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_4\}),((\vartheta_2/0.4,s,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3\}),\\ ((\vartheta_2/0.4,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_6\}),((\vartheta_2/0.4,u,1),\{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_3/0.5,s,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_5\}),((\vartheta_3/0.5,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_3/0.5,u,1),\{\tilde{\oe}_1\ \tilde{\oe}_3,\ \tilde{\oe}_6\}),((\vartheta_1/0.2,s,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_1/0.2,t,0),\{\tilde{\oe}_1,\ \tilde{\oe}_4,\ \tilde{\oe}_5\}),((\vartheta_1/0.2,u,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_2/0.4,s,0),\{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),((\vartheta_2/0.4,t,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_6,\ \tilde{\oe}_4\}),\\ ((\vartheta_2/0.4,u,0),\{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),((\vartheta_3/0.5,s,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_4,\ \tilde{\oe}_6\}),\\ ((\vartheta_3/0.5,t,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3\}),((\vartheta_3/0.5,u,0),\{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_5,\ \tilde{\oe}_4\}), \end{array}\right\}.$$

Definition 11. *Fuzzy Parameterized Hypersoft Expert Subset*
A FPHSE-set $(\mho_1, \check{\mathscr{J}})$ *is said to be FPHSE-subset of* $(\mho_2, \check{\mathbb{R}})$ *over* $\hat{\triangle}$*, if (i)* $\check{\mathscr{J}} \subseteq \check{\mathbb{R}}$*, (ii)* $\forall\ \gamma \in \check{\mathscr{J}}, \mho_1(\gamma) \subseteq \mho_2(\gamma)$ *shown by* $(\mho_1, \check{\mathscr{J}}) \subseteq (\mho_2, \check{\mathbb{R}})$.

Example 12. *Dealing with Example 10, suppose*

$$\check{\mathscr{J}}_1 = \begin{Bmatrix} (\vartheta_1/0.2, s, 1), (\vartheta_3/0.5, s, 0), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.5, t, 1), \\ (\vartheta_3/0.5, t, 0), (\vartheta_1/0.2, u, 0), (\vartheta_3/0.5, u, 1) \end{Bmatrix}$$

$$\check{\mathscr{J}}_2 = \begin{Bmatrix} (\vartheta_1/0.2, s, 1), (\vartheta_3/0.5, s, 0), (\vartheta_3/0.5, s, 1), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.5, t, 1), \\ (\vartheta_1/0.2, t, 0), (\vartheta_3/0.5, t, 0), (\vartheta_1/0.2, u, 0), (\vartheta_3/0.5, u, 1), (\vartheta_1/0.2, u, 1) \end{Bmatrix}.$$

It is clear that $\check{\mathscr{J}}_1 \subset \check{\mathscr{J}}_2$*. Suppose* $(\mho_1, \check{\mathscr{J}}_1)$ *and* $(\mho_2, \check{\mathscr{J}}_2)$ *be defined as following*

$$(\mho_1, \check{\mathscr{J}}_1) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_1/0.2, t, 1), \{\tilde{œ}_1,\ \tilde{œ}_3,\ \tilde{œ}_5\}), \\ ((\vartheta_3/0.5, t, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_3/0.5, u, 1), \{\tilde{œ}_1,\ \tilde{œ}_3,\ \tilde{œ}_6\}), \\ ((\vartheta_1/0.2, u, 0), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_4,\ \tilde{œ}_6\}), \\ ((\vartheta_3/0.5, t, 0), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3\}), \end{array} \right\}$$

$$(\mho_2, \check{\mathscr{J}}_2) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_1/0.2, t, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_5\}), \\ ((\vartheta_3/0.5, s, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_3/0.5, t, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_1/0.2, u, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_3/0.5, u, 1), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_6\}), \\ ((\vartheta_1/0.2, u, 0), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4,\ \tilde{œ}_5\}), \\ ((\vartheta_1/0.2, t, 0), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_4,\ \tilde{œ}_6\}), \\ ((\vartheta_3/0.5, t, 0), \{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}) \end{array} \right\}$$

$\Rightarrow (\mho_1, \check{\mathscr{J}}_1) \subseteq (\mho_2, \check{\mathscr{J}}_2)$.

Definition 13. *Two FPHSE-set* $(\mho_1, \check{\mathscr{J}}_1)$ *and* $(\mho_2, \check{\mathscr{J}}_2)$ *over* $\hat{\triangle}$ *are said to be equal if* $(\mho_1, \check{\mathscr{J}}_1) \subseteq (\mho_2, \check{\mathscr{J}}_2)$ *and* $(\xi_2, \check{\mathscr{J}}_2) \subseteq (\xi_1, \check{\mathscr{J}}_1)$.

Definition 14. *A set* $(\mho, Æ)^c$ *representing the complement of FPHSE-set* $(\mho, Æ)$ *and is characterized by* $(\mho, Æ)^c = (\mho^c, \sim Æ)$ *such that* $\mho^c : \sim Æ \rightarrow P(\hat{\triangle})$*is defined by* $\mho^c(!') = \hat{\triangle} - \mho(\sim !')$ *and* $\sim !' \in \sim Æ$.

Example 15. *Examining the complement of FPHSE-set determined in example 10, we* *have,* $(\mho,\mathscr{S})^c =$

$$\left\{\begin{array}{l} ((\sim\vartheta_1/0.2,s,1),\{\tilde{\oe}_5,\ \tilde{\oe}_6\}),((\sim\vartheta_1/0.2,t,1),\{\tilde{\oe}_2,\ \tilde{\oe}_4,\ \tilde{\oe}_6\}),\\ ((\sim\vartheta_1/0.2,u,1),\{\tilde{\oe}_3,\ \tilde{\oe}_5,\ \tilde{\oe}_6\}),((\sim\vartheta_3/0.5,s,1),\{\tilde{\oe}_3,\ \tilde{\oe}_4,\ \tilde{\oe}_6\}),\\ ((\sim\vartheta_3/0.5,t,1),\{\tilde{\oe}_5,\ \tilde{\oe}_6\}),((\sim\vartheta_3/0.5,u,1),\{\tilde{\oe}_2,\ \tilde{\oe}_4,\ \tilde{\oe}_5\}),\\ ((\sim\vartheta_2/0.4,s,1),\{\tilde{\oe}_4,\ \tilde{\oe}_5,\ \tilde{\oe}_6\}),((\sim\vartheta_2/0.4,t,1),\{\tilde{\oe}_3,\ \tilde{\oe}_4,\ \tilde{\oe}_5\}),\\ ((\sim\vartheta_2/0.4,u,1),\{\tilde{\oe}_1,\ \tilde{\oe}_5,\ \tilde{\oe}_6\}),((\sim\vartheta_1/0.2,s,0),\{\tilde{\oe}_5,\ \tilde{\oe}_6\}),\\ ((\sim\vartheta_1/0.2,t,0),\{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_6\}),((\sim\vartheta_1/0.2,u,0),\{\tilde{\oe}_5,\ \tilde{\oe}_6\}),\\ ((\sim\vartheta_3/0.5,s,0),\{\tilde{\oe}_3,\ \tilde{\oe}_5\}),((\sim\vartheta_3/0.5,t,0),\{\tilde{\oe}_4,\ \tilde{\oe}_5,\ \tilde{\oe}_6\}),\\ ((\sim\vartheta_3/0.5,u,0),\{\tilde{\oe}_1,\ \tilde{\oe}_6\}),((\sim\vartheta_2/0.4,s,0),\{\tilde{\oe}_1,\ \tilde{\oe}_5,\ \tilde{\oe}_6\}),\\ ((\sim\vartheta_2/0.4,t,0),\{\tilde{\oe}_3,\ \tilde{\oe}_5\}),((\sim\vartheta_2/0.4,u,0),\{\tilde{\oe}_2,\ \tilde{\oe}_5,\ \tilde{\oe}_6\}) \end{array}\right\}.$$

Definition 16. *An agree-FPHSE-set* $(\mho,\mathscr{S})_{ag}$ *over* $\hat{\triangle}$*, is a FPHSE-subset of* $(\mho,\mathscr{S})$ *and is characterized as* $(\mho,\mathscr{S})_{agree} = \{\mho_{agree}(\vec{\beta}) : \vec{\beta} \in \times\mathscr{I} \times \{1\}\}$.

Example 17. *Finding agree-FPHSE-set determined in example 10, we get*

$$(\mho,\mathscr{S}) = \left\{\begin{array}{l} ((\vartheta_1/0.2,s,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),((\vartheta_1/0.2,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_5\}),\\ ((\vartheta_1/0.2,u,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_4\}),((\vartheta_2/0.4,s,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3\}),\\ ((\vartheta_2/0.4,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_6\}),((\vartheta_2/0.4,u,1),\{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_3/0.5,s,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_5\}),((\vartheta_3/0.5,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_3/0.5,u,1),\{\tilde{\oe}_1\,\tilde{\oe}_3,\ \tilde{\oe}_6\}), \end{array}\right\}.$$

Definition 18. *A disagree-FPHSE-set* $(\mho,\mathscr{S})_{dagree}$ *over* $\hat{\triangle}$*, is a FPHSE-subset of* $(\mho,\mathscr{S})$ *and is characterized as* $(\mho,\mathscr{S})_{dagree} = \{\mho_{dagree}(\vec{\beta}) : \vec{\beta} \in \times\mathscr{I} \times \{0\}\}$.

Example 19. *Getting disagree-FPHSE-set determined in example10,* $(\mho,\mathscr{S}) =$

$$\left\{\begin{array}{l} ((\vartheta_1/0.2,s,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_1/0.2,t,0),\{\tilde{\oe}_1,\ \tilde{\oe}_4,\ \tilde{\oe}_5\}),((\vartheta_1/0.2,u,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_2/0.4,s,0),\{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),((\vartheta_2/0.4,t,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_6,\ \tilde{\oe}_4\}),\\ ((\vartheta_2/0.4,u,0),\{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),((\vartheta_3/0.5,s,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_4,\ \tilde{\oe}_6\}),\\ ((\vartheta_3/0.5,t,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3\}),((\vartheta_3/0.5,u,0),\{\tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_5,\ \tilde{\oe}_4\}), \end{array}\right\}.$$

Definition 20. *Let* $\eta_{\mathscr{F}_1}$ *and* $\eta_{\mathscr{F}_2}$ *be the two FPHSE-sets, then their union* $\eta_{\mathscr{F}_1} \sqcup \eta_{\mathscr{F}_2}$ *is defined as for all* $\hat{g} \in$ *and* $\psi_{\mathfrak{I}_1}(\hat{g}) \sqcup \psi_{\mathfrak{I}_2}(\hat{g}) = \psi_{\mathfrak{I}_3}(\hat{g})$

1. $\zeta_{\psi_{\mathfrak{I}_1} \sqcup \psi_{\mathfrak{I}_2}}(\hat{g}) = max\{\zeta_{\psi_{\mathfrak{I}_1}(\hat{g})}, \zeta_{\psi_{\mathfrak{I}_2}(\hat{g})}\}$
2. $\psi_{\mathfrak{I}_3}(\hat{g}) = \left\{\begin{array}{cl} \psi_{\mathfrak{I}_1}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_1} \setminus \psi_{\mathfrak{I}_2} \\ \psi_{\mathfrak{I}_2}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_2} \setminus \psi_{\mathfrak{I}_1} \\ \psi_{\mathfrak{I}_1}(\hat{g}) \sqcup \psi_{\mathfrak{I}_2}(\hat{g}) & ; \hat{g} \in \psi_{\mathfrak{I}_1} \sqcap \psi_{\mathfrak{I}_2}. \end{array}\right.$

Example 21. *Dealing again Example 10, consider the following two sets*

$$(\mho_1,\check{\mathscr{J}}_1) = \left\{\begin{array}{l} ((\vartheta_1/0.2,s,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_1/0.2,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_5\}),\\ ((\vartheta_3/0.5,t,1),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_3/0.5,u,1),\{\tilde{\oe}_1,\ \tilde{\oe}_3,\ \tilde{\oe}_6\}),\\ ((\vartheta_1/0.2,u,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3,\ \tilde{\oe}_4\}),\\ ((\vartheta_3/0.5,t,0),\{\tilde{\oe}_1,\ \tilde{\oe}_2,\ \tilde{\oe}_3\}), \end{array}\right\},$$

$$(\mho_2,\check{\mathscr{J}}_2)=\left\{\begin{array}{l}((\vartheta_1/0.2,s,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_1/0.2,t,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_5\}),\\ ((\vartheta_3/0.5,s,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,t,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_1/0.2,u,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,u,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_6\}),\\ ((\vartheta_1/0.2,u,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4,\ \tilde{œ}_5\}),\\ ((\vartheta_1/0.2,t,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,t,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\end{array}\right\}.$$

Then $(\mho_1,\check{\mathscr{J}}_1)\sqcup(\mho_2,\check{\mathscr{J}}_2)=(\mho_3,\check{\mathscr{J}}_3)$

$$(\mho_3,\check{\mathscr{J}}_3)=\left\{\begin{array}{l}((\vartheta_1/0.2,s,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_1/0.2,t,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_5\}),\\ ((\vartheta_3/0.5,s,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,t,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_1/0.2,u,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,u,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_6\}),\\ ((\vartheta_1/0.2,u,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4,\ \tilde{œ}_5\}),\\ ((\vartheta_1/0.2,t,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,t,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\end{array}\right\}.$$

Proposition 22. *If* $(\aleph_1,\hat{\top}_1)$,$(\aleph_2,\hat{\top}_2)$ *and* $(\aleph_3,\hat{\top}_3)$ *are three FPHSE-sets over* $\hat{\triangle}$*, then*

(1) $(\aleph_1,\hat{\top}_1)\sqcup(\aleph_2,\hat{\top}_2)=(\aleph_2,\hat{\top}_2)\sqcup(\aleph_1,\hat{\top}_1)$

(2) $((\aleph_1,\hat{\top}_1)\sqcup(\aleph_2,\hat{\top}_2))\sqcup(\aleph_3,\hat{\top}_3)=(\aleph_1,\hat{\top}_1)\sqcup((\aleph_2,\hat{\top}_2)\sqcup(\aleph_3,\hat{\top}_3))$.

Proof. 1. Suppose $(\aleph_1,\hat{\top}_1)\sqcup(\aleph_2,\hat{\top}_2)=(\aleph_3,\hat{\top}_3)$, then using the definition union for all $\in\hat{\top}_3$, we get $\aleph_3()=\aleph_1()\sqcup\aleph_2()$. Since union of fuzzy sets also implies hypersoft expert set commutative property. Therefore $\aleph_3()=\aleph_1()\sqcup\aleph_2()=\aleph_2()\sqcup\aleph_1()$. Hence proved.

2. Its simple to proof like first. □

Definition 23. *Let* $\eta_{\mathscr{F}_1}$ *and* $\eta_{\mathscr{F}_2}$ *be the two FPHSE-sets, then their intersection* $\eta_{\mathscr{F}_1}\sqcap\eta_{\mathscr{F}_2}$ *is defined as for all* $\hat{g}\in$ *and* $\psi_{\mathfrak{I}_1}(\hat{g})\sqcap\psi_{\mathfrak{I}_2}(\hat{g})=\psi_{\mathfrak{I}_3}(\hat{g})$

1. $\zeta_{\psi_{\mathfrak{I}_1}\sqcap\psi_{\mathfrak{I}_2}}(\hat{g})=min\{\zeta_{\psi_{\mathfrak{I}_1}(\hat{g})},\zeta_{\psi_{\mathfrak{I}_2}(\hat{g})}\}$
2. $\psi_{\mathfrak{I}_3}(\hat{g})=\left\{\begin{array}{ll}\psi_{\mathfrak{I}_1}(\hat{g}) & ;\hat{g}\in\psi_{\mathfrak{I}_1}\setminus\psi_{\mathfrak{I}_2}\\ \psi_{\mathfrak{I}_2}(\hat{g}) & ;\hat{g}\in\psi_{\mathfrak{I}_2}\setminus\psi_{\mathfrak{I}_1}\\ \psi_{\mathfrak{I}_1}(\hat{g})\sqcap\psi_{\mathfrak{I}_2}(\hat{g}) & ;\hat{g}\in\psi_{\mathfrak{I}_1}\sqcap\psi_{\mathfrak{I}_2}.\end{array}\right.$

Example 24. *Referring to the Example 10, consider the sets*

$$(\mho_1,\check{\mathscr{J}}_1)=\left\{\begin{array}{l}((\vartheta_1/0.2,s,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_1/0.2,t,1),\{\tilde{œ}_1,\ \tilde{œ}_3,\ \tilde{œ}_5\}),\\ ((\vartheta_3/0.5,t,1),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,u,1),\{\tilde{œ}_1,\ \tilde{œ}_3,\ \tilde{œ}_6\}),\\ ((\vartheta_1/0.2,u,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3,\ \tilde{œ}_4\}),\\ ((\vartheta_3/0.5,s,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_4,\ \tilde{œ}_6\}),\\ ((\vartheta_3/0.5,t,0),\{\tilde{œ}_1,\ \tilde{œ}_2,\ \tilde{œ}_3\}),\end{array}\right\},$$

$$(\mho_2, \check{\mathscr{J}}_2) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_1/0.2, t, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_5\}), \\ ((\vartheta_3/0.5, s, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_3/0.5, t, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_1/0.2, u, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_3/0.5, u, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_6\}), \\ ((\vartheta_1/0.2, u, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4,\ \tilde{æ}_5\}), \\ ((\vartheta_1/0.2, t, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_4,\ \tilde{æ}_6\}), \\ ((\vartheta_3/0.5, t, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}) \end{array} \right\}.$$

Then $(\mho_1, \check{\mathscr{J}}_1) \sqcap (\mho_2, \check{\mathscr{J}}_2) = (\mho_3, \check{\mathscr{J}}_3)$

$$(\mho_3, \check{\mathscr{J}}_3) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_1/0.2, t, 1), \{\tilde{æ}_1,\ \tilde{æ}_3,\ \tilde{æ}_5\}), \\ ((\vartheta_3/0.5, t, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_3/0.5, u, 1), \{\tilde{æ}_1,\ \tilde{æ}_3,\ \tilde{æ}_6\}), \\ ((\vartheta_1/0.2, u, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_4,\ \tilde{æ}_6\}), \\ ((\vartheta_3/0.5, t, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3\}) \end{array} \right\}.$$

Definition 25. *If* $(\mho_1, \check{\mathscr{J}}_1)$ *and* $(\mho_2, \check{\mathscr{J}}_2)$ *are two FPHSE-sets over* $\hat{\triangle}$ *then* $(\mho_1, \check{\mathscr{J}}_1)$ *AND* $(\mho_2, \check{\mathscr{J}}_2)$ *denoted by* $(\mho_1, \check{\mathscr{J}}_1) \wedge (\mho_2, \check{\mathscr{J}}_2)$ *is defined by* $(\mho_1, \check{\mathscr{J}}_1) \wedge (\mho_2, \check{\mathscr{J}}_2) = (\mho_3, \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2)$, *while* $\mho_3(\triangleleft, \triangleright) = \mho_1(\triangleleft) \cap \mho_2(\gamma), \forall (\triangleleft, \triangleright) \in \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2$.

Example 26. *Taking Example 10, let two sets*
$\check{\mathscr{J}}_1 = \{(\vartheta_1/0.2, s, 1), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.5, s, 0)\}$, $\check{\mathscr{J}}_2 = \{(\vartheta_1/0.2, s, 1), (\vartheta_3/0.5, s, 0)\}$.
Suppose $(\mho_1, \check{\mathscr{J}}_1)$ *and* $(\mho_2, \check{\mathscr{J}}_2)$ *over* $\hat{\triangle}$ *are two FPHSE-sets such that*

$$(\mho_1, \check{\mathscr{J}}_1) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ ((\vartheta_1/0.2, t, 1), \{\tilde{æ}_1,\ \tilde{æ}_4,\ \tilde{æ}_5,\ \tilde{æ}_6\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_4,\ \tilde{æ}_6\}), \end{array} \right\},$$

$$(\mho_2, \check{\mathscr{J}}_2) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_5\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \end{array} \right\}.$$

Then $(\mho_3, \check{\mathscr{J}}_3) \wedge (\mho_2, \check{\mathscr{J}}_2) = (\mho_3, \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2)$,

$$(\mho_3, \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2) = \left\{ \begin{array}{l} (((\vartheta_1/0.2, s, 1), (\vartheta_1/0.2, s, 1)), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3\}), \\ (((\vartheta_1/0.2, t, 1), (\vartheta_1/0.2, s, 1)), \{\tilde{æ}_1,\ \tilde{æ}_5\}), \\ (((\vartheta_1/0.2, t, 1), (\vartheta_3/0.5, s, 0)), \{\tilde{æ}_1,\ \tilde{æ}_4\}), \\ (((\vartheta_1/0.2, s, 1), (\vartheta_3/0.5, s, 0)), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_3,\ \tilde{æ}_4\}), \\ (((\vartheta_3/0.5, s, 0), (\vartheta_1/0.2, s, 1)), \{\tilde{æ}_1,\ \tilde{æ}_2\}), \\ (((\vartheta_3/0.5, s, 0), (\vartheta_3/0.5, s, 0)), \{\tilde{æ}_1,\ \tilde{æ}_2,\ \tilde{æ}_4\}) \end{array} \right\}.$$

Definition 27. *If* $(\mho_1, \check{\mathscr{J}}_1)$ *and* $(\mho_2, \check{\mathscr{J}}_2)$ *are two FPHSE-sets over* $\hat{\triangle}$ *then* $(\mho_1, \check{\mathscr{J}}_1)$ *OR* $(\mho_2, \check{\mathscr{J}}_2)$ *denoted by* $(\mho_1, \check{\mathscr{J}}_1) \vee (\mho_2, \check{\mathscr{J}}_2)$ *is defined by* $(\mho_1, \check{\mathscr{J}}_1) \vee (\mho_2, \check{\mathscr{J}}_2) = (\mho_3, \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2)$, *while* $\mho_3(\triangleleft, \triangleright) = \mho_1(\triangleleft) \cap \mho_2(\gamma), \forall (\triangleleft, \triangleright) \in \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2$.

Example 28. *Taking Example 10, suppose the sets*

$$(\mho_1, \check{\mathscr{J}}_1) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4\}), \\ ((\vartheta_1/0.2, t, 1), \{\tilde{\ae}_1,\ \tilde{\ae}_4,\ \tilde{\ae}_5,\ \tilde{\ae}_6\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_4,\ \tilde{\ae}_6\}), \end{array} \right\},$$

$$(\mho_2, \check{\mathscr{J}}_2) = \left\{ \begin{array}{l} ((\vartheta_1/0.2, s, 1), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_5\}), \\ ((\vartheta_3/0.5, s, 0), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4\}), \end{array} \right\}$$

Then $(\mho_3, \check{\mathscr{J}}_3) \vee (\mho_2, \check{\mathscr{J}}_2) = (\mho_3, \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2)$,

$$(\mho_3, \check{\mathscr{J}}_1 \times \check{\mathscr{J}}_2) = \left\{ \begin{array}{l} (((\vartheta_1/0.2, s, 1), (\vartheta_1/0.2, s, 1)), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4,\ \tilde{\ae}_5\}), \\ (((\vartheta_1/0.2, t, 1), (\vartheta_1/0.2, s, 1)), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4,\ \tilde{\ae}_5,\ \tilde{\ae}_6\}), \\ (((\vartheta_1/0.2, t, 1), (\vartheta_3/0.5, s, 0)), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4,\ \tilde{\ae}_5,\ \tilde{\ae}_6\}), \\ (((\vartheta_1/0.2, s, 1), (\vartheta_3/0.5, s, 0)), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4\}), \\ (((\vartheta_3/0.5, s, 0), (\vartheta_1/0.2, s, 1)), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4,\ \tilde{\ae}_5,\ \tilde{\ae}_6\}), \\ (((\vartheta_3/0.5, s, 0), (\vartheta_3/0.5, s, 0)), \{\tilde{\ae}_1,\ \tilde{\ae}_2,\ \tilde{\ae}_3,\ \tilde{\ae}_4,\ \tilde{\ae}_5,\ \tilde{\ae}_6\}), \end{array} \right\}.$$

4. IMPLEMENTATION OF ALGORITHM

Figure 2 presents the flow chart of implementation of algorithm.

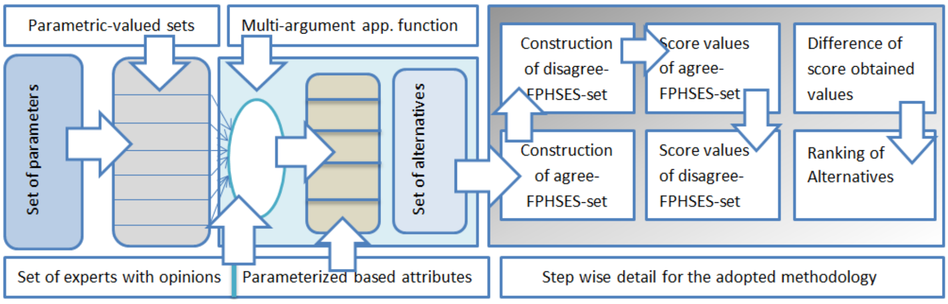

Figure 2: Flow chart of Implementation of algorithm

4.1. An Applications to Fuzzy Parameterized Hypersoft Expert Set

This section of the paper applies fuzzy parameterized hypersoft expert set theory to a problem of decision-making.

Statement of the problem

In a scenario involving product selection, purchasing an electronic item has evolved into a difficult issue for both an individual and an enterprise. Over the course of their lives, many adults make several computer purchases. A computer is a major acquisition. Its cost might be as much as several years' worth of disposable income. With so many computers to look through, selecting the ideal one to fit your budget can feel like navigating a minefield. In any case, it's not easy to figure out the constantly evolving list of item judgments. Work stations vary greatly in terms of their size, drive storage capacity, random access memory, central processing speed, and other features. Your computer demands could also be very different from those of another person, which would only add to the confusion. The majority of people require a laptop for personal, professional, and business purposes as well as for comforts and conveniences. Assume a bank office wants to purchase a computer for office use. The bank manager hires a computer expert team which consists of three experts such as $\otimes = \{\mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3\}$. Expert team try to complete the process taking the attributes of the product as weight of the computer, battery timing,random access memory, price, screen size and the objects (computers) of universe $\{\tilde{\ddot{u}}_1, \tilde{\ddot{u}}_2, \tilde{\ddot{u}}_3, \tilde{\ddot{u}}_4, \tilde{\ddot{u}}_5\}$. Let $\{\hat{\chi}_1/0.2, \hat{\chi}_2/0.4, \hat{\chi}_3/0.5, \hat{\chi}_4/0.7, \hat{\chi}_5/0.6\}$ be the fuzzy subset of I. Flow diagram of the algorithm is shown in Figure 3.

==

Proposed Algorithm I: Selection of Computer

==

▷ **Start**:

▷ **Input**:

———1. Considering expert's team $\{\mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3\}$

———2. Expert opinions $\{0, 1\}$

———3. Set of parameters $\{\mathtt{T}_1, \mathtt{T}_2, \mathtt{T}_3, \mathtt{T}_4, \mathtt{T}_5\}$

———4. Formation of FPHSE-set $(\mho, \mathbb{K})$

▷ **Construction**:

———-5. Calculation of an Agree-FPHSE-set and Disagree-FPHSE-set

▷ **Computation**:

———6. Compute $o_i = \sum_i \tilde{\ddot{u}}_{ij}$ for Agree-FPHSE-set

———7. Compute $\rho_i = \sum_i \tilde{\ddot{u}}_{ij}$ for Disagree-FPHSE-set

———8. Determine $g_i = |o_i - \rho_i|$ for Agree and Disagree-FPHSE-set

▷ **Output**:

———9. Calculate m for which P_m = max p_j for solution.

▷ **End**:

==

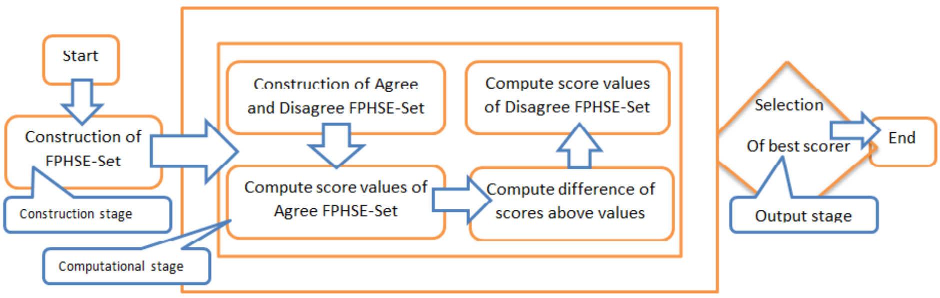

Figure 3: Flow chart of the Algorithm

4.2. Criteria for the attributes and sub-attributes

Many attributes have been found in the literature [47, 48, 49, 50, 51] for the selection of computer systems. A comparison of the options have been made in relation to the anticipated costs and benefits during the evaluation and decision process. In contrast to the cost, the benefit component is typically harder to quantify quantitatively or financially. In-depth analyses of these issues can be found in the literature, for example in [47, 48, 49, 50, 51]. In soft sets like models, we need only those attributes which have sub-attributes. There is no restriction for the choice of attributes whether they have no sub-attributes either they are in the form of sets or any other forms. For example, the operating system is the attribute which has no sub-attribute and colour is another attribute which has sub-attributes pink, green and red etc. Here we have also no need to see which attributes are low costly. But here we need only those attributes which have sub-attributes and these attributes have attribute valued sets (numerical values). These sets must be disjoint sets for the requirement of used structure FPHSES-sets. There is no structure in the literature which can describe the feasibility of parameters in such a way which we require. For example price is the attribute which have disjoint attributed valued sets like $5000Dollar$ and $6000Dollar$. Similarly such kind of the some attributes are given on the Figure 4.

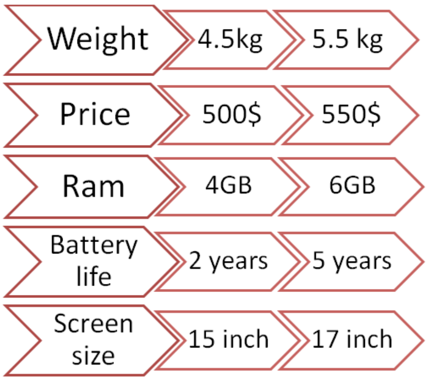

Figure 4: Some selected attributes with sub-attributed values

4.3. Operational role of the selected parameters

1. **Weight**: A computer can weigh between 20 and 40 pounds (9 kg 18 kg). Despite the fact that this varies depending on the case and components we use, the typical gaming PC (with the graphics card) weighs about 31 pounds (14 kg).
2. **Price**: Computers, as we are all aware, are now a necessary component of our daily life, and they are the only tools used for all tasks. Almost all tasks require a computer, including student education, online shopping, online money transfers, workplace work, school and college projects, and many more. As a result, buying a computer has become important in order to handle many duties in our everyday lives. When looking to buy a desktop or laptop, a variety of computers are available, each costing a different amount depending on the model and brand. Prior to the present, there was less demand for computers, which resulted in lower prices than there are now.

3. **Ram**: The hardware of a computing device stores the operating system, application programmes, and data that are currently in use so that the processor of the device may access them rapidly. It serves as a computer's primary memory. Compared to other forms of storage, such as a hard drive, solid-state drive, or optical drive, it is significantly faster to read from and write to.
4. **Battery life**: The battery in your computer is vital. You can only use your computer by plugging it in if the battery dies or malfunctions, which isn't always practical or handy. After all, the key feature of such a device is portability. The operational longevity of a computer battery is influenced by a number of factors, including the battery's construction quality, how well it has been cared for, its brand, and how frequently it has been exposed to high temperatures. Even now and then, it seems that your battery's longevity is somewhat influenced by chance.
5. **Screen size**: The majority of widely used computers typically have screens that are between 13 and 15 inches in size. Although 13 inches is the norm, there are several models that tend to be on the smaller or larger side, ranging from 11 to 17 inches. Laptops are available in a range of sizes to suit a range of lifestyle requirements, including portability, price, and processing speed. Their screen sizes often follow this range, with smaller screens typically having slower processing and storage but being more portable, and vice versa.

Step-1

The attribute-valued sets for prescribed attributes are given as :

T_1= Weight = $\{4.5kg = \Upsilon_1, 5.5kg = \Upsilon_2\}$,

T_2 = Price = $\{500Dollar = \Upsilon_3, 550Dollar = \Upsilon_4\}$

T_3 = Battery Life = $\{2years = \Upsilon_5, 5years = \Upsilon_6\}$,

$\mathsf{T}_4 = Ram = \{4GB = \Upsilon_7, 6GB = \Upsilon_8\}$,

T_5 = Screen Size = $\{15inch = \Upsilon_9, 17inch = \Upsilon_{10}\}$

and then $\mathsf{T} = \mathsf{T}_1 \times \mathsf{T}_2 \times \mathsf{T}_3 \times \mathsf{T}_4 \times \mathsf{T}_5$

$$\mathsf{T} = \left\{ \begin{array}{l} (\Upsilon_1, \Upsilon_3, \Upsilon_5, \Upsilon_7, \Upsilon_9), (\Upsilon_1, \Upsilon_3, \Upsilon_5, \Upsilon_7, \Upsilon_{10}), (\Upsilon_1, \Upsilon_3, \Upsilon_5, \Upsilon_8, \Upsilon_9), \\ (\Upsilon_1, \Upsilon_3, \Upsilon_5, \Upsilon_8, \Upsilon_{10}), (\Upsilon_1, \Upsilon_3, \Upsilon_6, \Upsilon_7, \Upsilon_9), (\Upsilon_1, \Upsilon_3, \Upsilon_6, \Upsilon_7, \Upsilon_{10}), \\ (\Upsilon_1, \Upsilon_3, \Upsilon_6, \Upsilon_8, \Upsilon_9), (\Upsilon_1, \Upsilon_3, \Upsilon_6, \Upsilon_8, \Upsilon_{10}), (\Upsilon_1, \Upsilon_4, \Upsilon_5, \Upsilon_7, \Upsilon_9), \\ (\Upsilon_1, \Upsilon_4, \Upsilon_5, \Upsilon_7, \Upsilon_{10}), (\Upsilon_1, \Upsilon_4, \Upsilon_5, \Upsilon_8, \Upsilon_9), (\Upsilon_1, \Upsilon_4, \Upsilon_5, \Upsilon_8, \Upsilon_{10}), \\ (\Upsilon_1, \Upsilon_4, \Upsilon_6, \Upsilon_7, \Upsilon_9), (\Upsilon_1, \Upsilon_4, \Upsilon_6, \Upsilon_7, \Upsilon_{10}), (\Upsilon_1, \Upsilon_4, \Upsilon_6, \Upsilon_8, \Upsilon_9), \\ (\Upsilon_1, \Upsilon_4, \Upsilon_6, \Upsilon_8, \Upsilon_{10}), (\Upsilon_2, \Upsilon_3, \Upsilon_5, \Upsilon_7, \Upsilon_9), (\Upsilon_2, \Upsilon_3, \Upsilon_5, \Upsilon_7, \Upsilon_{10}), \\ (\Upsilon_2, \Upsilon_3, \Upsilon_5, \Upsilon_8, \Upsilon_9), (\Upsilon_2, \Upsilon_3, \Upsilon_5, \Upsilon_8, \Upsilon_{10}), (\Upsilon_2, \Upsilon_3, \Upsilon_6, \Upsilon_7, \Upsilon_9), \\ (\Upsilon_2, \Upsilon_3, \Upsilon_6, \Upsilon_7, \Upsilon_{10}), (\Upsilon_2, \Upsilon_3, \Upsilon_6, \Upsilon_8, \Upsilon_9), (\Upsilon_2, \Upsilon_3, \Upsilon_6, \Upsilon_8, \Upsilon_{10}), \\ (\Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_7, \Upsilon_9), (\Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_7, \Upsilon_{10}), (\Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_8, \Upsilon_9), \\ (\Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_8, \Upsilon_{10}), (\Upsilon_2, \Upsilon_4, \Upsilon_6, \Upsilon_7, \Upsilon_9), (\Upsilon_2, \Upsilon_4, \Upsilon_6, \Upsilon_7, \Upsilon_{10}), \\ (\Upsilon_2, \Upsilon_4, \Upsilon_6, \Upsilon_8, \Upsilon_9), (\Upsilon_2, \Upsilon_4, \Upsilon_6, \Upsilon_8, \Upsilon_{10}) \end{array} \right\}$$

and now take $\mathbb{K} \subseteq \mathsf{T}$ as

$\mathbb{K} = \{\hat{\chi}_1/0.2 = (\Upsilon_1, \Upsilon_3, \Upsilon_5, \Upsilon_7, \Upsilon_9)$,

$\hat{\chi}_2/0.4 = (\Upsilon_1, \Upsilon_3, \Upsilon_6, \Upsilon_7, \Upsilon_{10})$,

$\hat{\chi}_3/0.5 = (\Upsilon_1, \Upsilon_4, \Upsilon_6, \Upsilon_8, \Upsilon_9)$,

$\hat{\chi}_4/0.7 = (\Upsilon_2, \Upsilon_3, \Upsilon_6, \Upsilon_8, \Upsilon_9)$,

$\hat{\chi}_5/0.6 = (\Upsilon_2, \Upsilon_4, \Upsilon_6, \Upsilon_7, \Upsilon_{10})\}$ and $(\mho, \mathbb{K}) =$

$$\left\{\begin{array}{l}
\left((\hat{\chi}_1/0.2,\mathbb{E}_1,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_4,\ \tilde{\ddot{u}}_5\right\}\right),\left((\hat{\chi}_1/0.2,\mathbb{E}_2,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_1/0.2,\mathbb{E}_3,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_4\right\}\right),\left((\hat{\chi}_2/0.4,\mathbb{E}_1,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_5\right\}\right),\\
\left((\hat{\chi}_2/0.4,\mathbb{E}_2,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\left((\hat{\chi}_2/0.4,\mathbb{E}_3,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4,\ \tilde{\ddot{u}}_5\right\}\right),\\
\left((\hat{\chi}_3/0.5,\mathbb{E}_1,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_5\right\}\right),\left((\hat{\chi}_3/0.5,\mathbb{E}_2,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_3/0.5,\mathbb{E}_3,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4,\ \tilde{\ddot{u}}_5\right\}\right),\left((\hat{\chi}_4/0.7,\mathbb{E}_1,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_4/0.7,\mathbb{E}_2,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_4\right\}\right),\left((\hat{\chi}_4/0.7,\mathbb{E}_3,1),\left\{\tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_5/0.6,\mathbb{E}_1,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\left((\hat{\chi}_5/0.6,\mathbb{E}_2,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2\right\}\right),\\
\left((\hat{\chi}_5/0.6,\mathbb{E}_3,1),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_3\right\}\right),\left((\hat{\chi}_1/0.2,\mathbb{E}_1,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_1/0.2,\mathbb{E}_2,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2\right\}\right),\left((\hat{\chi}_1/0.2,\mathbb{E}_3,0),\left\{\tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_2/0.4,\mathbb{E}_1,0),\left\{\tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_5\right\}\right),\left((\hat{\chi}_2/0.4,\mathbb{E}_2,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2\right\}\right),\\
\left((\hat{\chi}_2/0.4,\mathbb{E}_3,0),\left\{\tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_5\right\}\right),\left((\hat{\chi}_3/0.5,\mathbb{E}_1,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_3/0.5,\mathbb{E}_2,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_3\ \tilde{\ddot{u}}_5\right\}\right),\left((\hat{\chi}_3/0.5,\mathbb{E}_3,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_4/0.7,\mathbb{E}_1,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\left((\hat{\chi}_4/0.7,\mathbb{E}_2,0),\left\{\tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\right),\\
\left((\hat{\chi}_4/0.7,\mathbb{E}_3,0),\left\{\tilde{\ddot{u}}_2,\ \tilde{\ddot{u}}_5\right\}\right),\left((\hat{\chi}_5/0.6,\mathbb{E}_1,0),\left\{\tilde{\ddot{u}}_6,\ \tilde{\ddot{u}}_7\right\}\right),\\
\left((\hat{\chi}_5/0.6,\mathbb{E}_2,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_4\right\}\}\right),\left((\hat{\chi}_5/0.6,\mathbb{E}_3,0),\left\{\tilde{\ddot{u}}_1,\ \tilde{\ddot{u}}_3,\ \tilde{\ddot{u}}_5\right\}\right),
\end{array}\right\}$$

is a fuzzy parameterized hypersoft expert set.

Step-2

Tables 1 and 2 are constructed for Agree-FPHSE-set and Disagree-FPHSE-set respectively with the condition that if $\tilde{\ddot{u}}_i \in \mho_1(\beta)$ then $\tilde{\ddot{u}}_{ij} \in [0,1]$ otherwise $\tilde{\ddot{u}}_{ij} = 0$, and if $\tilde{\ddot{u}}_i \in \mho_0(\beta)$ then $\tilde{\ddot{u}}_{ij} \in [0,1]$, otherwise $\tilde{\ddot{u}}_{ij} = 0$ where $\tilde{\ddot{u}}_{ij}$ are showing the entries in Tables 1 and 2.

Step-3

Table 3 presents $o_i = \sum_i \tilde{\ddot{u}}_{ij}$ for Agree-FPHSE-set, $\rho_i = \sum_i \tilde{\ddot{u}}_{ij}$ for Disagree-FPHSE-set, $g_j = |o_j - \rho_j|$ for Agree and Disagree-FPHSE-sets, and then find m for which p_m= max p_j.

Table 1: Agree-FPHSE-set

Z	$\tilde{\ddot{u}}_1$	$\tilde{\ddot{u}}_2$	$\tilde{\ddot{u}}_3$	$\tilde{\ddot{u}}_4$	$\tilde{\ddot{u}}_5$
$(\hat{\chi}_1/0.2,\mathbb{E}_1)$	1	1	0	1	1
$(\hat{\chi}_1/0.2,\mathbb{E}_2)$	1	0	0	1	0
$(\hat{\chi}_1/0.2,\mathbb{E}_3)$	1	1	0	1	0
$(\hat{\chi}_2/0.4,\mathbb{E}_1)$	1	1	0	0	1
$(\hat{\chi}_2/0.4,\mathbb{E}_2)$	1	0	1	1	0
$(\hat{\chi}_2/0.4,\mathbb{E}_3)$	1	0	1	1	1
$(\hat{\chi}_3/0.5,\mathbb{E}_1)$	0	1	1	0	1
$(\hat{\chi}_3/0.5,\mathbb{E}_2)$	1	1	1	1	0
$(\hat{\chi}_3/0.5,\mathbb{E}_3)$	1	1	1	1	1
$(\hat{\chi}_4/0.7,\mathbb{E}_1)$	1	0	1	1	0
$(\hat{\chi}_4/0.7,\mathbb{E}_2)$	1	0	0	1	1
$(\hat{\chi}_4/0.7,\mathbb{E}_3)$	0	1	1	1	0
$(\hat{\chi}_5/0.6,\mathbb{E}_1)$	1	0	0	0	0
$(\hat{\chi}_5/0.6,\mathbb{E}_2)$	1	1	0	0	1
$(\hat{\chi}_5/0.6,\mathbb{E}_3)$	1	1	1	0	0
$o_j = \sum_i \tilde{\ddot{u}}_{ij}$	$o_1 = 6$	$o_2 = 4.7$	$o_3 = 4.2$	$o_4 = 4.5$	$o_5 = 3.3$

Table 2: Disagree-FPHSE-set

Z	$\tilde{\ddot{u}}_1$	$\tilde{\ddot{u}}_2$	$\tilde{\ddot{u}}_3$	$\tilde{\ddot{u}}_4$	$\tilde{\ddot{u}}_5$
$(\hat{\chi}_1/0.2, \mathbb{E}_1)$	0	0	1	1	0
$(\hat{\chi}_1/0.2, \mathbb{E}_2)$	0	1	0	0	0
$(\hat{\chi}_1/0.2, \mathbb{E}_3)$	1	0	1	1	0
$(\hat{\chi}_2/0.4, \mathbb{E}_1)$	0	0	1	0	1
$(\hat{\chi}_2/0.4, \mathbb{E}_2)$	1	1	0	0	0
$(\hat{\chi}_2/0.4, \mathbb{E}_3)$	1	1	0	0	1
$(\hat{\chi}_3/0.5, \mathbb{E}_1)$	1	1	0	1	0
$(\hat{\chi}_3/0.5, \mathbb{E}_2)$	1	1	0	1	1
$(\hat{\chi}_3/0.5, \mathbb{E}_3)$	1	0	1	1	0
$(\hat{\chi}_4/0.7, \mathbb{E}_1)$	1	1	1	1	0
$(\hat{\chi}_4/0.7, \mathbb{E}_2)$	0	0	1	1	0
$(\hat{\chi}_4/0.7, \mathbb{E}_3)$	0	1	0	0	1
$(\hat{\chi}_5/0.6, \mathbb{E}_1)$	1	1	0	0	0
$(\hat{\chi}_5/0.6, \mathbb{E}_2)$	0	0	1	0	1
$(\hat{\chi}_5/0.6, \mathbb{E}_3)$	1	0	1	0	1
$\rho_i = \sum_i \tilde{\ddot{u}}_{ij}$	$\rho_1 = 3.4$	$\rho_2 = 4.0$	$\rho_3 = 3.9$	$\rho_4 = 3.3$	$\rho_5 = 3.2$

Table 3: Optimal

$o_i = \sum_i \tilde{\ddot{u}}_{ij}$	$\rho_i = \sum_i \tilde{\ddot{u}}_{ij}$	$g_j = \lvert o_j - \rho_j \rvert$
$o_1 = 6$	$\rho_1 = 3.4$	$g_1 = 2.6$
$o_2 = 4.7$	$\rho_2 = 4.0$	$g_2 = 0.7$
$o_3 = 4.2$	$\rho_3 = 3.9$	$g_3 = 0.3$
$o_4 = 4.5$	$\rho_4 = 3.3$	$g_4 = 1.2$
$o_5 = 3.3$	$\rho_5 = 3.2$	$g_5 = 0.1$

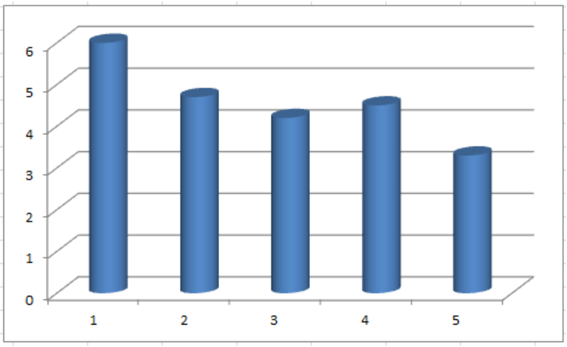

Figure 5: Ranking of Alternative for First Algorithm

Decision

Since g_1 is at its highest, $\tilde{\ddot{u}}_1$ is recommended as the best category for purchases and has been pictured in Figure 5.

4.4. Analysis and discussion

The fuzzy parameterized hypersoft expert set (FPHSE-set) is an advanced mathematical tool designed to handle uncertainty and ambiguity in multi-attribute decision-making scenarios. It integrates expert opinions with fuzzy parameterization, extending the capabilities of existing structures like FPHS-sets and FPSS. Comparing FPHSE-structure to the currently available soft set-like structures, the former offers the best rapport, accuracy, and agreeability. Comparing FPHSE-set to other models will demonstrate this. Due to the inclusion of the multi-argument approximate function, which is very useful in decision-making situations, this proposed model is more beneficial to others because this involves the domain of experts opinions as compared to the soft set like structures. Also it involves the property of multi-objective function, while in soft set like structure there is only single objective functions. In this model, multi-experts opinions related to the choice of the parameters have been introduced which is not found in soft sets like models. This is the main reason behind the superiority of the model. In Table 4, a sensitivity analysis is done. From the analysis, we see that ranking of the alternatives has not been disturbed after taking Pythagorean means. Actually three different means like arithmetic, geometric and harmonic means have been applied to the ranking obtained. But we see that ranking has not been disturbed. It shows that results obtained related to the alternatives are good and accurate. Moreover the comparisons with the previous structures have been shown in Figure 6.

Table 4: Sensitivity analysis

Type	g_1	g_2	g_3	g_4	g_5	Ranking
A.M	4.7	4.05	3.9	4.35	3.25	g_1, g_4, g_2, g_3, g_5
G.M	4.5166	4.0472	3.8536	4.3359	3.2496	g_1, g_4, g_2, g_3, g_5
H.M	4.3404	4.0444	3.8077	4.3248	3.2492	g_1, g_4, g_2, g_3, g_5
Pro.	2.6	0.7	0.3	1.2	0.1	g_1, g_4, g_2, g_3, g_5

5. WEIGHTED FUZZY PARAMETERIZED HYPERSOFT EXPERT SET (WFPHSE-SET)

In this portion, we present the concept of WFPHSE-set by using the concept of FPHSE-set and compare their results.

Definition 29. *A triple $(\mho, \mathscr{S}, \mathscr{W})$ is called a WFPHSE-set with $(\mathscr{S}, \mathscr{W})$ is a FPHSE-set over $\hat{\triangle}$, and $\mathscr{W}$ is a mapping given by $\mathscr{W} : \mathbb{G} \to [0,1]$ with $\mathbb{G}_i = \mathscr{W}(\alpha_i)$ for each attribute $\alpha_i \in \otimes$.*

From definition, it is clear that every FPHSE-set is a WFPHSE-set and is the extension of weighted fuzzy parameterized soft expert set. The concept of WFPHSE-set gives a mathematical structure for modeling and considering the DMPs in which all the choice experts may not be the same importance. The weight function of WFPHSE-set is applied to characterize the differences between the importance of experts.

Example 30. *Reconsidering the Example 10 with following weights assigned to the experts. Suppose weight* 0.3 *is assigned to expert* $\mathbb{E}_1$, 0.5 *to expert* $\mathbb{E}_2$ *and 0.8 to expert* $\mathbb{E}_3$

Feature	FPHSE-sets	FPHS-sets	FPS-sets
Basic Components	Incorporates multiple experts and their weightages along with parameterized fuzzy sets over multi-sub attributes.	Uses fuzzy parameterization for multi-sub attributes without expert integration.	Combines fuzzy sets with parameters but lacks multi-sub attribute structure.
Attribute Parameterization	Multi-layered parameterization across attributes and sub-attributes with expert opinions.	Focuses on attributes and sub-attributes without explicit expert involvement.	Parameterizes attributes directly without sub-attributes.
Expert Contribution	Explicitly models expert weightages and consensus mechanisms.	Lacks explicit modeling of experts or their influence.	No expert-related features.
Real-World Applications	Effective in scenarios requiring group decision-making, such as healthcare, project evaluation, and industrial optimization.	Useful in multi-criteria decision-making without expert involvement, e.g., product ranking.	Limited to simpler applications like preference ranking.

Figure 6: Comparison with previous structures

respectively. The following algorithm may be used to meet the requirements(purchase of computer).

==

Proposed Algorithm II : Selection of Best Product

==

▷ ***Start***:

▷ ***Input***:

———*1. Considering expert's team* $\{\mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3\}$ *with weights*

———*2. Expert opinions* $\{0,1\}$

———*3. Set of parameters* $\{\mathsf{T}_1, \mathsf{T}_2, \mathsf{T}_3, \mathsf{T}_4, \mathsf{T}_5\}$

———*4. Formation of WFPHSE-set* $(\mho, \mathbb{K})$

▷ ***Construction***:

———*-5. Calculation of an Agree-WFPHSE-set and Disagree-WFPHSE-set*

▷ ***Computation***:

———*6. Compute* $o_i = \sum_i \tilde{\ddot{u}}_{ij}$ *for Agree-WFPHSE-set*

———*7. Compute* $\rho_i = \sum_i \tilde{\ddot{u}}_{ij}$ *for Disagree-WFPHSE-set*

———*8. Determine* $g_i = |o_i - \rho_i|$ *for Agree-WFPHSE-set and Disagree-WFPHSE-set*

▷ ***Output***:

———*9. Calculate m for which* P_m= *max* p_j *for solution.*

▷ ***End***:

==

Flow diagram for second algorithm has been shown in Figure 7. Table 4 *shows an Agree-WFPHSE-set and Table* 5 *presents a Disagree-WFPHSE-set respectively, such that if* $\tilde{\ddot{u}}_i \in \mho_1(\beta)$ *then* $\tilde{\ddot{u}}_{ij} = 1$ *otherwise* $\tilde{\ddot{u}}_{ij} = 0$, *and similarly for* $\tilde{\ddot{u}}_i \in \mho_0(\beta)$ *then* $\tilde{\ddot{u}}_{ij} \in [0,1]$.

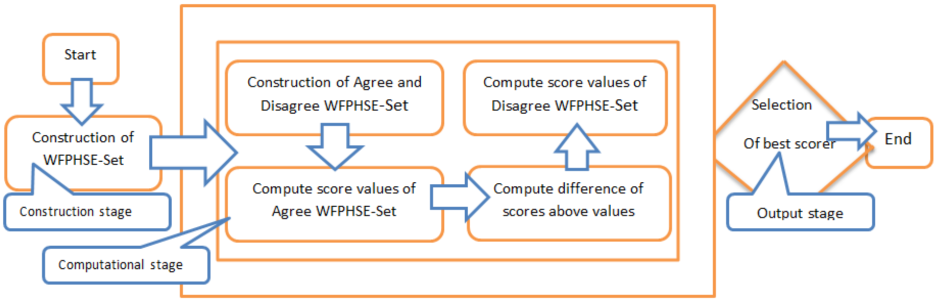

Figure 7: Detail view of Algorithm

Table 5: Agree-WFPHSE-set

Z	$\tilde{\ddot{u}}_1$	$\tilde{\ddot{u}}_2$	$\tilde{\ddot{u}}_3$	$\tilde{\ddot{u}}_4$	$\tilde{\ddot{u}}_5$
$(\hat{\chi}_1/0.2, \mathbb{E}_1/0.3)$	1	1	0	1	1
$(\hat{\chi}_1/0.2, \mathbb{E}_2/0.5)$	1	0	0	1	0
$(\hat{\chi}_1/0.2, \mathbb{E}_3/0.8)$	1	1	0	1	0
$(\hat{\chi}_2/0.4, \mathbb{E}_1/0.3)$	1	1	0	0	1
$(\hat{\chi}_2/0.4, \mathbb{E}_2/0.5)$	1	0	1	1	0
$(\hat{\chi}_2/0.4, \mathbb{E}_3/0.8)$	1	0	1	1	1
$(\hat{\chi}_3/0.5, \mathbb{E}_1/0.3)$	0	1	1	0	1
$(\hat{\chi}_3/0.5, \mathbb{E}_2/0.5)$	1	1	1	1	0
$(\hat{\chi}_3/0.5, \mathbb{E}_3/0.8)$	1	1	1	1	1
$(\hat{\chi}_4/0.7, \mathbb{E}_1/0.3)$	1	0	1	1	0
$(\hat{\chi}_4/0.7, \mathbb{E}_2/0.5)$	1	0	0	1	1
$(\hat{\chi}_4/0.7, \mathbb{E}_3/0.8)$	0	1	1	1	0
$(\hat{\chi}_5/0.6, \mathbb{E}_1/0.3)$	1	0	0	0	0
$(\hat{\chi}_5/0.6, \mathbb{E}_2/0.5)$	1	1	0	0	1
$(\hat{\chi}_5/0.6, \mathbb{E}_3/0.8)$	1	1	1	0	0
$o_j = \sum_i \tilde{\ddot{u}}_{ij}$	$o_1 = 3.04$	$o_2 = 2.27$	$o_3 = 2.57$	$o_4 = 2.61$	$o_5 = 1.95$

Decision

Since g_4 is at its highest, $\tilde{\ddot{u}}_4$ is recommended as the best category for purchases and has been pictured in Figure 8.

6. COMPARISON ANALYSIS, SENSITIVITY ANALYSIS, AND DISCUSSION

Comparing FPHSE-structure to the currently available soft set-like structures, the former offers the best rapport, accuracy, and agreeability. Comparing FPHSE-set to other models will demonstrate this. Due to the inclusion of the multi-argument approximate function, which is very useful in decision-making situations, this proposed model is more beneficial to others. In Table 8, comparison analysis is displayed. Here, a helpful exposition of this structure-FPHSE-set has been developed.

1. If expert set is eliminated, it transforms into fuzzy parameterized hypersoft set.
2. When fuzzy parameterization is disregarded, it transforms into the hypersoft expert set.

Table 6: Disagree-WFPHSE-set

Z	$\tilde{\tilde{u}}_1$	$\tilde{\tilde{u}}_2$	$\tilde{\tilde{u}}_3$	$\tilde{\tilde{u}}_4$	$\tilde{\tilde{u}}_5$
$(\hat{\chi}_1/0.2, \mathbb{E}_1/0.3)$	0	0	1	1	0
$(\hat{\chi}_1/0.2, \mathbb{E}_2/0.5)$	0	1	0	0	0
$(\hat{\chi}_1/0.2, \mathbb{E}_3/0.8)$	1	0	1	1	0
$(\hat{\chi}_2/0.4, \mathbb{E}_1/0.3)$	0	0	1	0	1
$(\hat{\chi}_2/0.4, \mathbb{E}_2/0.5)$	1	1	0	0	0
$(\hat{\chi}_2/0.4, \mathbb{E}_3/0.8)$	1	1	0	0	1
$(\hat{\chi}_3/0.5, \mathbb{E}_1/0.3)$	1	1	0	1	0
$(\hat{\chi}_3/0.5, \mathbb{E}_2/0.5)$	1	1	0	1	1
$(\hat{\chi}_3/0.5, \mathbb{E}_3/0.8)$	1	0	1	1	0
$(\hat{\chi}_4/0.7, \mathbb{E}_1/0.3)$	1	1	1	1	0
$(\hat{\chi}_4/0.7, \mathbb{E}_2/0.5)$	0	0	1	1	0
$(\hat{\chi}_4/0.7, \mathbb{E}_3/0.8)$	0	1	0	0	1
$(\hat{\chi}_5/0.6, \mathbb{E}_1/0.3)$	1	1	0	0	0
$(\hat{\chi}_5/0.6, \mathbb{E}_2/0.5)$	0	0	1	0	1
$(\hat{\chi}_5/0.6, \mathbb{E}_3/0.8)$	1	0	1	0	1
$\rho_i = \sum_i \tilde{\tilde{u}}_{ij}$	$\rho_1 = 2.35$	$\rho_2 = 1.97$	$\rho_3 = 2.08$	$\rho_4 = 1.58$	$\rho_5 = 2.03$

Table 7: Optimal(weighted)

$o_i = \sum_i \tilde{\tilde{u}}_{ij}$	$\rho_i = \sum_i \tilde{\tilde{u}}_{ij}$	$g_j = \lvert o_j - \rho_j \rvert$
$o_1 = 3.04$	$\rho_1 = 2.35$	$g_1 = 0.69$
$o_2 = 2.27$	$\rho_2 = 1.97$	$g_2 = 0.3$
$o_3 = 2.57$	$\rho_3 = 2.08$	$g_3 = 0.49$
$o_4 = 2.61$	$\rho_4 = 1.58$	$g_4 = 1.03$
$o_5 = 1.95$	$\rho_5 = 2.03$	$g_5 = 0.08$

3. When fuzzy parameterization and the expert set are not included, it becomes the hypersoft set.
4. It becomes simpler construct a fuzzy parameterized soft expert set if single argument approximate functions rather than multi-argument approximate functions are used.
5. The soft set is fuzzy parameterized if both the expert set and the multi-argument approximate functions are ignored.
6. when fuzzy parameterization is not considered then this converts into soft set.

As no researcher has ever chosen the computer using the recommended model, the FPHSE-set, the statistical results of the most recent work cannot be compared to any earlier struc-

Table 8: Comparison with particular characteristics

Particular Characteristics	FPSS	FPSES	FPSHS	FPHSE-set
Multi Decisive Opinion	No	No	No	Yes
Multi Argument App. Function	No	No	Yes	Yes
Single Argument App. Function	Yes	Yes	Yes	Yes
Weight for Experts	No	No	No	Yes
Ranking	No	Yes	No	Yes

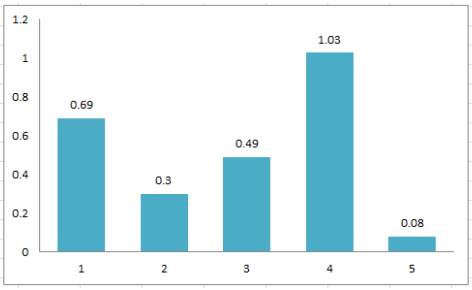

Figure 8: Ranking of Alternative for Second Algorithm

Table 9: Sensitivity analysis (weighted)

Type	g_1	g_2	g_3	g_4	g_5	Ranking
A.M	2.325	2.095	2.12	2.695	1.99	g_4, g_1, g_3, g_2, g_5
G.M	2.3121	2.0307	2.1147	2.6728	1.9896	g_4, g_1, g_3, g_2, g_5
H.M	2.2992	1.9892	2.1094	2.6508	1.9684	g_4, g_1, g_3, g_2, g_5
Pro.	0.69	0.3	0.49	1.03	0.08	g_4, g_1, g_3, g_2, g_5

ture. Nonetheless, a few features such as the weight of experts, multi-decisive opinions, single, and multi-argument-functions are believed to be adequate for contrasting the suggested model with the most relevant continuing structures. In multi-attribute decision-making, it is typical to see that certain experts decide against rendering any expert opinions regarding the attributes of the objects being studied. The decision-makers' impartiality is guaranteed by the professional viewpoints that are taken into consideration. In a similar vein, expert opinions guarantee the acceptability of the expert assessments acquired. A judgment that is prejudiced could arise from the lack of these qualities. The comparison of this structure is based on Table 8, which makes it evident that while the existing models lack one or more aspects, the recommended model satisfies every need separately. Consequently, it is reasonable to conclude that compared to the current models, the suggested model is more reliable, flexible, and generalized. Table 8 employs the terms ”Yes” and ”No” to signify the presence or absence of specific traits or elements in the structures under comparison. An parameter's presence or existence is shown by yes, and its lack or non-existence is indicated by no. Statistical approaches such as the Pythagorean mean (arithmetic, geometric, and harmonic means) are used to analyze the sensitivity of the rating values of substitutes produced from the approximation of sub-parametric multi-arguments in order to rank them and assess their fluctuation. Table 9's findings show that the ordering of alternatives did not change even after ratings were calculated using Pythagorean means.

6.1. Superiority of the Model

The following Table 10 illustrates how superior this structure is. The FPHSE-set is contrasted with the degree of parameterization ($\mathscr{DOP}$), the single argument approximate function ($\mathscr{SAAF}$), the multi-argument approximate function ($\mathscr{MAAF}$), and the multi-decisive opinions ($\mathscr{MDOS}$) of existing structures in Table 10.

Table 10: Compared to various structures with specific properties

Authors	Models	$\mathscr{DOP}$	$\mathscr{SAAF}$	$\mathscr{MAAF}$	$\mathscr{MDOS}$
Molodstov [22]	S-set	No	No	No	No
Maji et al.[23]	FS-set	No	No	No	No
Çağman et al. [41]	FPFS-set	Yes	No	No	No
Bashir et al. [44]	FPFSE-set	Yes	No	No	Yes
Rahman et al. [45]	FPH-set	Yes	Yes	Yes	No
Ihsan et al.[46]	HSE-set	No	Yes	Yes	Yes
Proposed Structure	FPHSE-set	Yes	Yes	Yes	Yes

It is evident from the above Table 10 that our proposed model is more generalised than the models previously.

7. LIMITATIONS OF THE STUDY

The limitations of the study are described below:

1. **Complexity of Parameterization:** The parameterization process in hypersoft sets, particularly under a fuzzy environment, can become computationally intensive when dealing with a large number of attributes or complex decision-making scenarios. This complexity might limit the model's scalability for extremely large data sets, potentially affecting its efficiency and applicability in real-time decision-making processes.
2. **Dependency on Expert Judgments:** The accuracy and reliability of the decision-making process are heavily dependent on the quality of expert input. Inconsistent or biased expert judgments can lead to skewed results, as the model assumes that experts can provide accurate and unbiased evaluations of each attribute. This may impact the overall validity of the final decision.
3. **Handling of Extreme Uncertainty:** While the fuzzy environment aids in managing uncertainty, the model may still encounter difficulties in situations with extremely high levels of uncertainty or insufficient data. In these situations, the model may find it difficult to accurately represent extremely confusing or contradicting data, which could lead to less trustworthy decision outcomes.
4. **Handling of Extreme Uncertainty:** While the fuzzy environment aids in managing uncertainty, the model may still encounter difficulties in situations with extremely high levels of uncertainty or insufficient data. In these situations, the model may find it difficult to accurately represent extremely confusing or contradicting data, which could lead to less trustworthy decision outcomes.
5. **Sensitivity to Parameter Weights:** The model's output could change depending on how different decision criteria are given different weights. The approach is heavily reliant on how accurately the decision-makers assess the value of each criterion, as even little adjustments to these weights might result in different rankings or judgments. This sensitivity may result in less stable decisions in circumstances where decisions are dynamic or changing.

Impact on Results: In some situations, these limitations might have an impact on the accuracy, effectiveness, and dependability of the findings. Processing durations could be prolonged or results could be less dependable due to computational complexity and reliance on expert input. Extreme ambiguity and sensitivity to weight distributions can also cause instability or fluctuation in the results of decisions. Notwithstanding these difficulties, the model's capacity to handle fuzziness and multi-criteria evaluation offers insightful information in situations involving decision-making when complexity and uncertainty are prevalent.

8. CONCLUSION

The principles of the FPHSE-set are covered in this paper, along with some generalised operations subset, equal set, agree and disagree sets, operations (union, intersection, complement, AND, and OR), and laws relevant to this idea like commutative, associative, and distributive. Two new proposed algorithm are built to describe decision-making applications for selecting the best product. Regarding the preferred features of the suggested structure, the following circumstances could make use of it:

1. In order to address contemporary decision-making challenges, the proposed method emphasised the significance of the concept of parameterization along the HSE-Set. The considered parameterization reflects the possibility of the lifestyles of the level of acceptance and forgiveness; thus, this association has the wonderful capacity alongside the real representation in the domain of computational invasions.
2. The current approach emphasises the primary investigation of parameters together with sub-parameters under the several conclusive viewpoints, making the decision-making process the best, most flexible, and most dependable.
3. The suggested structure includes all the elements and characteristics of models like the FPHS-set, FPS-set, FPSE-set, HSE-set, SE-set, and S-set that are already in existence.

Although this structure has advantages, however there are certain limitations in this model. It is unable to handle the data of periodic nature. It is not capable to deal with the three dimensional data i.e., membership, non-membership and indeterminate type data. The creation of more effective algorithms to improve the model's scalability for big data sets and intricate decision scenarios is one potential course for the future. The multi-decisive hypersoft set model can be integrated with machine learning, optimization methods, or hybrid frameworks to increase prediction and decision accuracy. Future studies can also investigate applications for real-time decision-making and use AI or data-driven systems to automate the gathering and analysis of expert information. The model's wider usefulness would be shown by expanding it to various domains like supply chain management, urban planning, and health care. Improving methods for quantifying uncertainty in the fuzzy environment may help to increase the reliability of decisions. Furthermore, performing comparative research with alternative decision-making frameworks and creating easily navigable software tools for real-world application could improve the model and confirm its benefits over current approaches.

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