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Research Article

A ROBUST ALGORITHMIC FRAMEWORK FOR THE EVALUATION OF WORLD HAPPINESS RANKING BASED ON Q RUNG ORTHOPAIR TRIANGULAR FUZZY NEUTROSOPHIC SET WITH POSSIBILITY SETTING (PQ-RTFNS)

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Abstract: This paper presents a robust algorithmic framework for evaluating and ranking world happiness using a novel model based on q-rung orthopair triangular fuzzy neutro-

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sophic sets (PQ-RTFNs) in a possibility setting. Traditional methods of ranking countries by happiness often face challenges in handling the complexity and uncertainty of the various social, economic, and environmental factors that influence well-being. To address this, the authors integrate the flexible and powerful PQ-RTFN model, which allows for better representation of indeterminate and inconsistent data. This approach enhances decision-making accuracy by effectively managing the fuzziness and ambiguity inherent in world happiness metrics. Through a comprehensive evaluation, the proposed framework demonstrates improved performance in ranking nations compared to existing models, offering a more reliable tool for policymakers and researchers to assess global happiness indices.

Keywords: Fuzzy set, neutrosophic set, q-rung fuzzy neutrosophic set,triangular fuzzy neutrosophic set, basic operations, optimization, DM.

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1. INTRODUCTION

World happiness, as one of the maximum essential elements of world wellbeing, captures enormous interest and hobby from people across the world.The rating of nations primarily based totally on their happiness ranges holds widespread significance in information societal development and comparing the great of existence skilled via way of means while present happiness indexes, along with the World Happiness Report, offer treasured insights into the relative happiness ranges of nations, there's a developing want for a greater complete technique that considers a extensive variety of things to make sure a greater correct and holistic evaluation of a nation's happiness rating.

In this studies article, we suggest a complete technique to assessing the happiness rating of nations world wide. By incorporating numerous parameters, inclusive of financial prosperity, social cohesion, environmental great, fitness and wellbeing indicators, and cultural elements, we intention to beautify the objectivity and effectiveness of happiness scores. This complete technique acknowledges the multi-dimensional nature of happiness and recognizes that different factors make contributions to the general well being and pride of people inside a society. The proposed technique seeks to deal with positive boundaries of present happiness indexes, which frequently depend closely on conventional metrics along with GDP in step with capita or existence expectancy. While those metrics provide treasured insights, they'll now no longer absolutely seize the complicated interaction of things that affect happiness ranges. By incorporating extra parameters along with social aid networks, environmental sustainability measures, get right of entry to healthcare and education, cultural diversity, and subjective wellbeing assessments, we intentionto offer a greater nuanced and complete assessment of nations' happiness ranges. Furthermore, this studies article pursuits to make contributions to the continued discourse on happiness scores and their implications for policy-making, societal development, and character wellbeing.

Theoretical framework for decision support system to super market investment risk [1] [2]. By using a data-pushed technique and accomplishing an intensive evaluation of numerous socio-financial and cultural indicators, we are able to derive treasured insights into the elements that make contributions to a country's happiness rating and its standard great of existence for its citizens. Zadeh [3] is widely acknowledged as the pioneer of the fuzzy set concept, which emerged in 1965. While credited with this ground breaking development, it's noteworthy that certain notions resembling fuzzy sets were initially proposed by Black [4] in 1937. Zadeh's [3] seminal work developed the notion of a MF, operating within the range [0, 1] across a potential parameters, to define fuzzy sets [5]. Importantly, membership in a FS isn't a binary affirmation or denial but rather exists along a continuum. The integration of artificial intelligence in the achievement of group decision-making for a given project is felt to enhance the healthcare supply chain management extensively. The suggested approach improves the precision and speed of decision making by utilizing Sugeno–Weber triangular norms in a dual hesitant q-rung orthopair fuzzy environment [6]. It is for this purpose that a new methodology of multiple attribute decision making has been proposed based on q-rung orthopair fuzzy sets. This analytics applies Dombi–Archimedean aggregation operators to improve the accuracy of the decision-making process [7]. The prioritized ordered operators of complex q-rung picture fuzzy Aczel-Alsina have been applied to examine adverse drug reactions. This method allows for improving the targeted approach to discussing pharmacological therapies at the level of structure [8].

There is a multiple criteria decision making to evaluate the artificial intelligence based technologies for tackling the problem of segregation of solid wastes. The assessment uses q-rung picture fuzzy Frank aggregation operators to capture the complexity of the scenarios investigated to make the findings very accurate [9]. A decision analytics framework has been presented to make sustainable urbanization on the basis of q-rung orthopair fuzzy soft sets. This approach uses the aggregation operators developed by Aczel and Alsina to improve the accuracy and efficiency of the urban planning decision [10]. The authors, Hussain and Ullah, discussed on how Sugeno-Weber norms were applied to the domain of spherical fuzzy sets for tackling with the difficult issue in decision making process. Their work also involved showing how these norms can be used practically in a real world intelligent decision support system [11]. To extend the existing CODAS method, Kannan et al. suggested using the linear Diophantine fuzzy sets as an enhanced solution. It was used selectively to enhance decision making in the choice of the specialists in logistics [12]. A crisp hybrid decision-making framework has been constructed from compounds of q-rung picture fuzzy sets and Einstein averaging operators. This method enriches decision-making analysis by increasing the level of accuracy needed in solving decision-making problems that involve multiple criteria [13].

Of considerable importance is the view of dualism of forces as an active function of Zoroastrian world view marked by constant struggle between Good and Evil. This interplay gives the religion a dualist view on the worlds in which dualism, harmony and conflict of opposites form existence and moral decisions [14]. The idea of the LR–type fully Pythagorean fuzzy linear programming deals with optimization problems with equality constraints. This approach optimizes for improved precision and flexibility in using linear programming to solve difficult problems through application of Pythagorean fuzzy sets [14]. Thus, the study examines potential tourism strategies that are antifragile and prioritisation of those strategies using the Neutrosophic approach. This strategy is said to increase the 'robustness' of the tourism development strategies in environments characterized by a great deal of volatility [15]. Smart TOPSIS is the improvement of the TOPSIS method by the integration of a neural network with neutrosophic triplets. This original approach is used in green supplier selection for sustainable manufacturing settings in order to enhance decision-making reliability and sustainability-enhancing efficacy [16]. Thus, this work introduces a fuzzy parameterized single-valued neutrosophic soft set model for choosing hard disks. This method makes the decision process better since it is able to handle uncertainties and give better results on which hard disk to select [17]. The study aims at analyzing the applicability of incorporating neutrosophic sets with deep learning models for improving breast cancer classification. They hope to enhance the specificity and sensitivity of cancer identification since the medical data involves uncertainties [18]. The paper proposes a tool that demonstrates factors that affect supply chain resilience using hesitant fuzzy TOPSIS with meta-synthesis. This integrative approach enriches the analysis and appraisal of the important antecedents to resilience in supply chain management [19]. The study aims at the analysis of geometrical properties of PF topology that leads to the decomposition of the Pythagorean fuzzy topological space for the improved understanding of spatial relations. This method is used to define the high level of orientation of the indeterminate spatial objects thus enhancing analysis in uncertain condition [20]. To rectify for these deficits, the study innovatively integrates new distance measures of a q-rung orthopair multi-fuzzy system in formulating enhanced cybersecurity strategies for MNCs. These measures are intended to enhance the tactical choices and threat sovereign tasks in intricate cyber security situations [21]. The research proposes an extended MADM framework based on Lq^* q-rung orthopair multi-fuzzy soft set for healthcare supplier selection. It provides a positive way of handling uncertainties in the selection process enabling the decision maker to make better, and more accurate decisions [22]. This work aims at comparing a number of methods for image enhancement based on neutrosophic sets applied to X-ray images. This approach tests the ability to increase image quality in medical imaging by reducing uncertainty and imprecision [23]. The study focuses on selecting sustainable materials for constructing drone aerodynamic wings using the Neutrosophic RAWEC method. This approach helps evaluate material options by considering uncertainty and imprecision in the decision-making process [24].

Human intuitionistic data based epmloyee performance evaluation with similarity measure. [25] Zadeh's contributions laid the foundation for a rich array of methodologies and tools tailored to handling moderate membership and non-statistical inconstancy. Essentially, the idea of Fuzzy sets(FSs) extends the traditional crisp set framework [26]. Over time, the field has undergone significant expansion, with Zadeh's original concept branching out into various directions. Various sets was introduced in the literature, including LFSs [27], flou sets [28], interval-valued fuzzy sets(IVFSs), intuitionistic fuzzy sets(IFSs), two-fold fuzzy sets(TFFSs) [29], IVFSs, and intuitionistic L-fuzzy sets(ILFSs) among others. These sets serve as mathematical frameworks for modeling uncertainty and imprecision in different applications [30]. IVFSs are considered a special case of LFSs according to Goguen [31] and also a anomaly of type TFSs. Atanassov [32] and Gargov [33] noticed the mathematical equivalence between IFS and IVFSs. Wang [34] and he demonstrated the equivalence between IFS and ILFSs, as well as LFSs. Kerre [35] summarized the connections between FS and other mathematical tools, such as TFFs, and LFSs. Deschrijver [36] introduced the relationships among IFS, LFS, IVFSs.

However, Dubois [37] et al. denounce the definition of IFSs as it contradicts the correct usage proposed by Takeuti and Titani [38], suggesting the term IFS instead. Smarandache [39] introduced NSs as a generalization of IFS, incorporating the degree of imprecise as an unconventional component. Georgiev [40] explored properties of neutrosophic logic and defined simplified neutrosophic set, concluding that neutrosophic logic lacks normalization rules for its components [41]. Smarandache further developed NS theory to handle unsure, inconsistent, and indeterminate information. NSs theory has found applications in many fields, including image processing, artificial intelligence, applied physics, topology, and social science. Several extensions of NSs theory have been developed in the composition, such as SVNSs, IVNSs, NSSs, NSESs, RNSs, INRSs, CNSs, BNSs, and neutrosophic cube set. These extensions further enrich the capabilities of NS assumption and find applications in diverse areas.Such as a novel fuzzy parametrized FS and summability approach based decision support system for diagnosis of disease [42] [43]. In this paper, we focus on applying SVNS [44], a order of NS theory, which characterizes every member of the world using validity membership,and inaccuracy membership levels that fall inside the actual unit range. SVNS gained awareness in various research areas, including similarity measure, medical recognition, and MCDM [45].

The assembling of information represented by SVNS has emerged as a critical area of research, particularly in the context of MADM scenarios. In such scenarios, the appraisal of alternative options often involves rating values shown in terms of SVNSs. AO play a vital role in fusing individual data points, typically properties by truth, indeterminacy, and falsity membership degrees, into a unified assessment [41]. Various AO have been developed in the literature to handle SVNS information effectively. Ye [46] introduced possibility arithmetic and geometric average operators for simplified NSs, while Peng [47] et al. developed a range of AO tailored for single-valued neutrosophic information. These operators include SNNWA, SNNWG, and others. The application of these operators has been demonstrated in MCGDM contexts, aiding in the selection of the most suitable alternative. Additionally, Liu et al. defined neutrosophic Hamacher AO for MAGDM, and Liu and Wang [48] proposed a single-valued neutrosophic standarized possibility Bonferroni mean operator. These developments have significantly contributed to the application of SVNS in solving complex decision-making problems.Despite the extensive research on SVNS in MADM, challenges persist in representing SVNS attributes with interval numbers, particularly in uncertain and complex situations. Triangular fuzzy numbers offer a robust approach for handling fuzzy data, and their combination with SVNS presents an effective strategy for managing incomplete, indeterminate, and uncertain information.

Recently, Ye introduced trapezoidal fuzzy neutrosophic sets and developed corresponding aggregation operators, such as trapezoidal fuzzy neutrosophic number possibility arithmetic averaging and TFNN possibility geometric averaging. These operators have proven effective in addressing MADM problems. While existing rules, such as those proposed by Zhang and Liu [49], focus on aggregating TFIF information, they may fall short in handling decision-making scenarios involving indeterminacy. As such, there is a need for novel approaches capable of addressing indeterminacy. In this study, we introduce TFNNS, along with their associated score and accuracy functions. We extend existing aggregation methods for TFIF information to develop TFNNPAA operator and TFNNPG operators for aggregating TFNNSs. These operators offer enhanced flexibility

and robustness compared to their fuzzy and intuitionistic fuzzy counterparts, enabling effective handling of variability and indeterminacy in DM rule.
The following are this paper's contributions as well as:

- This case develop a novel algorithmic structure based on possibility q-RTFNS for the evaluation of world happiness. This innovative approach provides a more comprehensive and accurate assessment of happiness levels across diverse populations.
- The proposed PQ-RTFNS-based framework demonstrates great achievement contrast to conventional techniques by achieving higher authenticity and regularity in assess world happiness. It addresses the inherent uncertainty in happiness assessment by incorporating positive, indeterminate, and negative membership degrees.
- The execution of the developed technique holds significant potential in boost up the evaluation of world happiness, leading to more nuanced and insightful analyses. It facilitates a seamless ranking experience by capturing the complex and multifaceted nature of happiness.
- To address the challenges posed by uncertainty in happiness evaluation, we introduce a composite value of possibility with QRN approach, resulting in a ambiguous framework referred to as Possibility Q-rung orthopair triangular fuzzy Neutrosophic set (PQ-RTFNs). This framework provides a robust foundation for capturing and analyzing the diverse dimensions of happiness.

The prevail part of article as well as: Section 2 dispense an outline of basic preliminaries and introduces a novel structure called PQ-RTFNs, which forms the foundation for our proposed framework. In Section 3, we delve into the fundamental operations of PQ-RTFNs, elucidating their key properties and functionalities. Section 4 explores various AO, inclusive of PQ-RTFNs controller for summing such as AO and GO. We analyze their initial properties and applicability in the context of our framework. Section 5 presents a systematic approach for addressing decision-making challenges using PQTFN operators, offering a step-by-step methodology for effective decision support. To demonstrate the practical utility of our framework, Section 6 focuses on the application of our methodology to evaluate the happiness levels of different countries' populations depend on PQTFNSs. In Part 7, we conduct susceptivity review with respect to varying values of "q" and illustrate the results through graphical representations for both AO and GO. Part 8 provides a relative study between our proposed model and existing approaches to showcase its superior performance and efficacy in evaluating world happiness. Finally, in Section 9, we present our conclusions and outline potential avenues for future research in this domain.

1.1. Motivation

The motivation for this research comes from the realization that measuring happiness on a global scale is incredibly complex. Happiness is influenced by a wide range of factors social, economic, environmental, and emotional all of which can vary greatly from country to country. Existing methods for ranking world happiness often struggle to capture the full picture because they can't effectively handle the uncertainty and ambiguity surrounding these factors. The authors were driven by the need for a more accurate and reliable way to evaluate happiness across nations. They turned to q-rung orthopair triangular fuzzy neutrosophic sets (PQ-RTFNs) because this approach allows for better handling of uncertainty and incomplete information. By applying this advanced mathematical model in a possibility setting, they aimed to create a framework that provides clearer, more trustworthy rankings of global happiness. Ultimately, the goal is to give policymakers and researchers a tool that can offer deeper insights into happiness data, helping them make informed decisions that can truly improve the well-being of societies.

1.2. Problem Statement

Accurately evaluating and ranking world happiness is a challenging task due to the complex and multifaceted nature of happiness. Factors such as economic conditions, social well-being, environmental quality, and psychological health all contribute to a nation's overall happiness, and these factors are often uncertain, ambiguous, and difficult to quantify. Traditional methods of ranking world happiness struggle to manage this complexity, leading to inconsistent and sometimes unreliable results.The problem lies in the inability of existing models to effectively handle the fuzziness, indeterminacy, and incompleteness of the data that influences happiness. Without a method that can address these uncertainties, policymakers and researchers are left with an incomplete or skewed understanding of global happiness metrics, which can lead to less effective or misinformed decisions. To address this, the authors propose the use of q-rung orthopair triangular fuzzy neutrosophic sets (PQ-RTFNs) within a possibility framework to develop a more robust algorithmic approach. This approach is designed to better manage the uncertainties and offer a clearer, more accurate picture of happiness across nations, thereby providing a stronger foundation for decision-making and policy formulation aimed at improving societal well-being.

1.3. Main Objective

The main objective of this research is to develop a more accurate and reliable framework for evaluating and ranking world happiness by addressing the limitations of existing methods. Traditional approaches often fail to account for the uncertainty, fuzziness, and indeterminacy in the factors that influence happiness, such as social, economic, and environmental conditions. By leveraging q-rung orthopair triangular fuzzy neutrosophic sets (PQ-RTFNs) within a possibility setting, this study aims to create a robust algorithmic model capable of managing complex, ambiguous data. The goal is to improve the accuracy of global happiness rankings, providing decision-makers and policymakers with a more comprehensive tool for understanding happiness metrics. Ultimately, this enhanced framework will support better-informed decisions and strategies that promote the well-being of societies across the world.

1.4. Research Gap

Existing methods for evaluating and ranking world happiness face significant limitations due to their inability to handle the inherent uncertainty, fuzziness, and indeterminacy present in happiness-related factors such as economic conditions, social well-being, and environmental quality. These conventional approaches often oversimplify complex data, resulting in inconsistent and less reliable happiness rankings. The gap in the current body of research lies in the absence of a comprehensive model that can effectively account for and process this uncertainty. While some models have been proposed to address specific aspects of fuzziness, there is no robust framework that integrates these advanced mathematical concepts in a meaningful way to improve happiness rankings. This research aims to fill this gap by introducing q-rung orthopair triangular fuzzy neutrosophic sets (PQ-RTFNs) within a possibility setting. This model provides a more refined approach to managing ambiguity and uncertainty, thereby enhancing the accuracy and reliability of happiness evaluations. By addressing this gap, the study offers a new and improved framework for understanding and ranking global happiness.

2. PRELIMINARIES

The principles of Pq-RTFNs and their historical background will be covered in this section. We will provide definitions for terminology like FSs and their characteristics, q-ROFSs and their possibilities, NSs and their attributes, etc. In order to address the imprecise character of value, FS was first developed by L.A Zadeh in 1965. [50].

Definition 1. *[51] The principle of FS* ζ *defined as* $\zeta = \{(\pounds, C_\zeta(\pounds)) | \pounds \in W'\}$ *as* $C_\zeta : W' \to I$ *where* $C_\zeta(\pounds)$ *indicates the be the possession value of* $\pounds \in \zeta$.

Definition 2. ***(Characteristic of fuzzy sets)****[52] Assume that two FSs say* ζ *and* ℓ *then* $\forall$ $\pounds \in W'$*, now*
(i) $\zeta \cup \ell = \{(\pounds, max\{C_\zeta(\pounds), C_\ell(\pounds)\})\}$
(ii) $\zeta \cap \ell = \{(\pounds, min\{C_\zeta(\pounds), C_\ell(\pounds)\})\}$
(iii) $\zeta^c = \{(\pounds, 1 - C_\zeta(\pounds)) | \pounds \in W'\}$
While fuzzy sets place a strong focus on degree of membership when handling a situation that is unclear, there many situations where NMD should be take into consideration in order to appropriately relate FSs to such senarios.

Definition 3. *[53] Let's examine the representation of the universal discourse set by* F', *a q-ROFS on* F' *is defined as:*

$$B'_1 = (\Theta, \{\ni_{B'_1}(\Theta), \dashv_{B'_1}(\Theta)\} | \Theta \in F'),$$

$\ni_{B'_1}(\Theta)$ *A bounded interval is known as the veracity degree of* B'_1, $\dashv_{B'_1}(\Theta)$ *is shows as degree of falsity of* B'_1 *and* $\ni_{B'_1}(\Theta)$, $\dashv_{B'_1}(\Theta)$ *maintains the relationship:* $0 \leq \ni_{B'_1}(\Theta)^q + \dashv_{B'_1}(\Theta)^q \leq 1$ *for* $\forall \Theta \in B'_1$. *Then ID of* Θ *in* B'_1 *is indicated by* $\vec{\pi}_{B'_1}(\Theta) = (\ni_{B'_1}(\Theta)^q + \dashv_{B'_1}(\Theta)^q - \ni_{B'_1}(\Theta)^q \dashv_{B'_1}(\Theta)^q)^{1/q}$ *[54].*

Definition 4. *[55, 56] A NS defined on* F' *is describe here:*

$$B_2 = (\Theta, \{\ni_{B_2}(\Theta), \Gamma_{B_2}(\Theta), \dashv_{B_2}(\Theta)\} | \Theta \in F'),$$

$\ni_{B_2}(\Theta) \in$ *[0, 1]is known as the degree of truth membership of* B_2, $\Gamma_{B_2}(\Theta) \in$ *[0, 1] is inscribe by NMD of* B_2 *and* $\dashv_{B_2}(\Theta) \in$ *[0, 3] is known as FMD of* B_2 *and* $\ni_{B_2}(\Theta), \Gamma_{B_2}(\Theta), \dashv_{B_2}(\Theta)$ *grab the later diseases :* $0 \leq \ni_{B_2}(\Theta) + \Gamma_{B_2}(\Theta) + \dashv_{B_2}(\Theta) \leq 3$ *for* $\forall \Theta \in B_2$. *Then* $\pi_{B_2}(\Theta) = 1 - \ni_{B_2}(\Theta) + \Gamma_{B_2}(\Theta) + \dashv_{B_2}(\Theta)$ *is referred to as the rejection-membership level of* Θ *in* B_2.

Definition 5. *[57] The NS $\grave{T}$ is lie in another NS $\grave{L}$, showed by $\grave{T} \subseteq \grave{L}$, if and only if $infT\grave{T}(\Theta) \leq infT\grave{L}(\Theta)$, $supT\grave{T}(\Theta) \leq supT\grave{T}(\Theta)$, $infI\grave{T}(\Theta) \geq infI\grave{L}(\Theta)$, $supI\grave{T}(\Theta) \geq supI\grave{L}(\Theta)$, $infF\grave{T}(\Theta) \geq infF\grave{L}(\Theta)$ and $supF\grave{T}(\Theta) \geq supF\grave{L}(\Theta)$ for any $\ddot{u} \in F'$.*

Definition 6. *[58] A q-RNS on F' is describe in below:*

$$B_3 = (\Theta, \{\eth_{B_3}(\Theta), \Gamma_{B_3}(\Theta), \dashv_{B_3}(\Theta)\} | \Theta \in F'),$$

$\eth_{B_3}(\Theta) \in$ [0, 1] is mentioned to as the positively MD of B_3, $\Gamma_{B_3}(\Theta) \in [0,1]$ is referred to as the ID of B_3 and $\dashv_{B_3}(\Theta) \in [0,1]$ is referred to as the negatively MD of B_3 and $\eth_{B_3}(\Theta), \Gamma_{B_3}(\Theta), \dashv_{B_3}(\Theta)$ convince the later conditions: $0 \leq \eth_{B_3}(\Theta)^q + \Gamma_{B_3}(\Theta)^q + \dashv_{B_3}(\Theta)^q \leq 3$ for $\forall\Theta \in B_3$. Then $\pi_{B_3}(\Theta) = (1 - (\eth_{B_3}(\Theta))^q + (\Gamma_{B_3}(\Theta))^q + (\dashv_{B_3}(\Theta))^q)^{1/q}$ is known as the RMD Θ in B_3.

Definition 7. *Suppose that a q-RLNS L on F' is shown in below:*

$$B_4 = (\Im(\Theta), \{\eth_{A_4}(\Theta), \Gamma_{A_4}(\Theta), \dashv_{A_4}(\Theta)\} | \Theta \in F'),$$

Definition 8. *Suppose that X be a universal set and $F[0,1]$ be the set of all TFNs on $[0,1]$. A TFNNS ω in X is showned by*

$\omega = \{(x, T_\omega(x), I_\omega(x), F_\omega(x)) \,|\, x \in X\}$ where $T_\omega(x) : X \to [0,1]$, $I_\omega(x) : X \to [0,1]$, and $F_\omega(x) : X \to [0,1]$.

The TFNs $T_\omega(x) = \left(T^1_\omega(x), T^2_\omega(x), T^3_\omega(x)\right)$, $I_\omega(x) = \left(I^1_\omega(x), I^2_\omega(x), I^3_\omega(x)\right)$, and $F_\omega(x) = \left(F^1_\omega(x), F^2_\omega(x), F^3_\omega(x)\right)$, respectively, denoted the truth MD, ID, and falsity MD of x in ω for every $x \in X$.

$$0 \leq T^3_\omega(x) + I^3_\omega(x) + F^3_\omega(x) \leq 3$$

For notational comfort, we consider
$\omega = ((i, j, k), (u, v, w), (x, y, z))$

$$(T^1_\omega(x), T^2_\omega(x), T^3_\omega(x)) = (i, j, k),$$
$$(I^1_\omega(x), I^2_\omega(x), I^3_\omega(x)) = (u, v, w),$$
$$\text{and } (F^1_\omega(x), F^2_\omega(x), F^3_\omega(x)) = (x, y, z).$$

3. BASIC OPERATIONS

Definition 9. ***(possibility q-rung orthopair triangular fuzzy neutrosophic set)*** *Let us X be the finite universal set and $F[0,1]$ be the set of all TFNs on $[0,1]$. A TFNNS ω in X is constitute by*

$\omega = \left\{ \frac{P_i}{(x, T_\omega(x), I_\omega(x), F_\omega(x))} \middle| x \in X \right\}$ where $T_\omega(x) : X \to [0,1]$, $I_\omega(x) : X \to [0,1]$, $F_\omega(x) : X \to [0,1]$, and $P_i \in [0,1]$. ω represent the possibility q-rung orthopair triangular fuzzy neutrosophic set.

The TFNs
$T_\omega(x) = \left(T^1_\omega(x), T^2_\omega(x), T^3_\omega(x)\right)$, $I_\omega(x) = \left(I^1_\omega(x), I^2_\omega(x), I^3_\omega(x)\right)$,

and $F_\omega(x) = \left(F^1_\omega(x), F^2_\omega(x), F^3_\omega(x)\right)$*, respectively, denoted the TMD, ID, and FMD of* x *in* ω *for every* $x \in X$.

$$0 \leq T^q_\omega(x) + I^q_\omega(x) + F^q_\omega(x) \leq 3 \tag{1}$$

For notational ease, we consider
$\omega = ((i, j, k), (u, v, w), (x, y, z))$

$(T^1_\omega(x), T^2_\omega(x), T^3_\omega(x)) = (i, j, k)$,
$(I^1_\omega(x), I^2_\omega(x), I^3_\omega(x)) = (u, v, w)$,
and $(F^1_\omega(x), F^2_\omega(x), F^3_\omega(x)) = (x, y, z)$.

The modified versions of the accuracy function and ow score function are [59, 60].

Definition 10. *Suppose that* $\omega = \{(i, j, k), (u, v, w), (x, y, z)\}$ *be a TFNNVs in the set of real numbers. The SF* $S(\omega_1)$ *of* ω_1 *is defined as follows:*

$$S(\omega_1) = \frac{1}{12}\left[8 + (i_1 + 2j_1 + k_1) - (u_1 + 2v_1 + w_1) - (x_1 + 2y_1 + z_1)\right] \tag{2}$$

The value of SF of
For the TFNNV $\omega^+ = (1,1,1), (0,0,0), (0,0,0)$*, the score function is:* $S(\omega^+) = 1$.
For the TFNNV $\omega^- = (0,0,0), (1,1,1), (1,1,1)$*, the accuracy function is:* $S(\omega^-) = -1$.

Definition 11. *Assume that* $\omega = \{(i, j, k), (u, v, w), (x, y, z)\}$ *be a TFNNVs in the set of real numbers. The AF* $H(\omega_1)$ *of* ω_1 *is defined as follows:*

$$H(\omega_1) = \frac{1}{4}\left[(i_1 + 2j_1 + k_1) - (x_1 + 2y_1 + z_1)\right]$$

The AF $H(\omega_1) \in [-1, 1]$ *affect the difference in between truth and falsity. A larger difference reflects the more affirmative nature of the TFNNV.*
For the TFNNV $\omega^+ = (1,1,1), (0,0,0), (0,0,0)$*, the accuracy function is:* $H(\omega^+) = 1$.
For the TFNNV $\omega^- = (0,0,0), (1,1,1), (1,1,1)$*, the accuracy function is:* $H(\omega^-) = -1$.

Definition 12. *[61] Assume that* $\ddot{\eth}_1 = (s_{\mathfrak{Z}_1}, \{\partial_1, \Gamma_1, \dashv_1\})$ *and* $\ddot{\eth}_2 = (s_{\mathfrak{Z}_2}, \{\partial_2, \Gamma_2, \dashv_2\})$ *be any two q-RLNN, Scr(*$\ddot{\eth}_1$*) and Scr(*$\ddot{\eth}_2$*) is a SF of* $\ddot{\eth}_1$ *and* $\ddot{\eth}_2$*, H(*$\ddot{\eth}_1$*) and H(*$\ddot{\eth}_2$*) is an AF of* $\ddot{\eth}_1$ *and* $\ddot{\eth}_2$.

1. *Assume that Scr(*$\ddot{\eth}_1) > Scr(\ddot{\eth}_2)$*, which shows that* $\ddot{\eth}_1 > \ddot{\eth}_2$
2. *Assume that Scr(*$\ddot{\eth}_1) = Scr(\ddot{\eth}_2)$*, which shows that* $\ddot{\eth}_1 = \ddot{\eth}_2$
3. *Assume that H(*$\ddot{\eth}_1) > H(\ddot{\eth}_2)$*, which shows that* $\ddot{\eth}_1 > \ddot{\eth}_2$
4. *Assume that H(*$\ddot{\eth}_1) = H(\ddot{\eth}_2)$*, which shows that* $\ddot{\eth}_1 = \ddot{\eth}_2$.

Depends on the t-Conorm and t-Norm of Archimedian[62], The last q-RLPF operational principles are enhanced to more inclusive form in this part. For this, a bijective-function $g : [i, j] \subseteq \mathbb{R}$ quantity of measurement defined as $g(t) = \frac{t - i_1}{t - i_2}, \forall t \in [i_1, i_2]$, is utilized. Letting $i_1 = 0, i_2 = \ddot{\xi}$, then $g : [0, \ddot{\xi}]$ quantity of measurement.

Definition 13. *Let* $\omega_1 = ((i_1, j_1, k_1), (u_1, v_1, w_1), (x_1, y_1, z_1))$ *and* $\omega_2 = ((i_2, j_2, k_2), (u_2, v_2, w_2), (x_2, y_2, z_2))$ *be two PQRTFNNVs* (1) *within the real number set. Then, the procedures listed below are described as:*

1. Additive operation:

$$\omega_1 \oplus \omega_2 = \left(\frac{P_1 + P_2}{(i_1 + i_2 - i_1 i_2, j_1 + j_2 - j_1 j_2, k_1 + k_2 - k_1 k_2), (u_1 u_2, v_1 v_2, w_1 w_2)} \right);$$

2. Multiplication:

$$\omega_1 \otimes \omega_2 = \left(\frac{P_1 \otimes P_1}{(i_1 i_2, j_1 j_2, k_1 k_2), (u_1 + u_2 - u_1 u_2, v_1 + v_2 - v_1 v_2, w_1 + w_2 - w_1 w_2)} \right);$$

3. Scalar-multiplication:

$$\lambda \omega_1 = \left(\frac{\lambda P_1}{(1 - (1 - i_1)^\lambda, 1 - (1 - j_1)^\lambda, 1 - (1 - k_1)^\lambda), (u_1^\lambda, v_1^\lambda, w_1^\lambda)} \right);$$

$for \lambda > 0$

4. Power operation:

$$\omega_1^\lambda = \left(\frac{P_1^\lambda}{(i_1^\lambda, j_1^\lambda, j_1^\lambda), (1 - (1 - u_1)^\lambda, 1 - (1 - v_1)^\lambda, 1 - (1 - w_1)^\lambda)} \right).$$

$\lambda > 0$

Theorem 14. *Suppose that* $\wp = (\acute{s}_{\mathfrak{Z}}, \{\eth, \Gamma, \daleth\})$, $\wp_1 = (\acute{s}_{\mathfrak{Z}_1}, \{\eth_1, \Gamma_1, \daleth_1\})$ *and* $\wp_2 = (\acute{s}_{\mathfrak{Z}_2}, \{\eth_2, \Gamma_2, \daleth_2\})$ *be any three q-RLNNs and* $\sqsupset_1, \sqsupset_2, \sqsupset_3 \geq 0$*, then the Several guidelines need to apply to these three scalar values.*

1. $\wp_1 \oplus \wp_2 = \wp_2 \oplus \wp_1$
2. $\wp_1 \otimes \wp_2 = \wp_2 \otimes \wp_1$
3. $(\sqsupset_1 \odot \wp) \oplus (\sqsupset_2 \odot \wp) = (\sqsupset_1 + \sqsupset_2) \odot \wp$
4. $(\wp_1 \otimes \wp_2)^{\sqsupset} = (\wp_2)^{\sqsupset} \otimes (\wp_1)^{\sqsupset}$
5. $\sqsupset \odot (\wp_1 \oplus \wp_2) = (\sqsupset \odot \wp_2) \oplus (\sqsupset \odot \wp_1)$
6. $(\wp)^{\sqsupset_1} \otimes (\wp)^{\sqsupset_2} = (\wp)^{(\sqsupset_1 + \sqsupset_2)}$

Theorem 15. *Let us suppose that* $\wp = (\acute{s}_{\mathfrak{Z}}, \{\eth, \Gamma, \daleth\})$, $\wp_1 = (\acute{s}_{\mathfrak{Z}_1}, \{\eth_1, \Gamma_1, \daleth_1\})$ *and* $\wp_2 = (\acute{s}_{\mathfrak{Z}_2}, \{\eth_2, \Gamma_2, \daleth_2\})$ *of three q-RLNS and* $\sqsupset$*, thus, for addition and multiplication, the concurrent terms are:*

1. $\wp_0 \otimes (\wp_1 \otimes \wp_2) = (\wp_1 \otimes \wp_0) \otimes \wp_3$.
2. $(\wp_0 \oplus \wp_1) \oplus \wp_2 = \wp_1 \oplus (\wp_0 \oplus \wp_2)$;

4. Pq-RTFNS AOs

In this part, we inquire into the possibility q-Rung orthopair triangular fuzzy nuetrosophic operators according to defined average A and G operations.

4.1. Possibility q-Rung orthopair triangular fuzzy Neutrosophic number Averaging Aggregation Operator

Definition 16. *[63] Let $\omega_t = \{(i_t, j_t, k_t), (u_t, v_t, w_t), (x_t, y_t, z_t)\}$ for $t = 1, 2, \ldots, r$ of the multitude of PQRTFNNS in the set of real numbers. Let $PQRTFNA : \mathbb{Z}^n \to \mathbb{Z}$. The TFNNAOs indicate by $TFNNA(\omega_1, \omega_2, \ldots, \omega_n)$ is defined as:*

$$TFNNA(\omega_1, \omega_2, \ldots, \omega_n) = p_1\omega_1 \oplus p_2\omega_2 \oplus \ldots \oplus p_n\omega_n = \bigoplus_{t=1}^{r}(p_t\omega_t),$$

where $p_t \in [0,1]$ is the hazard vector of ω_t for $t = 1, 2, \ldots, r$ such that $\sum_{t=1}^{r} p_t = 1$.

If $p = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, then the TFNNPAOs reduces to the TFNNAOs:

$$\omega_1, \omega_2, \ldots, \omega_n = \frac{1}{n}(\omega_1 \oplus \omega_2 \oplus \ldots \oplus \omega_n)$$

The following theorem may now be established using the fundamental operations of TFNNVs defined in Definition 13.

Theorem 17. *Let $\omega_t = \{(i_t, j_t, k_t), (u_t, v_t, w_t), (x_t, y_t, z_t)\}$ for $t = 1, 2, \ldots, r$ be a collection of PQRTFNNS in the set of real numbers. Then the aggregated value acquire by TFNNPA, is also a $TFNNA(\omega_1, \omega_2, \ldots, \omega_n)$ and*

$$\Pi \qquad textTFNNA(\omega_1, \omega_2, \ldots, \omega_n) = p_1\omega_1 \oplus p_2\omega_2 \oplus \ldots \oplus p_n\omega_n = \bigoplus_{t=1}^{r}(p_t\omega_t),$$

$$\begin{bmatrix} 1 - \prod_{t=1}^{r}(1-i_t)^{p_t} & 1 - \prod_{t=1}^{r}(1-j_t)^{p_t} & 1 - \prod_{t=1}^{r}(1-k_t)^{p_t} \\ \prod_{t=1}^{r} u_t^{p_t} & \prod_{t=1}^{r} v_t^{p_t} & \prod_{t=1}^{r} w_t^{p_t} \\ \prod_{t=1}^{r} x_t^{p_t} & \prod_{t=1}^{r} y_t^{p_t} & \prod_{t=1}^{r} z_j^{p_j} \end{bmatrix} \tag{3}$$

where $p_t \in [0,1]$ is the possibility vector of ω_t for $t = 1, 2, \ldots, r$ such that $\sum_{t=1}^{r} p_t = 1$.

Proof. By Mathematical-induction.
(1) When n=1 it is a minor case
When n=2 we have

The direct sum of two matrices ω_1 and ω_2, weighted by p_1 and p_2 respectively, can be expressed as:

$$\bigoplus_{t=1}^{2} p_t\omega_t = p_1\omega_1 \oplus p_2\omega_2$$

(2) The direct sum of matrices ω_1 and ω_2, weighted by p_1 and p_2 respectively, can be expressed as:

$$\left\{\begin{pmatrix} 1-(1-i_1)^{p_t} & 1-(1-t_1)^{p_t} & 1-(1-k_1)^{p_t} \\ u_1^{p_t} & v_1^{p_t} & w_1^{p_t} \\ x_1^{p_t} & y_1^{p_t} & z_1^{p_t} \end{pmatrix}\right\} \oplus \left\{\begin{pmatrix} 1-(1-i_2)^{p_t} & 1-(1-j_2)^{p_t} & 1-(1-k_2)^{p_t} \\ u_2^{p_2} & v_2^{p_2} & w_2^{p_2} \\ x_2^{p_2} & y_2^{p_2} & z_2^{p_2} \end{pmatrix}\right\}$$

The direct sum of matrices ω_1 and ω_2, weighted by p_1 and p_2 respectively, can be expressed as:

$$\left\{\begin{pmatrix} (1-(1-i_1)^{p_1})+(1-(1-i_2)^{p_2})-(1-(1-i_1)^{p_1})(1-(1-i_2)^{p_2}) \\ (1-(1-j_1)^{p_1})+(1-(1-j_2)^{p_2})-(1-(1-j_1)^{p_1})(1-(1-j_2)^{p_2}) \\ (1-(1-k_1)^{p_1})+(1-(1-k_2)^{p_2})-(1-(1-k_1)^{p_1})(1-(1-k_2)^{p_2}) \\ (u_1^{p_1}u_2^{p_2}, v_1^{p_1}v_2^{p_2}, w_1^{p_1}w_2^{p_2}), (x_1^{p_1}x_2^{p_2}, y_1^{p_1}y_2^{p_2}, z_1^{p_1}z_2^{p_2}) \end{pmatrix}\right\}$$

$$\begin{pmatrix} 1-\prod_{t=1}^{2}(1-i_j)^{p_t} & 1-\prod_{t=1}^{2}(1-j_t)^{p_t} & 1-\prod_{t=1}^{2}(1-k_t)^{p_t} \\ \prod_{t=1}^{2}u_t^{p_t} & \prod_{t=1}^{2}v_t^{p_t} & \prod_{t=1}^{2}w_t^{p_t} \\ \prod_{t=1}^{2}x_t^{p_t} & \prod_{t=1}^{2}y_t^{p_t} & \prod_{t=1}^{2}z_t^{p_t} \end{pmatrix}$$

Thus the theorem is true for $n=2$.

(3) When $n=k$ we suppose that this is also true

The expression $TFNNWA(\omega_1, \omega_2, \omega_3, \ldots, \omega_k) = p_1\omega_1 \oplus p_2\omega_2 \oplus \ldots \oplus p_n\omega_n = \bigoplus_{t=1}^{r}(p_t\omega_t)$ can be represented in Latex :

$$TFNNWA(\omega_1, \omega_2, \omega_3, \ldots, \omega_k) = p_1\omega_1 \oplus p_2\omega_2 \oplus \ldots \oplus p_n\omega_n = \bigoplus_{t=1}^{k}(p_t\omega_r)$$

$$\begin{pmatrix} 1-\prod_{t=1}^{r}(1-i_t)^{p_t} & 1-\prod_{t=1}^{r}(1-j_t)^{p_t} & 1-\prod_{t=1}^{r}(1-k_t)^{p_t} \\ \prod_{t=1}^{r}u_t^{p_t} & \prod_{t=1}^{r}v_t^{p_t} & \prod_{t=1}^{r}w_t^{p_t} \\ \prod_{t=1}^{r}x_t^{p_t} & \prod_{t=1}^{r}y_t^{p_t} & \prod_{t=1}^{r}z_t^{p_t} \end{pmatrix}$$

(4) When $n=r+1$, we have

The expression $TNFNWA(\omega_1, \omega_2, \ldots, \omega_{k+1}) = \bigoplus_{t=1}^{r}(w_t\omega_t) \oplus (w_{k+1}\omega_{k+1})$ can be represented in LaTeX as:

$$TNFNWA(\omega_1, \omega_2, \ldots, \omega_{r+1}) = \bigoplus_{t=1}^{r}(w_t\omega_t) \oplus (w_{r+1}\omega_{r+1})$$

$$\begin{pmatrix} 1-(\prod_{t=1}^{r}(1-i_t)^{p_t})-(1-a_{r+1})^{p_{r+1}}-1-(\prod_{t=1}^{r}(1-i_t)^{p_t})\cdot(1-i_{r+1})^{p_{r+1}} \\ 1-(\prod_{t=1}^{r}(1-j_t)^{p_t})-(1-j_{r+1})^{p_{r+1}}-1-(\prod_{t=1}^{r}(1-j_t)^{p_t})\cdot(1-j_{r+1})^{p_{r+1}} \\ 1-(\prod_{t=1}^{r}(1-k_t)^{p_t})-(1-k_{r+1})^{p_{r+1}}-1-(\prod_{t=1}^{r}(1-k_t)^{p_t})\cdot(1-k_{r+1})^{p_{r+1}} \\ \prod_{j=1}^{r}u_t^{p_t}\cdot u_{r+1}^{p_{r+1}} \prod_{t=1}^{r}v_t^{p_t}\cdot v_{r+1}^{p_{r+1}} \prod_{t=1}^{r}w_t^{p_t}\cdot w_{r+1}^{p_{r+1}} \\ \prod_{j=1}^{r}x_t^{p_t}\cdot x_{r+1}^{p_{r+1}} \prod_{t=1}^{r}y_t^{p_t}\cdot y_{r+1}^{p_{r+1}} \prod_{t=1}^{r}z_t^{p_t}\cdot z_{r+1}^{p_{r+1}} \end{pmatrix}$$

$$\begin{pmatrix} 1-\left(\prod_{t=1}^{r+1}(1-i_t)^{p_t}\right) & 1-\left(\prod_{t=1}^{r+1}(1-j_t)^{p_t}\right) & 1-\left(\prod_{t=1}^{r+1}(1-k_t)^{p_t}\right) \\ \prod_{t=1}^{k+1}u_t^{p_t} & \prod_{t=1}^{k+1}v_t^{p_t} & \prod_{t=1}^{r+1}w_t^{p_t} \\ \prod_{t=1}^{k+1}x_t^{p_t} & \prod_{t=1}^{k+1}y_t^{p_t} & \prod_{t=1}^{r+1}z_t^{p_t} \end{pmatrix}$$

we observe that the theorem is true for n.Therefore by mathematical induction ,we can say Equation grasp for all worth of n.as the element of MF of ,then following connection are reasonable.

$$0 \leq \left[1 - \prod_{t=1}^{r}(1-k_t)^{p_t}\right] \leq 1$$

$$0 \leq \left[\prod_{t=1}^{r} w_t^{p_t}\right] \leq 1$$

$$0 \leq \left[\prod_{t=1}^{r} z_t^{p_t}\right] \leq 1$$

it follows that the relation

$$0 \leq \left[1 - \sum_{t=1}^{r}(1-k_t)^{p_t} + \sum_{t=1}^{r} w_t^{p_t} + \sum_{t=1}^{r} z_t^{p_t}\right] \leq 3$$

is also valid This complete the proof of theorem 1.Now , we highlight some compulsory effects of TFNNWA operator. □

Theorem 18. *(Idempotency)*

If all ω_j *for* $t = 1,2,3,\ldots,r$ *are equal, i.e.,* $\omega_t = \omega = \begin{pmatrix} i & j & k \\ u & v & w \\ x & y & z \end{pmatrix}$ *for all j, then* $TFNNWA(\omega_1, \omega_2, \ldots, \omega_k) = \omega$.

Proof. We have

$$TFNNWA(\omega_1, \omega_2, \ldots, \omega_n) = TFNNWA(\omega, \omega, \ldots, \omega) = \bigoplus_{t=1}^{r}(w_t \omega)$$

$$\begin{pmatrix} 1 - \prod_{j=1}^{n}(1-i)^{p_j} & 1 - \prod_{t=1}^{r}(1-t)^{p_t} & 1 - \prod_{t=1}^{r}(1-k)^{p_t} & \\ \sum_{t=1}^{r} u^{p_t} & \sum_{t=1}^{r} v^{p_t} & \sum_{t=1}^{r} w^{p_t} & \\ \sum_{t=1}^{r} x^{p_t} & \sum_{t=1}^{r} x^{p_t} & \sum_{t=1}^{r} y^{p_t} & \sum_{t=1}^{r} z^{p_t} \end{pmatrix}$$

$$\begin{pmatrix} 1 - (1-i)^{\sum_{t=1}^{r} p_t} & 1 - (1-t)^{\sum_{t=1}^{r} p_t} & 1 - (1-k)^{\sum_{t=1}^{r} p_t} \\ u^{\sum_{t=1}^{r} p_t} & v^{\sum_{t=1}^{r} p_t} & w^{\sum_{t=1}^{r} p_t} \\ x^{\sum_{t=1}^{r} p_t} & y^{\sum_{t=1}^{r} p_t} & z^{\sum_{t=1}^{r} p_t} \end{pmatrix}$$

Let $\omega = \begin{pmatrix} i & j & k \\ u & v & w \\ x & y & z \end{pmatrix}$.

This completes the proof of property 1. □

Theorem 19. *(Monotonicity) suppose that*

If $\omega_t^1 = \begin{pmatrix} i_t^1 & j_t^1 & k_t^1 \\ u_t^1 & v_t^1 & w_t^1 \\ x_t^1 & y_t^1 & z_t^1 \end{pmatrix}$ *and* $\omega_t^2 = \begin{pmatrix} i_t^2 & j_t^2 & k_t^2 \\ u_t^2 & v_t^2 & w_t^2 \\ x_t^2 & y_t^2 & z_t^2 \end{pmatrix}$ *for* $t = 1,2,3,\ldots,r$ *be a collection of two TFNNVs in the set of real numbers, and if* $\omega_t^1 \leq A_t^2$ *for* $t = 1,2,3,\ldots,r$*, then* $TFNNWA(\omega_1^1, \omega_2^1, \ldots, \omega_n^1) \leq TFNNWA(\omega_1^2, \omega_2^2, \ldots, \omega_n^2)$.

Proof. We first consider k_t^1, w_t^1, z_t^1 of ω_t^1 and k_t^2, w_t^2, z_t^2 of ω_t^2 to prove property 3. We can consider $k_t^1 \leq k_t^2$, $w_t^1 \geq w_t^2$, and $z_t^1 \geq z_t^2$ for $\omega_t^1 \leq \omega_t^2$ $(t = 1,2,3,\ldots,r)$. Then we have $(1-k_t^1)^{p_t} \geq (1-k_t^2)^{p_t}$, $(w_t^1)^{p_t} \geq (w_t^2)^{p_t}$, and $(z_t^1)^{p_t} \geq (z_t^2)^{p_t}$; $1-\prod_{t=1}^r (1-k_t^1)^{p_t} \leq 1-\prod_{t=1}^r (1-k_t^2)^{p_t}$, $(w_t^1)^{p_t} \geq (w_t^2)^{p_t}$, and $(z_t^1)^{p_t} \geq (z_t^2)^{p_t}$.

Therefore,
$1-\prod_{t=1}^r (1-k_t^1)^{p_t} < 1-\prod_{t=1}^r (1-k_t^2)^{p_t}$; $\sum_{t=1}^r (w_t^1)^{p_t} > \sum_{t=1}^r (w_t^2)^{p_t}$,
and $\sum_{t=1}^r (z_t^1)^{p_t} > \sum_{t=1}^r (z_t^2)^{p_t}$.

Similarly, we can show
$1-\prod_{t=1}^r (1-i_j^1)^{p_j} \leq 1-\prod_{j=1}^n (1-i_j^2)^{p_j}$; $\sum_{j=1}^n (u_j^1)^{p_j} > \sum_{j=1}^n (u_j^2)^{p_j}$,
and $\sum_{t=1}^r (x_t^1)^{p_t} \geq \sum_{t=1}^r (x_t^2)^{p_t}$; $1-\prod_{t=1}^r (1-j_t^1)^{p_t} \leq 1-\prod_{t=1}^r (1-j_t^2)^{p_t}$;
$\sum_{t=1}^r (v_t^1)^{p_t} > \sum_{t=1}^r (v_t^2)^{p_t}$, and $\sum_{t=1}^r (y_t^1)^{p_t} \geq \sum_{t=1}^r (y_t^2)^{p_t}$.

Assume that
$\omega^1 = TFNNWA(\omega_1^1, \omega_2^1, \ldots, \omega_n^1) = \{(i^1, j^1, k^1), (u^1, v^1, w^1), (x^1, y^1, z^1)\}$ and
$\omega^2 = TFNNWA(\omega_1^2, \omega_2^2, \ldots, \omega_n^2) = \{(i^2, j^2, k^2), (u^2, v^2, w^2), (x^2, y^2, z^2)\}$, where $i^s = 1 - \prod_{t=1}^r (1-i_t^s)^{p_t}$, $t^s = 1-\prod_{t=1}^r (1-j_t^s)^{p_t}$, $k^s = 1-\prod_{t=1}^r (1-k_t^s)^{p_t}$; $u^s = \sum_{t=1}^r (u_t^s)^{p_t}$, $v^s = \sum_{t=1}^r (v_t^s)^{p_t}$, $w^s = \sum_{t=1}^r (w_t^s)^{p_t}$, and $x^s = \sum_{t=1}^r (x_t^s)^{p_t}$, $y^s = \sum_{t=1}^r (y_t^s)^{p_t}$, $z^s = \sum_{t=1}^r (z_t^s)^{p_t}$. for $s = 1,2$. Now we consider the score function $S(\omega^1) = \frac{1}{12}[8+(i^1+2j^1+k^1)-(u^1+2v^1+w^1)-(x^1+2y^1+z^1)] \leq \frac{1}{12}[8+(i^2+2j^2+k^2)-(u^2+2v^2+w^2)-(x^2+2y^2+z^2)] = S(\omega^2)$.

Now we examine the retinue two cases:

(Case 1) .
If $S(\omega^1) < S(\omega^2)$, from the definition we have
$TNFNWA(\omega_1^1, \omega_2^1, \ldots, \omega_n^1) < TNFNWA(\omega_1^2, \omega_2^2, \ldots, \omega_n^2)$.

(Case 2) . If $S(\omega^1) = S(\omega^2)$, then by equation we can rewrite:

$$\frac{1}{12}\Big[8+(i^1+2j^1+k^1)-(u^1+2v^1+w^1)-(x^1+2y^1+z^1)\Big] =$$

$$\frac{1}{12}\Big[8+(i^2+2j^2+k^2)-(u^2+2v^2+w^2)-(x^2+2y^2+z^2)\Big].$$

Thus, for ω_t^1 is less than or equal to ω_t^2 $(t = 1,2,\ldots,r)$, i.e., for $i_t^1 \leq i_t^2$, $j_t^1 \leq j_t^2$, $k_t^1 \leq k_t^2$, $u_t^1 \geq u_t^2$, $v_t^1 \geq v_t^2$, $w_t^1 \geq w_t^2$, $x_t^1 \leq x_t^2$, $y_t^1 \geq y_t^2$, and $z_t^1 \geq z_t^2$, we have:

$$i^1 = i^2, \quad j^1 = j^2, \quad k^1 = k^2, \quad u^1 = u^2, \quad v^1 = v^2, \quad w^1 = w^2, \quad x^1 = x^2, \quad y^1 = y^2, \quad z^1 = z^2 .$$

Then the accuracy function of ω^1 yields:

$$H(\omega^1) = \frac{1}{4}\left[(i^1+2j^1+k^1)-(x^1+2y^1+z^1)\right] = \frac{1}{4}\left[(i^2+2j^2+k^2)-(x^2+2y^2+z^2)\right] = H(\omega^2).$$

Thus, from the definition, we have:

$$TNFNWA(\omega_1^1, \omega_2^1, \ldots, \omega_n^1) = TNFNWA(\omega_1^2, \omega_2^2, \ldots, \omega_n^2).$$

Finally, from both equations, we have the following result:

$$TFNNWA(\omega_1^1, \omega_2^1, \ldots, \omega_n^1) \leq TFNNWA(\omega_1^1, \omega_2^1, \ldots, \omega_n^2).$$

This complete the proof of property 2. □

Theorem 20. *(Boundedness)*
Let $\omega_t = \{(i_j, j_j, k_j), (u_j, v_j, w_j), (x_j, y_j, z_j)\}$*, where* $t = 1, 2, 3, \ldots, r$*, be a collection of TFNNVs in the set of real numbers.*
Assume $\omega^+ = \{(\max_t(i_t), \max_t(j_t), \max_t(k_t)), (\min_t(u_t), \min_t(v_t), \min_t(w_t)), (\min_t(x_t), \min_t(y_t), \min_t(z_t))\}$ *and* $\omega^- = \{(\min_t(i_t), \min_t(j_t), \min_t(k_t)), (\max_t(u_t), \max_t(v_t), \max_t(w_t)), (\max_t(x_t), \max_t(y_t), \max_t(z_t))\}$ *for all* $t = 1, 2, \ldots, r$*.*
Then $\omega^- \leq TFNNWA(\omega_1, \omega_2, \ldots, \omega_n) \leq \omega^+$*.*
Proof: we have Let $\min_t(k_t) \leq k_t \leq \max_t(k_t)$*,* $\min_t(w_t) \leq w_t \leq \max_t(w_t)$*, and* $\min_t(z_t) \leq z_t \leq \max_t(z_t)$ *for* $t = 1, 2, \ldots, r$*. Then*

$$\begin{aligned}
&1 - \prod_{t=1}^{r} (1 - \min_t(k_t))^{p_t} \\
&\leq 1 - \prod_{t=1}^{r} (1 - k_t)^{p_t} \\
&\leq 1 - \prod_{t=1}^{r} (1 - \max_t(k_t))^{p_t} \\
&= 1 - \Big((1 - \min_t(k_t))\Big)^{\sum_{t=1}^{r} p_t} \\
&\leq 1 - \prod_{t=1}^{r} (1 - k_t)^{p_t} \\
&\leq 1 - \Big((1 - \max_t(k_t))\Big)^{\sum_{t=1}^{r} p_t} \\
&= \min_t(k_t) \leq 1 - \prod_{t=1}^{r} (1 - k_t)^{p_t} \leq \max_t(k_t).
\end{aligned}$$

Again from the equation, we have for $t = 1, 2, \ldots, r$:

$$\prod_{t=1}^{r}(\min_t(w_t))^{p_t}$$
$$\leq \prod_{t=1}^{r} w_t^{p_t}$$
$$\leq \prod_{t=1}^{r}(\max_t(w_t))^{p_t}$$
$$= (\min_t(w_t))^{\sum_{t=1}^{r} p_t}$$
$$\leq \prod_{t=1}^{r} w_t^{p_t}$$
$$\leq (\max_t(w_t))^{\sum_{t=1}^{r} p_t}$$
$$= \min_t(w_t)$$
$$\leq \prod_{t=1}^{r} e^{p_t}$$
$$\leq \max_t(w_t);$$

and

$$\prod_{t=1}^{r}(\min_t(z_t))^{p_t}$$
$$\leq \prod_{t=1}^{r} z_t^{p_t}$$
$$\leq \prod_{t=1}^{r}(\max_t(z_t))^{p_t}$$
$$= (\min_t(z_t))^{\sum_{t=1}^{r} p_t}$$
$$\leq \prod_{t=1}^{r} z_t^{p_t}$$
$$\leq (\max_t(z_t))^{\sum_{t=1}^{r} p_t}$$
$$= \min_t(z_t)$$
$$\leq \prod_{t=1}^{t} z_t^{p_t}$$
$$\leq \max_t(z_t).$$

For $t = 1, 2, \ldots, r$:

$$\min_t(i_t) \le 1 - \prod_{t=1}^{r}(1 - i_j)^{p_t} \le \max_t(i_t),$$
$$\min_t(j_t) \le 1 - \prod_{t=1}^{r}(1 - j_j)^{p_t} \le \max_t(j_t),$$
$$\min_t(u_t) \le 1 - \prod_{t=1}^{r}(1 - u_j)^{p_t} \le \max_t(u_t),$$
$$\min_t(v_t) \le 1 - \prod_{t=1}^{r}(1 - v_j)^{p_t} \le \max_t(v_t),$$
$$\min_t(x_t) \le 1 - \prod_{t=1}^{r}(1 - x_j)^{p_t} \le \max_t(x_t),$$
$$\min_t(y_t) \le 1 - \prod_{t=1}^{r}(1 - y_j)^{p_t} \le \max_t(y_t).$$

Assume that $TFNNWA_p(\omega_1, \omega_2, \ldots, \omega_n) = \{(i, j, k), (u, v, w), (x, y, z)\}$, then the score function of ω, $S(\omega)$, is:

$$S(\omega) = \frac{1}{12}\left[8 + (i + 2j + k) - (u + 2v + w) - (x + 2y + z)\right]$$

is less than or equal to

$$\frac{1}{12}\Big[8 + \Big(\max_t(i_t) + \max_t(2j_t) + \max_t(k_t)\Big) - \Big(\min_t(u_t) + \min_t(2v_t) + \min_t(w_t)\Big) - \Big(\min_t(x_t) + \min_t(2y_t) + \min_t(z_t)\Big)\Big] = S(\omega^+). \tag{4}$$

Similarly, the score function of ω, $S(\omega)$, is:

$$S(\omega) = \frac{1}{12}\left[8 + (i + 2j + k) - (u + 2v + w) - (x + 2y + z)\right]$$

is greater than or equal to

$$\frac{1}{12}\Big[8 + \Big(\min_t(i_t) + 2\min_t(j_t) + \min_t(k_t)\Big) - \Big(\max_t(u_t) + 2\max_t(v_t) + \max_t(w_t)\Big) - \Big(\max_t(x_t) + 2\max_t(y_t) + \max_t(z_t)\Big)\Big] = S(\omega^-). \tag{5}$$

Now, we consider the following cases.

(Case 1) *.*
If $S(\omega) < S(\omega^+)$ and $S(\omega) > S(\omega^-)$, then we have $\omega^- < TFNNWA(\omega_1, \omega_2, \ldots, \omega_n) < \omega^+$.

(Case 2) *.*
If $S(\omega) = S(\omega^+)$, then we can take:

$$\frac{1}{12}\left[8+(i+2j+k)-(u+2v+w)-(x+2y+z)\right]$$

and

$$\frac{1}{12}\left[8+\left(\max_t(i_t)+2\max_t(j_t)+\max_t(k_t)\right)-\left(\min_t(u_t)+2\min_t(v_t)+\min_t(w_t)\right)-\left(\min_t(x_t)+2\min_t(y_t)+\min_t(z_t)\right)\right].$$

(case 3) *.*

it follows that

$$(i+2j+k)=(\max_t(i_t)+2\max_t(j_t)+\max_t(k_t)),$$

$$(u+2v+w)=(\min_t(u_t)+2\min_t(v_t)+\min_t(w_t)),$$

$$(x+2y+z)=(\min_t(x_t)+2\min_t(y_t)+\min_t(z_t)).$$

Therefore, the accuracy function of ω *is given by:*

$$H(\omega)=\frac{1}{4}\left[(i+2j+k)-(x+2y+z)\right]$$

$$=\frac{1}{4}\left[(\max_t(i_t)+2\max_t(j_t)+\max_t(k_t))-(\min_t(x_t)+\min_t(2y_t)+\min_t(z_t))\right]=H(\omega^+).$$

Thus, we have:

$$TFNNWA(\omega_1,\omega_2,\ldots,\omega_n)=\omega^+.$$

Similarly, for $S(\omega)=S(\omega^-)$*, the accuracy function of* ω *is:*

$$H(\omega)=\frac{1}{4}\left[(i+2j+k)-(x+2y+z)\right]$$

$$=\frac{1}{4}\left[(\min_t(i_t)+\min_t(2j_t)+\min_t(k_t))-(\max_t(x_t)+\max_t(2y_t)+\max_t(z_t))\right]=H(\omega^-).$$

From these equations, we obtain:

$$TFNNWA(\omega_1,\omega_2,\ldots,\omega_n)=\omega^-.$$

Combining the equations, we conclude:

$$\omega^-\leq TFNNWA(\omega_1,\omega_2,\ldots,\omega_n)\leq\omega^+.$$

This proves the property 3.

4.2. Possiility q-Rung orthopair triangular fuzzy Neutrosophic Geometric Aggregation Operator

Definition 21. *Suppose that* $\omega_t = (i_t, j_t, k_t), (u_t, v_t, w_t), (x_t, y_t, z_t)$, $(t = 1, 2, \cdots, r)$ *be a collection TFNNVswithin the collection of actual numbers and TFNNWG* $:\Theta^n \to \Theta$. *The TFNNPG operator shows that by* $TFNNWG_p(\omega_1, \omega_2, \cdots, \omega_n)$ *is defined as follows:*
$TFNNWG_p(\omega_1, \omega_2, \cdots, \omega_n) = \omega_1^{p_1} \otimes \omega_2^{p_2} \otimes \cdots \otimes \omega_n^{p_n}$

$$= \bigotimes_{t=1}^{r} (\omega_t^{p_t})$$

where $p_t \in [o, 1]$ *is exponential weight operator of* $\omega_t (t = 1, 2, \cdots, r)$ *s.t* $\sum_{t=1}^{r} p_t = 1$. *In specific, if* $p = (1/n, 1/n, \cdots, 1/n)^T$ *then the* $TFNNWG_p(\omega_1, \omega_2, \cdots, \omega_n)$*operator transforms into a triangular fuzzy geometric (TNFG) operator.:*

$$TFNNWG_p(\omega_1, \omega_2, \cdots, \omega_n) = (\omega_1 \otimes \omega_2 \otimes \cdots \otimes \omega_n)^{\frac{1}{n}}.$$

We now use the fundamental TFNNV operations to demonstrate the following theorem. defined in Definition 10.

Theorem 22. $\omega_j = \{(i_t, j_t, k_t), (u_t, v_t, w_t), (x_t, y_t, z_t)\}$, $(t = 1, 2, \cdots, r)$ *be a collection TFNNVs in the set of real numbers. Then the*

$$= \Big[\{(i_1^{p_1}, j_1^{p_1}, k_1^{p_1}), (1-(1-u_1)^{p_1}, 1-(1-v_1)^{p_1}, 1-(1-w_1)^{p_1}), (1-(1-x_1)^{p_1}, 1-(1-y_1)^{p_1}, 1-(1-z_1)^{p_1})\}$$
$$\otimes \{(i_2^{p_1}, j_2^{p_1}, k_2^{p_1}), (1-(1-u_2)^{p_1}, 1-(1-v_2)^{p_1}, 1-(1-w_2)^{p_1}), (1-(1-x_2)^{p_1}, 1-(1-y_2)^{p_1}, 1-(1-z_2)^{p_1})\}\Big]$$

aggregated value from TFNNPG, which is likewise a TFNNV, and after that, we have
$TFNNWG_p(\omega_1, \omega_2, \cdots, \omega_n)$
$= \omega_1^{p_1} \otimes \omega_2^{p_2} \otimes \cdots \otimes \omega_n^{p_n}$
$= \bigotimes_{j=1}^{n} (\omega_j^{p_j})$

$$\begin{aligned} = \Bigg\langle & \left[\prod_{t=1}^{r} i_t^{p_t}, \prod_{t=1}^{r} j_t^{p_t}, \prod_{t=1}^{r} k_t^{p_t}\right], \\ & \left[1 - \prod_{t=1}^{r}(1-u_t)^{p_t}, 1 - \prod_{t=1}^{r}(1-v_t)^{p_t}, 1 - \prod_{t=1}^{r}(1-w_t)^{p_t}\right], \\ & \left[1 - \prod_{t=1}^{r}(1-x_t)^{p_t}, 1 - \prod_{t=1}^{r}(1-y_t)^{p_t}, 1 - \prod_{t=1}^{r}(1-z_t)^{p_t}\right] \Bigg\rangle \end{aligned} \tag{6}$$

where $p_t \in [o, 1]$ *is the poss vector of TFNNV* $\omega_t (t = 1, 2, \cdots, r)$ *such that* $\sum_{t=1}^{r} p_t = 1$.

We may also use mathematical induction to establish the theorem, just like we would with an arithmetic averaging operator.
1. When $n = 1$*, the theorem is true.*
2. When $n = 2$*, we have*

$$= \bigotimes_{t=1}^{2}(\omega_t^{p_t}) = \omega_1^{p_1} \otimes \omega_2^{p_2}$$

$$= \Big\langle (a_1^{p_1}a_2^{p_2}, b_1^{p_1}b_2^{p_2}, c_1^{p_1}c_2^{p_2}), \Big[(1-(1-e_1)^{p_1}) + (1-(1-e_2)^{p_2}) - (1-(1-e_1)^{p_1}).(1-(1-e_2)^{p_2}),$$
$$(1-(1-f_1)^{p_1}) + (1-(1-f_2)^{p_2}) - (1-(1-f_1)^{p_1}).(1-(1-f_2)^{p_2}),$$
$$(1-(1-c_1)^{p_1}) + (1-(1-c_2)^{p_2}) - (1-(1-c_1)^{p_1}).(1-(1-c_2)^{p_2})\Big]$$
$$\Big[(1-(1-r_1)^{p_1}) + (1-(1-r_2)^{p_2}) - (1-(1-r_1)^{p_1}).(1-(1-r_2)^{p_2}),$$
$$(1-(1-s_1)^{p_1}) + (1-(1-s_2)^{p_2}) - (1-(1-s_1)^{p_1}).(1-(1-s_2)^{p_2}),$$
$$(1-(1-t_1)^{p_1}) + (1-(1-t_2)^{p_2}) - (1-(1-t_1)^{p_1}).(1-(1-t_2)^{p_2})\Big]\Big\rangle$$

$$\begin{aligned} = \Bigg\langle & \left[\prod_{t=1}^{2} a_t^{p_t}, \prod_{t=1}^{2} b_t^{p_t}, \prod_{t=1}^{2} c_t^{p_t}\right], \\ & \left[1-\prod_{t=1}^{2}(1-e_t)^{p_t}, 1-\prod_{t=1}^{2}(1-f_t)^{p_t}, 1-\prod_{t=1}^{2}(1-g_t)^{p_t}\right], \\ & \left[1-\prod_{t=1}^{2}(1-r_t)^{p_t}, 1-\prod_{t=1}^{2}(1-s_t)^{p_t}, 1-\prod_{t=1}^{2}(1-t_t)^{p_t}\right]\Bigg\rangle \end{aligned} \tag{7}$$

3. When $n = r$*, we assume that Eq.(49) is true then,*
$TNFNWG_p(\omega_1, \omega_2, \cdots, \omega_k) = (\omega_1^{p_1} \otimes \omega_2^{p_2} \otimes \cdots \otimes \omega_k^{p_k})$

$$\begin{aligned} = \Bigg\langle & \left[\prod_{t=1}^{r} a_t^{p_t}, \prod_{t=1}^{r} b_t^{p_t}, \prod_{t=1}^{r} c_t^{p_t}\right], \\ & \left[1-\prod_{t=1}^{r}(1-e_t)^{p_t}, 1-\prod_{t=1}^{k}(1-f_t)^{p_t}, 1-\prod_{t=1}^{r}(1-g_t)^{p_t}\right], \\ & \left[1-\prod_{t=1}^{r}(1-r_t)^{p_t}, 1-\prod_{t=1}^{k}(1-s_t)^{p_t}, 1-\prod_{t=1}^{r}(1-t_t)^{p_t}\right]\Bigg\rangle \end{aligned} \tag{8}$$

4. When $n = r+1$*, we can consider the following expression:*

$$TNFNPG_p(\omega_1, \omega_2, \cdots, \omega_{r+1}) = \bigotimes_{t=1}^{r} (\omega_t)^{p_t} \otimes (\omega_{r+1})^{pr+1}$$

$$\begin{aligned}
&= \Bigg\langle \left[\prod_{t=1}^{r} a_t^{p_t} \cdot a_{r+1}^{p_{r+1}}, \prod_{t=1}^{r} b_t^{p_t} \cdot b_{r+1}^{p_{r+1}}, \prod_{t=1}^{r} c_t^{p_t} \cdot c_{r+1}^{p_{r+1}} \right], \\
&\Bigg[\left[1 - \prod_{t=1}^{r} (1-e_t)^{p_t} \right] + (1-(1-e_{r+1})^{p_{r+1}}) \\
&- \left[1 - \prod_{t=1}^{r} (1-e_t)^{p_t} \right] \cdot (1-(1-e_{r+1})^{p_{r+1}}), \\
&\left[1 - \prod_{t=1}^{r} (1-f_t)^{p_t} \right] + (1-(1-f_{r+1})^{p_{r+1}}) \\
&- \left[1 - \prod_{t=1}^{r} (1-f_t)^{p_t} \right] \cdot (1-(1-f_{r+1})^{p_{r+1}}), \\
&\left[1 - \prod_{t=1}^{r} (1-g_t)^{p_t} \right] + (1-(1-g_{r+1})^{p_{r+1}}) \\
&- \left[1 - \prod_{t=1}^{r} (1-g_t)^{p_t} \right] \cdot (1-(1-g_{r+1})^{p_{r+1}}) \Bigg], \\
&\Bigg[\left[1 - \prod_{t=1}^{r} (1-r_t)^{p_t} \right] + (1-(1-r_{r+1})^{p_{r+1}}) \\
&- \left[1 - \prod_{t=1}^{r} (1-r_t)^{p_t} \right] \cdot (1-(1-r_{r+1})^{p_{r+1}}), \\
&\left[1 - \prod_{t=1}^{r} (1-s_t)^{p_t} \right] + (1-(1-s_{r+1})^{p_{r+1}}) \\
&- \left[1 - \prod_{t=1}^{r} (1-s_t)^{p_t} \right] \cdot (1-(1-s_{r+1})^{p_{r+1}}), \\
&\left[1 - \prod_{t=1}^{r} (1-t_t)^{p_t} \right] + (1-(1-t_{r+1})^{p_{r+1}}) \\
&- \left[1 - \prod_{t=1}^{r} (1-t_t)^{p_t} \right] \cdot (1-(1-t_{r+1})^{p_{r+1}}) \Bigg]
\end{aligned} \tag{9}$$

$$= \langle \left[\prod_{t=1}^{r+1} a_t^{p_t}, \prod_{t=1}^{r+1} b_t^{p_t}, \prod_{t=1}^{r+1} c_t^{p_t} \right], \left[1 - \prod_{t=1}^{r+1} (1-e_t)^{p_t}, 1 - \prod_{t=1}^{r+1} (1-f_t)^{p_t}, 1 - \prod_{t=1}^{r+1} (1-g_t)^{p_t} \right], \left[1 - \prod_{t=1}^{r+1} (1-r_t)^{p_t}, 1 - \prod_{t=1}^{r+1} (1-s_t)^{p_t}, 1 - \prod_{t=1}^{r+1} (1-t_t)^{p_t} \right] \rangle \tag{10}$$

As we can see, the theory also holds for $n = r+1$.
Therefore, by mathematical induction, holds for all values of n.
Given that the elements of each of the three membership roles of $\omega_t (t = 1,2,\cdots,r)$ belongs to $[0,1]$ the following relations are valid

$$0 \le [\prod_{t=1}^{r} c_t^{p_t}] \le 1, 0 \le [1 - \prod_{t=1}^{r} (1-g_t)^{p_t}] \le 1, and 0 \le [1 - \prod_{t=1}^{r} (1-t_t)^{p_t}] \le 1$$

It follows that
$0 \le [\prod_{t=1}^{r} c_t^{p_t} + 1 - \prod_{t=1}^{r} (1-g_t)^{p_t} + 1 - \prod_{t=1}^{r} (1-t_t)^{p_t}] \le 3.$
This concludes Theorem 2's proof. We now go over a few key characteristics of the TFN-NWG operator for TFNNs.

Theorem 23. *(Boundedness) Let $\omega_t = \{(a_t, b_t, c_t), (e_t, f_t, g_t), (r_t, s_t, t_t)\}$, where $t = 1,2,\ldots,r$, be a collection of TFNNs in the set of real numbers. Suppose that:*

$$\omega^+ = \left\{ \left(\max_t(a_t), \max_t(b_t), \max_t(c_t) \right), \left(\min_t(e_t), \min_t(f_t), \min_t(g_t) \right), \left(\min_t(r_t), \min_t(s_t), \min_t(t_t) \right) \right\}$$

and

$$\omega^- = \left\{ \left(\min_t(a_t), \min_t(b_t), \min_t(c_t) \right), \left(\max_t(e_t), \max_t(f_t), \max_t(g_t) \right), \left(\max_t(r_t), \max_t(s_t), \max_t(t_t) \right) \right\}$$

for all $t = 1,2,\ldots,r$. Then,

$$\omega^- \le TNFNPG_p(\omega_1, \omega_2, \ldots, \omega_n) \le \omega^+.$$

Proof. The proof of the Property 5 is similar to property 2. □

Theorem 24. *(Monotonicity)*
Let $\omega_t^1 = \{(a_t^1, b_t^1, c_t^1), (e_t^1, f_t^1, g_t^1), (r_t^1, s_t^1, t_t^1)\}$ and $\omega_t^2 = \{(a_t^2, b_t^2, c_t^2), (e_t^2, f_t^2, g_t^2), (r_t^2, s_t^2, t_t^2)\}$, $(t = 1,2,\ldots,r)$ be a multitude of TFNNVs with the collection of actual numbers. If $\omega_t^1 \le \omega_t^2$ for $t = 1,2,\ldots,r$, then $TFNNWG_p(\omega_1^1, \omega_2^1, \ldots, \omega_n^1) \le TFNNWG_p(\omega_1^2, \omega_2^2, \ldots, \omega_n^2)$.

Proof. Property 6 can be proved by a similar argument of Property 3. Therefore, we do not discuss again to avoid repetition. □

Theorem 25. *(Idempotency)*
If all ω_t for $t = 1,2,\ldots,r$ are equal, that is $\omega_t = \{(a,b,c),(e,f,g),(r,s,t)\}$ for all t, then $TFNNWG_p(\omega_1, \omega_2, \ldots, \omega_n) = \omega$.

Proof.

$$
\begin{aligned}
&TFNNWG_p(\omega_1,\omega_2,\ldots,\omega_n) = TFNNWG_p(\omega,\omega,\ldots,\omega)\\
&= \prod_{t=1}^{r}(\omega_t)^{p_t}\\
&= \Bigg\{\left(\sum_{t=1}^{r} a^{p_t}, \sum_{t=1}^{r} b^{p_t}, \sum_{t=1}^{r} c^{p_t}\right),\\
&\quad \left(1-\sum_{t=1}^{r}(1-e)^{p_t}, 1-\sum_{t=1}^{r}(1-f)^{p_t}, 1-\sum_{t=1}^{r}(1-g)^{p_t}\right),\\
&\quad \left(1-\sum_{t=1}^{r}(1-r)^{p_t}, 1-\sum_{t=1}^{r}(1-s)^{p_t}, 1-\sum_{t=1}^{r}(1-t)^{p_t}\right)\Bigg\}\\
&= \Big\{\left(a^{\sum_{t=1}^{r} p_t}, b^{\sum_{t=1}^{r} p_t}, c^{\sum_{t=1}^{r} p_t}\right),\\
&\quad \left(1-(1-e)^{\sum_{t=1}^{r} p_t}, 1-(1-f)^{\sum_{t=1}^{r} p_t}, 1-(1-g)^{\sum_{t=1}^{r} p_t}\right),\\
&\quad \left(1-(1-r)^{\sum_{t=1}^{r} p_t}, 1-(1-s)^{\sum_{t=1}^{r} p_t}, 1-(1-t)^{\sum_{t=1}^{r} p_t}\right)\Big\}\\
&= \{(a,b,c),(e,f,g),(r,s,t)\} = \omega.
\end{aligned}
$$

This complete the property 3. □

5. AN INNOVATIVE TECHNIQUE FOR EFFECTIVE DECISION-MAKING

This section will provide an approach to MADM problem solution based on Pq-RTFN operators. Assume that we have P=$\{p_1, p_2, p_3, ..., p_k\}$ almost finite values of k substitute, here in our hand a limited number of attributes, such S=$\{s_1, s_2, s_3, ..., s_\ell\}$. Drawing from the possibility q-rung triangular fuzzy neutrosophic set, data will be gathered in the format of $\omega = \left\{ \frac{P_i}{(x, T_\omega(x), I_\omega(x), F_\omega(x))} \middle| x \in X \right\}$ is the prerequisite for the numerical portion of ω is $0 \le T_\omega^3(x) + I_\omega^3(x) + F_\omega^3(x) \le 3$.

1. Data collection in table 1:

   Compile the evaluation data from the decision-makers into a matrix. G=[P_{nm}] as

$$
\mathrm{G} = \begin{pmatrix} \mathrm{P}_{11} & \mathrm{P}_{12} & \cdots & \mathrm{P}_{1m} \\ \mathrm{P}_{21} & \mathrm{P}_{22} & \cdots & \mathrm{P}_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \mathrm{P}_{n1} & \mathrm{P}_{n2} & \cdots & \mathrm{P}_{nm} \end{pmatrix}
$$

2. Normalization in table 2:

   At this point, the decision matrix is employed. as $\mathscr{G} = [\mathscr{P}_{xy}]$ into the formalized matrix modification $\bar{\mathscr{G}} = [\bar{\mathscr{P}}_{xy}]$ by this criteria:

$$\bar{\mathscr{P}}_{xy} = \begin{cases} \mathscr{P}_{xy}, & \text{benefit payments} \\ (\mathscr{P}_{xy})^c, & \text{Cost-payments .} \end{cases}$$

here $\mathscr{P}^c_{xy}$ is impute to the commendation of $\mathscr{P}_{xy}$. It is important to remember that for each Pq-RTFN $\mathscr{P} = (s_{\ni}, \{\eth, \Gamma, \dashv\})$ its commendation can be calculated as

$$\mathscr{B}^c = (s_{t-\ni}, \{\dashv, \Gamma, \eth\}). \tag{11}$$

3. Aggregation:
   Collection of Pq-RTFNs P_{xy}(y=1, 2, 3,..., p) for parallel $P_x(x = 1, 2, 3, ..., q)$ into preference's of total worth P by using the The controllers of Pq-RTFNAA in equation (3) or Pq-RTFNGA in equation (6) the fact that were originally proposed.
   It may be stated mathematically as;

$$\begin{aligned} P_x &= Pq-RTFNsAA_{\delta'}(P_{x1}, P_{x2}, P_{x3}, ..., P_{xp}), \\ P_x &= Pq-RTFNsGA_{\delta'}(P_{x1}, P_{x2}, P_{x3}, ..., P_{xp}), \end{aligned}$$

   where $\aleph' = (\aleph'_1, \aleph'_2, ..., \aleph'_n)$ is the characteristics' likelihood vector that is used.

4. Identify the score values.:
   Here the Definition 10 and equation (2), find out the score attributes Sc(P_x)(x=1, 2, 3, ..., p) of all q-RLNs P_x(x=1, 2, 3, ..., p).

5. Ranking:
   Evaluate the choices to determine which is preferable. $t_x(x = 1, 2, 3, .., p)$ using this attributes Scr(P_x).

6. EXPLANATORY EXAMPLE

In part, an example concerning the proceed of classify of world happiness is to intricate on the imputation and viability of the proposed strategy. Happiness, in the context of MCDM, is a complex concept that involves the consideration of multiple factors and criteria. MCDM methods aim to analyze and evaluate different alternatives based on a set of criteria or objectives. When applying MCDM to happiness, the challenge lies in defining and quantifying the criteria that contribute to individual or societal well-being. These criteria may include income, health, education, social support, environmental sustainability, and personal freedom. MCDM technique can help decision-makers and prioritize these criteria, facilitating a systematic approach to understanding and enhancing happiness. By incorporating various dimensions of happiness into decision-making processes, MCDM provides a framework for considering the holistic well-being of individuals and societies, ultimately guiding choices that can lead to greater happiness and overall life satisfaction. For happiness their are different parameters include some benefits parameters strong social support, high life expectancy, low corruption, well-functioning government, access to quality education and health care, and a sense of community and some costs parameters are Harsh climate conditions, relatively high cost of living, Relatively high taxes, strict immigration policies, small population size and high housing prices The observes that the occur with qualities are used to investigate the number of countries: strong social support $(\mathscr{S}_1)$, high life expectancy$(\mathscr{S}_2)$, health care $(\mathscr{S}_3)$, Harsh climate condition $(\mathscr{S}_4)$ and Relatively high taxes $(\mathscr{S}_5)$. Then,at this stage, how I select which five top rated ranking countries. such as: Finland $(\mathscr{C}_1)$, Denmark$(\mathscr{C}_2)$, Switzerland $(\mathscr{C}_3)$, Iceland $(\mathscr{C}_4)$ and Norway $(\mathscr{C}_5)$. It is evident that the way batters interact is an MCDM problem with five possible solutions. $\{c_1, c_2, c_3, c_4, c_5\}$, four models $\{s_1, s_2, s_3, s_4, s_5\}$ and expert d. The optimal ordering at At that point, it may be found using the generated approach. It's crucial to remember that the precise likelihood given to these expenses and advantages factors may change depending on the corporation's ranking process responsible for world happiness ranking.

Flow chart for the suggested procedure is in Figure 1.

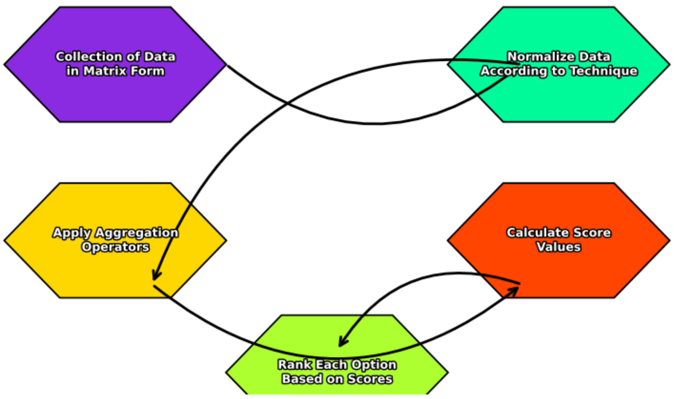

Figure 1: Flow chart for the suggested procedure

Step 1: Data collection in the matrix form (For q=2).

Table 1: DM of Pq-RTFNS taken by "D"

	$\mathscr{S}_1$	$\mathscr{S}_2$	$\mathscr{S}_3$	$\mathscr{S}_4$
$\mathscr{P}_1$	$\frac{0.4}{\{(0.8,0.6,0.7),(0.1,0.3,0.2),(0.4,0.6,0.2)\}}$	$\frac{0.3}{\{(0.9,0.7,0.5),(0.1,0.4,0.2),(0.8,0.6,0.5)\}}$	$\frac{0.2}{\{(0.7,0.8,0.6),(0.3,0.1,0.2),(0.6,0.5,0.7)\}}$	$\frac{0.1}{\{(0.6,0.9,0.8),(0.2,0.4,0.3),(0.7,0.4,0.5)\}}$
$\mathscr{P}_2$	$\frac{0.3}{\{(0.9,0.6,0.7),(0.3,0.2,0.1),(0.8,0.5,0.4)\}}$	$\frac{0.4}{\{(0.8,0.5,0.7),(0.4,0.1,0.3),(0.6,0.4,0.7)\}}$	$\frac{0.1}{\{(0.7,0.9,0.8),(0.2,0.3,0.1),(0.5,0.7,0.6)\}}$	$\frac{0.2}{\{(0.6,0.8,0.9),(0.1,0.4,0.2),(0.4,0.6,0.5)\}}$
$\mathscr{P}_3$	$\frac{0.4}{\{(0.8,0.5,0.7),(0.4,0.1,0.3),(0.6,0.4,0.7)\}}$	$\frac{0.3}{\{(0.9,0.6,0.7),(0.2,0.4,0.1),(0.8,0.5,0.4)\}}$	$\frac{0.2}{\{(0.7,0.9,0.8),(0.2,0.3,0.1),(0.5,0.7,0.6)\}}$	$\frac{0.1}{\{(0.6,0.8,0.9),(0.1,0.4,0.2),(0.4,0.6,0.5)\}}$
$\mathscr{P}_4$	$\frac{0.3}{\{(0.5,0.7,0.8),(0.3,0.4,0.1),(0.4,0.7,0.6)\}}$	$\frac{0.4}{\{(0.6,0.9,0.8),(0.1,0.2,0.4),(0.7,0.6,0.4)\}}$	$\frac{0.1}{\{(0.9,0.8,0.7),(0.2,0.3,0.1),(0.6,0.5,0.7)\}}$	$\frac{0.2}{\{(0.7,0.6,0.9),(0.4,0.2,0.3),(0.5,0.4,0.6)\}}$
$\mathscr{P}_5$	$\frac{0.3}{\{(0.7,0.6,0.9),(0.1,0.3,0.2),(0.5,0.7,0.4)\}}$	$\frac{0.4}{\{(0.5,0.7,0.8),(0.3,0.4,0.1),(0.6,0.4,0.8)\}}$	$\frac{0.1}{\{(0.9,0.8,0.7),(0.2,0.1,0.3),(0.7,0.5,0.6)\}}$	$\frac{0.2}{\{(0.8,0.6,0.9),(0.4,0.2,0.1),(0.5,0.6,0.9)\}}$

Step 2: Normalize the data according to proposed technique.

Table 2: Normalized Matrix

	$\mathscr{S}_1$	$\mathscr{S}_2$	$\mathscr{S}_3$	$\mathscr{S}_4$
$\mathscr{P}_1$	$\frac{0.4}{\{(0.4,0.6,0.2),(0.1,0.3,0.2),(0.8,0.6,0.7)\}}$	$\frac{0.3}{\{(0.8,0.6,0.5),(0.1,0.4,0.2),(0.9,0.7,0.5)\}}$	$\frac{0.2}{\{(0.6,0.5,0.7),(0.3,0.1,0.2),(0.7,0.8,0.6)\}}$	$\frac{0.1}{\{(0.7,0.4,0.5),(0.2,0.4,0.3),(0.6,0.9,0.8)\}}$
$\mathscr{P}_2$	$\frac{0.3}{\{(0.8,0.4,0.5),(0.3,0.2,0.1),(0.9,0.6,0.7)\}}$	$\frac{0.4}{\{(0.6,0.4,0.7),(0.4,0.1,0.3),(0.8,0.5,0.7)\}}$	$\frac{0.1}{\{(0.5,0.7,0.6),(0.2,0.3,0.1),(0.7,0.9,0.8)\}}$	$\frac{0.2}{\{(0.4,0.6,0.5),(0.1,0.4,0.2),(0.6,0.8,0.9)\}}$
$\mathscr{P}_3$	$\frac{0.4}{\{(0.6,0.4,0.7),(0.4,0.1,0.3),(0.8,0.5,0.7)\}}$	$\frac{0.3}{\{(0.8,0.5,0.4),(0.2,0.4,0.1),(0.9,0.6,0.7)\}}$	$\frac{0.2}{\{(0.5,0.7,0.6),(0.2,0.3,0.1),(0.7,0.9,0.8)\}}$	$\frac{0.1}{\{(0.4,0.6,0.5),(0.1,0.4,0.2),(0.6,0.8,0.9)\}}$
$\mathscr{P}_4$	$\frac{0.3}{\{(0.4,0.7,0.6),(0.3,0.4,0.1),(0.5,0.7,0.8)\}}$	$\frac{0.4}{\{(0.7,0.6,0.4),(0.1,0.2,0.4),(0.6,0.9,0.8)\}}$	$\frac{0.1}{\{(0.6,0.5,0.7),(0.2,0.3,0.1),(0.9,0.8,0.7)\}}$	$\frac{0.2}{\{(0.5,0.4,0.6),(0.4,0.2,0.3),(0.7,0.6,0.9)\}}$
$\mathscr{P}_5$	$\frac{0.3}{\{(0.5,0.7,0.4),(0.1,0.3,0.2),(0.7,0.6,0.9)\}}$	$\frac{0.4}{\{(0.6,0.4,0.8),(0.3,0.4,0.1),(0.5,0.7,0.8)\}}$	$\frac{0.1}{\{(0.7,0.5,0.6),(0.2,0.1,0.3),(0.9,0.8,0.7)\}}$	$\frac{0.2}{\{(0.5,0.6,0.4),(0.4,0.2,0.1),(0.8,0.6,0.9)\}}$

Step 3: In this stage, we employed aggregation algorithms (Pq-RTFNAA and q-RTFNGA) by utilizing alternatives that we already knew from the previous step:
We got outcomes:

- Pq-RTFNAA:
  $\mathscr{P}_1 = \{(0.6282, 0.5641, 0.4553), (0.1331, 0.2709, 0.2083), (0.7844, 0.6930, 0.6217)\}$,
  $\mathscr{P}_2 = \{(0.6398, 0.4839, 0.6012), (0.2597, 0.1812, 0.1782), (0.6870, 0.6155, 0.7455)\}$,
  $\mathscr{P}_3 = \{(0.6465, 0.5253, 0.5880), (0.2465, 0.2162, 0.1661), (0.7849, 0.6225, 0.7370)\}$,
  $\mathscr{P}_4 = \{(0.5787, 0.5932, 0.5421), (0.1965, 0.2561, 0.2169), (0.6100, 0.7601, 0.8086)\}$,
  and
  $\mathscr{P}_5 = \{(0.5654, 0.5487, 0.6280), (0.2198, 0.2782, 0.1370), (0.6443, 0.6561, 0.8376)\}$.

- Pq-RTFNGA:
  $\mathscr{P}_1 = \{(0.5641, 0.5550, 0.3701), (0.1545, 0.3071, 0.2162), (0.8110, 0.7212, 0.6442)\}$,
  $\mathscr{P}_2 = \{(0.5921, 0.4587, 0.5821), (0.2980, 0.2186, 0.2051), (0.8056, 0.6681, 0.7684)\}$,
  $\mathscr{P}_3 = \{(0.6054, 0.4982, 0.5543), (0.2782, 0.2720, 0.1951), (0.8112, 0.6901, 0.7527)\}$,
  $\mathscr{P}_4 = \{(0.5443, 0.5680, 0.5189), (0.2391, 0.2754, 0.2720), (0.6488, 0.8031, 0.8183)\}$,
  and
  $\mathscr{P}_5 = \{(0.5563, 0.5240, 0.5491), (0.2584, 0.3069, 0.1522), (0.6957, 0.6672, 0.8528)\}$.

Step 4: We determined the parameters of the scoring mechanism for each choice in this stage.

- Pq-RTFNNA:
  $Sc(\mathscr{P}_1) = 0.5212$, $Sc(\mathscr{P}_2) = 0.6058$, $Sc(\mathscr{P}_3) = 0.5975$, $Sc(\mathscr{P}_4) = 0.5701$ and $Sc(\mathscr{P}_5) = 0.5851$.

- Pq-RTFNNG:
  $Sc(\mathscr{P}_1) = 0.5401$, $Sc(\mathscr{P}_2) = 0.5962$, $Sc(\mathscr{P}_3) = 0.5601$, $Sc(\mathscr{P}_4) = 0.5423$ and $Sc(\mathscr{P}_5) = 0.5641$.

Step 5: Next, we gave each option a score and tabulated the results.

- P-qTFNAA:

$$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1. \tag{12}$$

- P-qTFNGA:

$$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1. \tag{13}$$

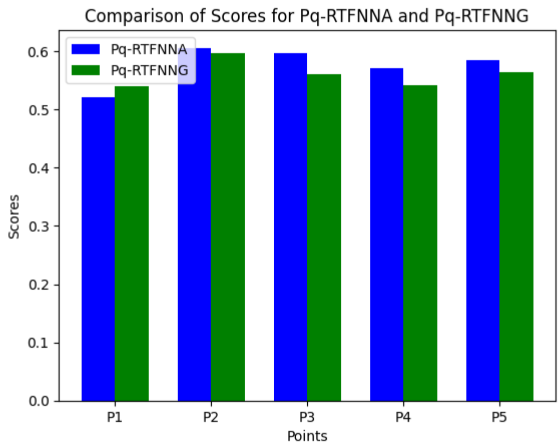

Figure 2: Graphically representation of our exaplanatory example ranking

The accumulation operator therefore display the finalized ratings in equation (11) and (12) and graphically in figure 2. The PQ-RTFNS indicates the prevalence of globalwide happiness rating that Finland has the best rating compared to different international locations. On the alternative hand, PQ-RTFNS indicates that Dussen has the best international locations rating as evaluate to different players. The effects for each operators to shut to every different however supply common effects. The furnished rating implies a hierarchy of fine among international locations. However, with out precise statistics approximately the international locations represented with the aid of using those labels, the conclusions drawn could be hypothetical. international locations ratings in international locations can vary, and for correct and up to date statistics, it's far advocated to seek advice from dependable reassets including the contry ratings. This ratings offer complete and dependable tests of u. s. performances and ratings, making sure correct statistics for international locations enthusiasts..

7. SENSITIVITY ANALYSIS

A sort of economic version known as sensitivity evaluation influences of modifications in enter elements on course elements.This method for waiting for a opportunities end result given a gaggle of extensive elements vulnerability evaluation is used to deal with the uncertainty in mathematics fashions, whilst the values for the version's given data can also additionally fluctuate. The are typically utilized in mixture on account that it's miles the analytical approach that is going in conjunction with uncertainty evaluation. All fashions built and research performed depend on supposing concerning the correctness of the given values utilized in submission to gain consequences or culmination for coverage decisions. Sensitivity evaluation is probably beneficial in numerous circumstances, which include estimating, waiting for, and spotting areas that want cycle improvements or modifications. However, utilising ancient information would possibly every so often bring about wrong projections on account that beyond occurrences do not always foretell destiny ones.

7.1. Sensitivity analysis according to parameter "q"

7.1.1. "q-RLNWAA" operator:

In this part, to discover the effect of various q attributes at the rating of the alternative opportunities, we really do the responded evaluation the usage of the PQ-RTFNSAA AO in Table 3, and table indicates that once we boom the asses of q, does not anything in reality adjustments. Additionally, we've got located that once the assess of q grew, the assess of rating feature of every desire have become extra modest. The surest desire stays the equal while q=2, q=4, and q=8, however it adjustments while q=10, q=12, and q=15 are inputs.in addition, we've got located that every choice behaved through searching on photo illustration of it is rating-asses in Figure 3. Here is a completely small alternate is performing in Figure 3. The parameter q is sort of a illustration of the DM's behaviour. The AO is suitable while managing vital DMs, at the same time as the PQ-RTFNSAA operator is beneficial while reflecting constructive DMs. Assume we use the PQ-RTFNSAA AO to accumulate statistics for the modern time from previous observation, better q values suggest that DMs have a extra poor behaviour for thats , at the same time as decrease values suggest a extra effective attitude. Therefore, exceptional DM can pick the maximum suitable value of q primarily based totally on their behaviour.

Table 3: A different ranking by altering the parameter values

q-values:	Values of score function	Ranking
q=2	$Sc(\mathscr{P}_1) = 2.0652$, $Sc(\mathscr{P}_2) = 2.8976$, $Sc(\mathscr{P}_3) = 2.6531$, $Sc(\mathscr{P}_4) = 2.0986$, $Sc(\mathscr{P}_5) = 2.2786$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
q=4	$Sc(\mathscr{P}_1) = 2.6972$, $Sc(\mathscr{P}_2) = 3.5021$, $Sc(\mathscr{P}_3) = 3.2760$, $Sc(\mathscr{P}_4) = 2.9850$, $Sc(\mathscr{P}_5) = 3.4986$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$
q=8	$Sc(\mathscr{P}_1) = 2.5102$, $Sc(\mathscr{P}_2) = 3.0198$, $Sc(\mathscr{P}_3) = 3.0056$, $Sc(\mathscr{P}_4) = 2.6501$, $Sc(\mathscr{P}_5) = 2.9856$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
q=10	$Sc(\mathscr{P}_1) = 2.4980$, $Sc(\mathscr{P}_2) = 3.6052$, $Sc(\mathscr{P}_3) = 3.2986$, $Sc(\mathscr{P}_4) = 2.5501$, $Sc(\mathscr{P}_5) = 3.1567$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
q=12	$Sc(\mathscr{P}_1) = 3.0086$, $Sc(\mathscr{P}_2) = 3.3147$, $Sc(\mathscr{P}_3) = 3.1976$, $Sc(\mathscr{P}_4) = 2.0198$, $Sc(\mathscr{P}_5) = 3.1765$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
q=15	$Sc(\mathscr{P}_1) = 3.0231$, $Sc(\mathscr{P}_2) = 3.7562$, $Sc(\mathscr{P}_3) = 3.3598$, $Sc(\mathscr{P}_4) = 3.0662$, $Sc(\mathscr{P}_5) = 3.5321$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$

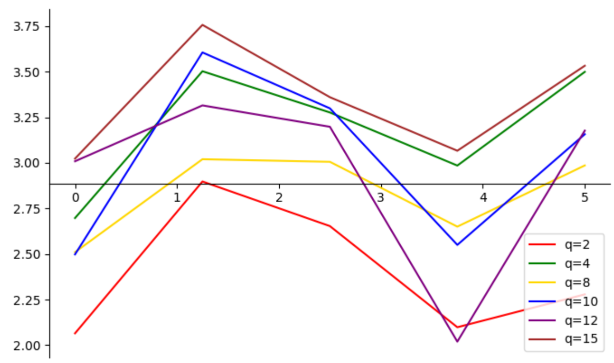

Figure 3: Using q-RLNWAA, The assessment of sensitivity shown visually with reference to the variables q

7.1.2. "PQ-RTFNSGA" operator:

Additionally, here we have PQ-RTFNSGA to regulate the assess of the attributes "q" and noticed how the rating of opportunity replaced. There is nearly no opportunity that happened, further to the PQ-RTFNSAA AO, because the price of parameter extrade need to apparent in Table 4.in this behavior of rating values, supposing equality,it may visible in Figure 4, in which minimum extrade is going on while q's asses alters. In this method for decision making to make the top-rated option, this AO is likewise very important.

Table 4: A different ranking by altering the parameter values

q-values:	Values of score function	Ranking
q=2	$Sc(\mathscr{P}_1) = 0.9866$, $Sc(\mathscr{P}_2) = 1.9852$, $Sc(\mathscr{P}_3) = 1.5321$, $Sc(\mathscr{P}_4) = 1.3590$, $Sc(\mathscr{P}_5) = 1.7740$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$
q=4	$Sc(\mathscr{P}_1) = 1.3521$, $Sc(\mathscr{P}_2) = 2.0896$, $Sc(\mathscr{P}_3) = 1.8650$, $Sc(\mathscr{P}_4) = 1.6632$, $Sc(\mathscr{P}_5) = 2.0731$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$
q=8	$Sc(\mathscr{P}_1) = 1.8990$, $Sc(\mathscr{P}_2) = 2.4690$, $Sc(\mathscr{P}_3) = 2.3854$, $Sc(\mathscr{P}_4) = 2.0752$, $Sc(\mathscr{P}_5) = 2.0981$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
q=10	$Sc(\mathscr{P}_1) = 1.6537$, $Sc(\mathscr{P}_2) = 2.8865$, $Sc(\mathscr{P}_3) = 2.0567$, $Sc(\mathscr{P}_4) = 1.8960$, $Sc(\mathscr{P}_5) = 2.5231$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$
q=12	$Sc(\mathscr{P}_1) = 2.5847$, $Sc(\mathscr{P}_2) = 3.1567$, $Sc(\mathscr{P}_3) = 2.9843$, $Sc(\mathscr{P}_4) = 2.7407$, $Sc(\mathscr{P}_5) = 3.0052$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$
q=15	$Sc(\mathscr{P}_1) = 2.3245$, $Sc(\mathscr{P}_2) = 3.2560$, $Sc(\mathscr{P}_3) = 3.0972$, $Sc(\mathscr{P}_4) = 2.6547$, $Sc(\mathscr{P}_5) = 2.9806$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$

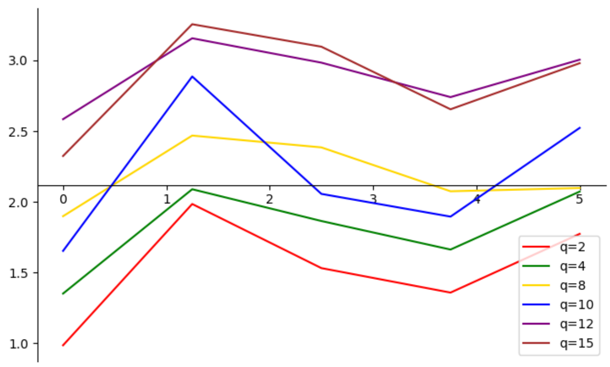

Figure 4: Using q-RLNGAA, Graphically representation of sensitivity analysis with regard to the parameter q

7.2. Analysis of sensitivity in relation to the possibilities degrees

Sensitivity evaluation w.r.t attributes opportunity is a way utilized in DM processes, specifically in multi-standards choice-making, to decide the effect of various possibilities assigned to the attributes which might be being evaluated. In such choice-making scenarios, a couple of attributes or standards are frequently evaluated, and those attributes may also have specific stages of significance or possibilities assigned to them. These opportunity are commonly assigned primarily based totally at the choice-maker's preferences, area knowledge, or different factors. Sensitivity evaluation with appreciate to attributes opportunity entails systematically various the opportunity assigned to every characteristic and studying the ensuing adjustments within side the universal choice or final results. The purpose of this evaluation is to evaluate how the choice or final results is to the opportunity assigned to every characteristic, and to pick out which attributes have the best effect at the choice. Sensitivity evaluation could contain various the opportunity assigned to every criterion and studying the ensuing adjustments within side the universal global international locations rankings. This evaluation should assist the universe decide which standards are maximum essential to their choice and modify their opportunities accordingly. Overall, it's far a beneficial method for comparing choice issues and assessing the robustness of selections to adjustments within side the significance of various attributes.

Table 5: Analysis of sensitivity in relation to possibilities degrees

Values of new Weights:	Values of score function	Ranking
$\{0.2830, 0.3124, 0.1971, 0.1659, 0.1456\}$	$Sc(\mathscr{P}_1) = 1.7432$, $Sc(\mathscr{P}_2) = 2.7536$, $Sc(\mathscr{P}_3) = 2.4053$, $Sc(\mathscr{P}_4) = 1.9768$, $Sc(\mathscr{P}_5) = 2.1560$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
$\{0.2651, 0.2475, 0.1676, 0.1969, 0.02958\}$	$Sc(\mathscr{P}_1) = 1.9652$, $Sc(\mathscr{P}_2) = 3.0732$, $Sc(\mathscr{P}_3) = 2.9473$, $Sc(\mathscr{P}_4) = 2.1687$, $Sc(\mathscr{P}_5) = 2.6524$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
$\{0.1986, 0.1730, 0.1403, 0.1719, 0.2380\}$	$Sc(\mathscr{P}_1) = 2.1056$, $Sc(\mathscr{P}_2) = 3.1215$, $Sc(\mathscr{P}_3) = 2.8248$, $Sc(\mathscr{P}_4) = 2.3539$, $Sc(\mathscr{P}_5) = 3.0098$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$
$\{0.1896, 0.1290, 0.1742, 0.2569, 0.1125\}$	$Sc(\mathscr{P}_1) = 0.8817$, $Sc(\mathscr{P}_2) = 1.8765$, $Sc(\mathscr{P}_3) = 1.6863$, $Sc(\mathscr{P}_4) = 1.2565$, $Sc(\mathscr{P}_5) = 1.3225$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_1 > \mathscr{P}_4$
$\{0.1075, 0.1863, 0.1151, 0.2685, 0.1801\}$	$Sc(\mathscr{P}_1) = 0.8652$, $Sc(\mathscr{P}_2) = 2.6532$, $Sc(\mathscr{P}_3) = 1.9450$, $Sc(\mathscr{P}_4) = 1.3210$, $Sc(\mathscr{P}_5) = 1.5621$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$

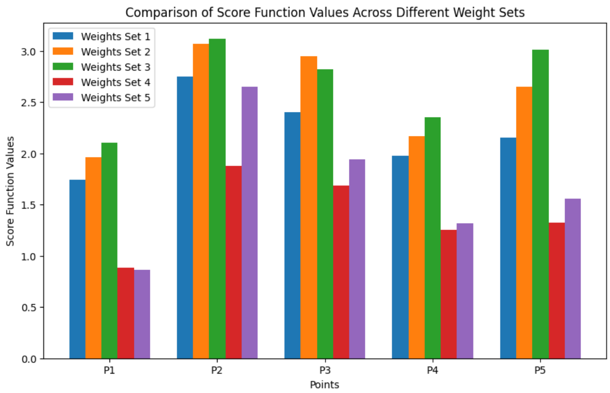

Figure 5: Graphically representation of Analysis of sensitivity in relation to possibilities degrees

The Table 5 indicates that, truly see that although we regulate the possibility of the qualities, the position of alternatives stays like as.so that is identify the efficiency of the aggregation attributes for generalised aggregation.as we like observed in Fig. 5, then i regulate the burden of standards, there is largely no alternate withinside the rating attributes. When using diverse possible qualities, in order of alternatives is always $\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$ or a little modification to it.

8. COMPARATIVE STUDY

Benchmarking is the exercise of reading or extra associated gadgets to peer how they're comparable and distinctive. By the use of it in quite a few conditions and industries, humans can higher apprehend the similarities and variations among distinctive things. It can assist companies make knowledgeable selections on vital issues. When used with clinical data, it may be used in beneficial methods. Information this is received thru a systematic take a look at and could be implemented for a particular reason is referred to as clinical data. It establishes the extent of consistency and accuracy of the data while used with clinical data. It additionally facilitates scientists make certain the validity and accuracy in their data. If we need to higher apprehend a subject or get solutions to vital questions, benchmarking is vital. These are the principle dreams for which businesses use benchmarking. It promotes in-intensity expertise of views associated with positive procedures, departments or enterprise units. Additionally, this studies guarantees that we're addressing the actual reasons of overall performance disparities. It is usually used as it facilitates apprehend the modern-day and beyond difficulties a enterprise faces. This technique affords objective, genuine facts approximately overall performance and indicates methods to enhance overall performance. To higher gift the blessings and benefits of the proposed techniques, we behavior the subsequent comparative comparison.

8.1. Comparison of proposed technique with method proposed by P. Biswas and S. Pramanik [64]

To address the previously mentioned issue, we employ Pranab Biswas and S. Pramanik's approach, with results shown in Table 6. Using the current TFNNWA and TFNNWG operators as a reference, we computed the rating values in Table 6 and compared the outcomes to the methodology suggested in this study. At this point, there has been no discernible change in the evaluations of any opportunity. The opportunity that ranks highest for each technique is the same. However, the technique cited by Pranab Biswas and S. Pramanik states, "In our recommended approach, the possibility characteristic became known, making it more flexible and affordable." The possibilities feature became well-known, indicating that choosing how to rank the options is quite simple.

Table 6: Evaluation of existing operators in comparison to current operators in figure 6 [64]

Different Operators :	Score function values	Ranking
TFNNWA **(Existing Operator)**[64]	$Sc(\mathscr{P}_1) = 0.5096$, $Sc(\mathscr{P}_2) = 0.5623$, $Sc(\mathscr{P}_3) = 0.5321$, $Sc(\mathscr{P}_4) = 0.5178$, $Sc(\mathscr{P}_5) = 0.5301$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
TFNNWG **(Existing Operator)**[64]	$Sc(\mathscr{P}_1) = 0.3180$, $Sc(\mathscr{P}_2) = 0.3986$, $Sc(\mathscr{P}_3) = 0.3755$, $Sc(\mathscr{P}_4) = 0.3393$, $Sc(\mathscr{P}_5) = 0.3523$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
Pq-RTFNNWA **(Proposed Operator)**	$Sc(\mathscr{P}_1) = 0.5824$, $Sc(\mathscr{P}_2) = 0.6871$, $Sc(\mathscr{P}_3) = 0.6383$, $Sc(\mathscr{P}_4) = 0.5933$ and $Sc(\mathscr{P}_5) = 0.6259$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$.
Pq-RTFNNWG **(Proposed Operator)**	$Sc(\mathscr{P}_1) = 1.0593$, $Sc(\mathscr{P}_2) = 1.9175$, $Sc(\mathscr{P}_3) = 1.8233$, $Sc(\mathscr{P}_4) = 1.3210$, $Sc(\mathscr{P}_5) = 1.3012$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_5 > \mathscr{P}_1$.

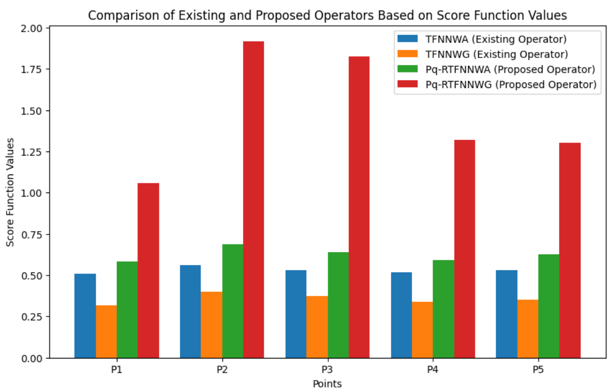

Figure 6: Graphically representation of Evaluation of existing operators in comparison to current operators

8.2. Comparison of proposed technique with method proposed by M. Riaz and H. Kamaci [65]

In addition, to demonstrate, in comparison to the methods of M. Riaz and H. Kamacicite, the efficacy of the suggested approach on this outdated one. Table 7 discusses the aforementioned example case using the SVNFWA and SVNFOWA operators. It also compares the suggested approach's consequences to the way cited in M. Riaz and H. Kamaci's study. We said that the options have almost the same ranking. Only a quantitative piece of the photo fuzzy set is present in the method that M. Riaz and H. Kamaci cite. In contrast, our proposed structure includes additional data about both the quantitative and qualitative components, which we refer to as the linguistic elements. We found that using the citation shows that our approach is more effective and powerful.

Table 7: Comparison of existing operators with current operators in figure 7. [65]

Different operators :	Score function values	Ranking
SVNFPA[65]	$Sc(\mathscr{P}_1) = 0.3953$, $Sc(\mathscr{P}_2) = 0.5208$, $Sc(\mathscr{P}_3) = 0.4765$, $Sc(\mathscr{P}_4) = 0.4121$, $Sc(\mathscr{P}_5) = 0.4596$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$
SVNFOPA[65]	$Sc(\mathscr{P}_1) = 0.3803$, $Sc(\mathscr{P}_2) = 0.4965$, $Sc(\mathscr{P}_3) = 0.4303$, $Sc(\mathscr{P}_4) = 0.4021$, $Sc(\mathscr{P}_5) = 0.4655$	$\mathscr{P}_2 > \mathscr{P}_5 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_1$
PQ-RTFNSAA (Proposed operator)	$Sc(\mathscr{P}_1) = 0.5905$, $Sc(\mathscr{P}_2) = 0.6357$, $Sc(\mathscr{P}_3) = 0.6321$, $Sc(\mathscr{P}_4) = 0.6275$, $Sc(\mathscr{P}_5) = 0.6186$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_4 > \mathscr{P}_5 > \mathscr{P}_1$
PQ-RTFNSGA (Proposed operator)	$Sc(\mathscr{P}_1) = 0.6923$, $Sc(\mathscr{P}_2) = 0.7567$, $Sc(\mathscr{P}_3) = 0.7305$, $Sc(\mathscr{P}_4) = 0.6986$ and $Sc(\mathscr{P}_5) = 0.7126$	$\mathscr{P}_2 > \mathscr{P}_3 > \mathscr{P}_5 > \mathscr{P}_4 > \mathscr{P}_1$

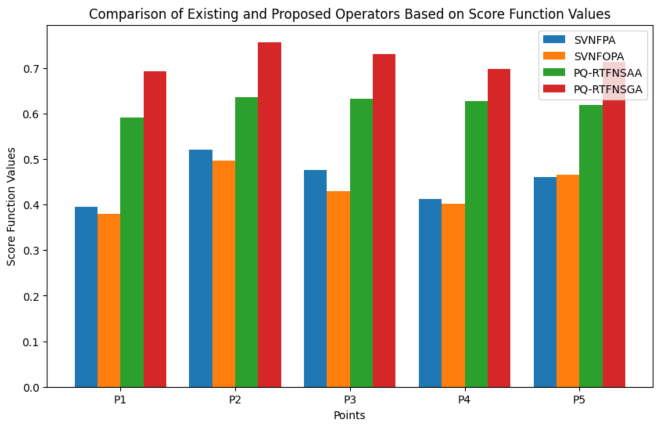

Figure 7: Graphically representation of Comparison of existing operators with current operators

The evaluation offered above demonstrates the effectiveness of our proposed techniques for fixing with the aid of using DM issues, mainly for MADM. Compared to different approaches, our strategies provide extra flexibility and Reason for addressing this matter. MADM demanding situations. This blessings are in large part shows to the usage of (PQ-RTFNS), thats permit DMs to expose the evaluations extra willingly at the same time as minimizing records loss. Our method additionally considers the quantitative assumptions that decision-makers have a tendency to make while making subjective decisions, taking PQ-RTFNS applicable and enough for representing opinions of opportunity attributes. Additionally, our MCDM approach is primarily based totally on PQ-

RTFNSAA or PQ-RTFNSGA, that denotes inner relations among distinctive attributes or criteria. This makes our technique extra powerful at recreate real-international MADM demanding situations at the same time as presenting DMs with a brand new device for speaking their additionals. Compared to different techniques, our method is broader, stronger, and extra flexible, showing it an powerful answer for talking of MCDM issues.

Table 8: Study Proposal Comparison with Current Relevant Structures in figure 8 and table 8

Name	Year	Structure	Ling	Mem	Ind	NMem	Possbility
L.A. Zadeh [66]	1965	FS	0	1	0	0	0
K.T. Atanassov [67]	1986	IFS	0	1	0	1	0
Yager et al. [68]	2013	PFS	0	1	0	1	0
Yager [69]	2016	q-RFS	0	1	0	1	0
Smarandache et al.[39]	2005	NS	0	1	1	1	0
Bhowmki et al. [70]	2009	INS	0	1	1	1	0
R. Jansi et al. [71]	2019	PNS	0	1	1	1	0
Ali et al.[62]	2022	q-RLPFS	1	1	1	1	0
Proposed Technique	2024	PQ-RTFNS	1	1	1	1	1

In Ali et al. development (GQRPFL) has a restriction. We inscribe the restriction of Ali et al. Structure. It offers whilst the prevailing version exist withinside the form of q-rung liguistic picture fuzzy set with the condition $0 \leq \eth_{B_2}(\Theta) + \Gamma_{B_2}(\Theta) + \daleth_{B_2}(\Theta) \leq 1$. But in our advance formation (PQ-RTFNS) deals when the $0 \leq \eth_{B_2}(\Theta) + \Gamma_{B_2}(\Theta) + \daleth_{B_2}(\Theta) \leq 3$,Then we generalized it through growing the strength of Mem, indt and NMem upto "q" to modify the cost in closed periods zero and 3.

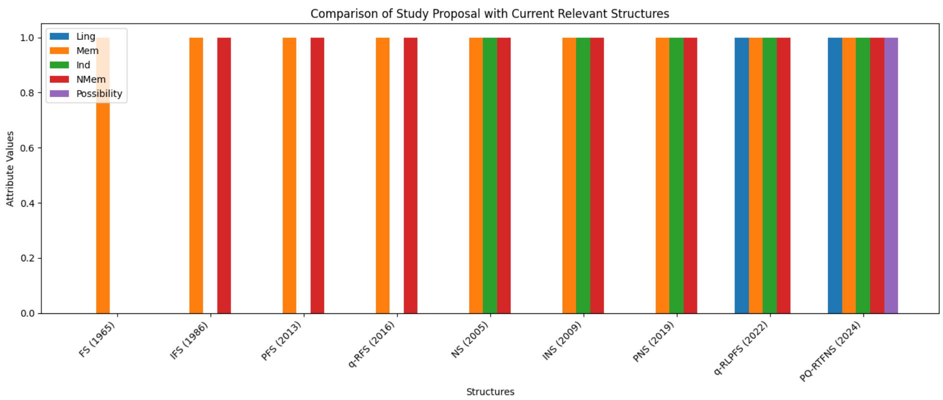

Figure 8: Graphical representation of study proposal comparison with current relevant structures

9. CONCLUSIONS AND FUTURE WORK

In conclusion, this research introduces a robust and innovative framework for evaluating world happiness rankings using q-rung orthopair triangular fuzzy neutrosophic sets (PQ-RTFNs) within a possibility setting. The proposed model effectively addresses the challenges of uncertainty, fuzziness, and indeterminacy that have hindered traditional methods of assessing global happiness. By offering a more accurate and comprehensive approach, the framework enhances the reliability of happiness evaluations, providing policymakers, researchers, and governments with deeper insights into the factors that shape happiness across different nations. This model not only improves the accuracy of rankings but also offers a practical tool for formulating policies aimed at improving societal well-being.

The results of this study clearly demonstrate that the PQ-RTFN model is well-suited to handle complex, multi-faceted data sets, offering greater flexibility in dealing with the uncertainty and ambiguity inherent in world happiness assessments. However, this research also opens up several avenues for future exploration.

In future work, several specific areas can be explored to further refine and expand this framework:

- **Incorporating more diverse datasets:** Future research could integrate additional factors beyond economic, social, and environmental variables, such as cultural and psychological dimensions of happiness. This could help capture a more holistic view of happiness and its drivers.
- **Application in regional or local settings:** The current model focuses on global happiness rankings, but future work could explore its application at regional or even local levels. Customizing the model to address specific local contexts could provide insights into targeted well-being policies.
- **Dynamic and real-time evaluations:** Developing a dynamic model that can process real-time data and offer continuous updates to happiness rankings would be an important step forward. This would allow governments and organizations to monitor happiness trends more effectively and respond to shifts more promptly.
- **Interdisciplinary integration:** Combining the PQ-RTFN model with data from other interdisciplinary fields — such as psychology, sociology, or public health — could provide richer and more nuanced insights into happiness. This could also lead to the development of more precise happiness indicators.
- **Machine learning integration:** Future work could explore integrating machine learning techniques with the PQ-RTFN model to automate the evaluation process, improving scalability and efficiency. This would allow the framework to handle larger and more complex datasets while maintaining high accuracy in happiness rankings.

In summary, while this research provides a significant improvement in the way happiness is evaluated on a global scale, there is considerable potential to extend and refine the model. By incorporating more diverse data, applying the framework to localized settings, and integrating advanced computational techniques, future studies can further enhance the ability to accurately assess and rank happiness, leading to better-informed policies and initiatives that improve the well-being of people around the world.

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