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Research Article

HYBRID SOLUTION (HYBSO) BASED ON HYBRIDS NORMALIZATION AND AGGREGATION FOR THE MULTI-CRITERIA DECISION-MAKING PROBLEMS

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Abstract: Real-world decision making problems often dictates to take into account several point of view that are objectively conflictuel. Many studies were dedicated to provide decision makers with methods for solving this type of highly complex problems. In this paper, we propose a new hybrid multi-criteria decision making method with a new hybrid normalization and aggregation strategies. Mainly, the proposed method introduces a new hybrid normalization between the distance measure and the ratio system, and also uses two hybrid equations to compute the weighted performance of alternatives as to improve the stability of the method and the flexibility of the results. Moreover, hybrid aggregation rule based on exponential and logarithmic functions is proposed to establish the final ranking of the alternatives. To assess the performance of the proposed method, we used two real problems: the logistic provider selection problem and the evaluation of microclimate in an office problem. Comparative results with eight state-of-the-art multi-criteria decision making methods and sensitivity analysis established its validity, in terms of performance and stability, for solving multi-criteria decision making problems.

Keywords: Multi-Criteria Decision-Making (MCDM), Multi-Attribute Decision-Making (MADM), Hybrid Solution (HybSo), Hybrid Normalization, Hybrid Aggregation.

MSC: 90B50, 91B06, 97M40.

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1. INTRODUCTION

The introduction of recent technological advancements into our day-to-day activities offers several alternatives to decision makers, which are often presented with different, yet contradictory, points of views. Eventually, they have to make critical decisions accordingly. Therefore, this makes decision making a very difficult and complex task. On the other hand, Operational Research, more precisely, Multi-criteria Decision Making (MCDM) provides us with robust methods for solving this type of decision making problems.

MCDM methods are mainly divided into two categories according to the nature of the alternatives of the initial problem [1, 2]: (1) discrete methods, namely, Multi-attribute Decision Making (MADM) methods, (2) continuous methods, namely, Multi-Objective Decision Making (MODM) methods. The objective of MODM methods is to find a set of efficient solutions, with values of decision variables to be determined in a continuous or integer domain [3, 4, 5]. Multi-attribute decision making (MADM) methods can be defined as techniques for ranking a finite number of decision alternatives (choices), these alternatives are judged according to their degree of satisfaction of a set of objectives called the criteria set [2, 6], decision-makers' preferences must also be taken into account, they are presented as weights of criteria. MADM problems are referred as MCDM problems in recent literature [2, 7]. The MCDM problem has many practical applications in several areas, including renewable energy [8], finance [9], industry [10], politics [11], portfolio selection [12], cloud services [13], risk management [14], military [15], sports [16], medicine [17], economics [18], human resources management [19], Architecture [20] and agricultural engineering [21].

Several methods have been proposed for solving MCDM problems in recent decades. Among them we can mention the taxonomy method proposed by Adanson in 1763 [22]. In 1971, Fonetla and Gabus have proposed the DEMATEL method [6, 23]. In 1975 Paelinck has proposed the QUALIFLEX method [24]. Keeney and Raiffa have proposed the MAUT method in 1976 [6, 25]. The ORESTE method was introduced by Roubens in 1980 [26]. Voogd proposed the EVAMIX method in 1982 [6, 27]. Hinloopen, Nijkamp, and Rietveld have proposed the REGIME method in 1983 [6, 28]. In 1986 Brans, Vincke and Mareschal have proposed the first version of PROMETHEE (PROMETHEE I) [29]. Winterfeldt and Edwards have proposed the SMART method in 1986 [30]. In 1990 Roy has proposed the first version of the ELECTRE method (ELECTRE I) [31]. The MACBETH method was proposed by Bana e Costa and Vansnick in 1990 [32]. An EXtension of the PROMethee methods (EXPROM) was proposed by Diakoulaki and Koumoutsos in 1991 [6]. In 1992 Gomes and Lima have proposed the TODIM method [33]. In 1994 Zavadskas, Kaklauskas, and Sarka have proposed the COmplex PRoportional ASsessment (COPRAS) method [34]. Diakoulaki, Mavrotas, and Papayannakis have proposed the CRITIC method in 1995 [6, 35]. Analytic Network Process (ANP) method was proposed by Martel, Kiss, and Rousseau in 1996 [36]. The VIKOR method was proposed by Opricovic in 1998 [37]. The Superiority and Inferiority Ranking (SIR) method was proposed by Xu in 2001 [6, 38]. The Multi-Objective Optimization Ratio Analysis (MOORA) method was proposed in 2004 by Brauers [39]. Kersuliene, Zavadskas, and Turskis have proposed the SWARA in 2010 [40]. In 2010 Zavadskas and Turskis have proposed the Additive Ratio ASsessment (ARAS) method [5]. In 2012 Zavadskas, Turskis, Antucheviciene, and Zakarevicius have proposed the Weighted Aggregates Sum Product Assessment (WASPAS) method [41]. The KEMIRA method was proposed by Krylovas, Zavadskas, Kosareva, and Dadelo in 2014 [42]. The EDAS method was proposed in 2015 by Keshavarz Ghorabaee, Zavadskas, Olfat, and Turskis [43]. The Multi-Attributive Border Approximation area Comparison (MABAC) method was proposed in 2015 by Pamucar and Cirovic [44]. In 2016 Zavadskas and Podvezko have proposed the Integrated Determination of Objective CRIteria Weights (IDOCRIW) method [6, 45]. A combined compromise solution (CoCoSo) method was proposed by Yazdani, Zarate, Zavadskas and Turskis in 2019 [2]. For further details, the reader my refer to the following documents: Zavadskas et al. 2014 [1] and Alireza & Javad 2019 [6].

Many research studies have been published in recent years focusing on the MCDM problem, its related problems and their applications. These include an assessment the performance of nine mutual funds in the Republic of Serbia in the period 2011-2013 [46].Pamučar, Ćirović and Božanić have proposed an application of Interval Valued Fuzzy-Rough Numbers in Multi-Criteria Decision Making [47]. Wan, Dong and Chen have proposed a linguistic hesitant fuzzy multi-criteria group decision making method based on the MULITMOORA (Multi-Objective Optimization on basis of a Ratio Analysis plus the Full Multiplicative form), the BWM (best and worst method), and the prospect theory (PT) [48]. Wan, Zeng, Dong and Hu have presented two methods for interactive multi-attribute group decision making with LHFSs based on comprehensive cloud power geometric aggregation operators [49]. Radovanović, Petrovski, Cirkin, Behlić, Jokić, Chemezov, Hashimov, Bouraima and Jana addresses the selection of the most favorable alternative in the form of assault rifles to meet the requirements arising from modern combat operations using Multi-Criteria Decision-making methods [15]. Wan, Dong and Zhang have proposed two-stage consensus reaching process for social network large group decision making considering self-adjustment and binding force of subgroup [50]. Kumar made a bibliometric analysis of multi-criteria decision-making applications in Agro-based industries [18]. Wan, Dong and Chen have presented a new intuitionistic fuzzy best-worst method for group decision making with intuitionistic fuzzy preference relations [51]. Salehzadeh and Ziaeian have presented a systematic literature review on the applications of AHP, FAHP, and ANP in human resource management [19]. Wan, Gao and Dong have presented a new method combining trapezoidal cloud and MULTIMOORA (Multi-Objective Optimization on the basis of Ratio Analysis plus full multiplicative form) for heterogeneous multi-criterion group decision-making (HMCGDM) to determine the optimal path of container multimodal transport [52]. Berčič, Bohanec, Marko and Lucija Ažman have proposed an integrating multi-criteria decision models in smart urban planning: a case study of architectural and urban design competitions [20]. Wan, Wu and Dong have presented the complex heterogeneous multi-attribute group decision-making characterized by two-layer decision-makers, individual attribute sets, complex relationships among attributes and heterogeneous evaluation information and application to photovoltaic power station site selection [53]. Heidarisoltanabadi, Elhami, Abdollah and Khadivi have presented a study of multi-criteria decision-making (MCDM) methods, including deterministic analytical hierarchy process (AHP) and fuzzy analytical hierarchy process (FAHP), technique for order of preference by similarity to ideal solution (TOPSIS), fuzzy TOP-

SIS (FTOPSIS), and analytic network process (ANP), were used to score and select the appropriate fertilizing method for apple trees based on the growers and expert's perspectives [21]. Gazi, Raisa, Biswas, Azizzadeh and Mondal have presented an approaches for finding the most important criterion in women's empowerment for sports using the decision-making trial and evaluation laboratory (DEMATEL) method [16]. Wan, Zou, Dong and Gao have proposed a dual strategies consensus reaching process for ranking consensus based probabilistic linguistic multi-criteria group decision-making [54].

Although numerous methods have been proposed in the literature for solving MCDM problems, the researchers are always interested to propose new methods for these problems owing of the shortcomings found in the state-of-the-art methods. A major shortcoming of MCDM methods is that they give different results when they are applied to solving the same problem [2]. Another shortcoming of state-of-the-art methods is the low resistance to small perturbations in the initial models of the problems solved (i.e., the rankings of the alternatives can change if any modifications are made to the initial values of the criteria). The development of new MCDM methods that use new strategies and overcome these shortcomings is a competitive field that has attracted a large number of researchers. Sensitivity analysis is one of the techniques used by decision-makers and MCDM method developers in order to test the resistance of methods to perturbations in initial models. In this paper, we focus on the development of a new method for solving MCDM problems based on new strategies and techniques, which gives good results in sensitivity analysis and attacks the shortcomings of literature methods. For this purpose, we propose a new hybrid multi-criteria decision making algorithm with a new hybrid normalization and hybrid aggregation strategies. The proposed normalization can be perceived as a hybridization between the distance measure and the ratio system normalizations. In order to improve the stability of the proposed method and the flexibility of the results, we used two hybrid equations to calculate the weighted performance of alternatives. Furthermore, the final ranking is based on hybrid aggregation rules relying on exponential and logarithmic functions to give even greater precision to the final results. To the best of our knowledge, there is no method that uses a hybrid normalization and a hybrid aggregation based on exponential and logarithmic functions, i.e., our method is the first one using these kinds of strategies and techniques. The rest of this paper is organized as follows: in Section 2, we present the proposed Hybrid solution algorithm (HybSo). In Section 3, a performance analysis and a comparative study with the state-of-the-art methods are presented, which is based on the logistic provider selection problem (Section 3.1) and the evaluation of microclimate in an office problems (Section 3.2). In the last section, a conclusion is given.

2. HYBRID SOLUTION (HYBSO) ALGORITHM FOR THE MULTI-CRITERIA DECISION-MAKING PROBLEMS

The newly proposed Hybrid solution algorithm (HybSo) uses a new hybrid normalization between the distance measure and the ratio system, and the weighted performance of alternatives is calculated using two hybrid equations, finally an hybrid aggregation based on exponential and logarithmic functions. Hybrid exponential and additive formulas are also used in the other processing phases. The general process of the proposed HybSo algorithm is defined by the following steps:

Step (0): Construct the initial decision-making matrix X as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \tag{1}$$

where, x_{ij} represents the performance measure of the i^{th} alternative on the j^{th} criterion, for each $i \in \{1,\dots,n\}$, and $j \in \{1,\dots,m\}$.

Step (1): Normalization of the decision-making matrix using a hybrid normalization between the compromise normalization equation [2, 55] and the ratio system used in the MOORA method [39]. This new normalization gives the proposed method more stability and more similarity with others (the existing methods) compared with the previously existing normalization. The normalized decision matrix (R) is defined as follows: let δ_j a normalization coefficient for a given criterion j, where

$$\delta_j = \sqrt{\sum_{i=1}^{n} (\max_i(x_{ij}) - x_{ij})^2}. \tag{2}$$

Hence, for each benefit criterion j, we have

$$r_{ij} = \frac{x_{ij} - \min_i(x_{ij})}{\delta_j}, \quad \forall i \in \{1,\dots,n\}. \tag{3}$$

Similarly, for each cost criterion j, we have

$$r_{ij} = \frac{\max_i(x_{ij}) - x_{ij}}{\delta_j}, \quad \forall i \in \{1,\dots,n\}. \tag{4}$$

Step (2): For each alternative, calculate the normalized sum of the power weight of the weighted sequence (S_i^1), the power sum of the weighted power products (S_i^2) and the weighted average comparability S_i^3. S_i^1 is a hybridization between the additive relative importance used in WASPAS [56], the sum of the power weight of comparability used in CoCoSo [2] and the unweighted normalization. S_i^2 is an aggregation of fuzzy categories [57] hybridized with the weighted sum. S_i^3 is a hybridization between the weighted sum and the generalized mean using the exponential and logarithmic functions. The indicators S_i^1, S_i^2 and S_i^3 are adjusted in this way to give more stability to the proposed method, S_i^1, S_i^2 and S_i^3 are defined by the following formulas:

$$S_i^1 = \frac{\sum_{j=1}^{m} (w_j r_{ij})^{w_j}}{\sum_{i=1}^{n} \sum_{j=1}^{m} (w_j r_{ij})^{w_j}}, \quad \forall i \in \{1,\dots,n\}, \tag{5}$$

This S_i^1 value is obtained by combining CoCoSo's grey relational generation approach and WASPAS's multiplicative relative importance.

$$S_i^2 = \left(\prod_{j=1}^{m} (r_{ij})^{w_j}\right)^{(1-\varepsilon)} + \left(\prod_{j=1}^{m} (1-r_{ij})^{w_j}\right)^{\varepsilon}, \quad \forall i \in \{1,\dots,n\}, \tag{6}$$

where, $\varepsilon \simeq 0$.

This S_i^2 value is obtained by combining t-norm, t-conorm and weighted sum using the product function.

$$S_i^3 = \ln\left(\sum_{j=1}^{m} \exp(w_j r_{ij})\right), \quad \forall i \in \{1,\dots,n\}. \tag{7}$$

This S_i^3 value is obtained according to the ordered weighted sum using the exponential function and its inverse the logarithmic function.

Step (3): Calculate the relative scores H_i^1 and H_i^2 of each alternative. H_i^1 is an aggregation of fuzzy categories [57] and H_i^2 is the rate of comparison of the distance to the worst with the distance between the best and the worst. The formulas of H_i^1 and H_i^2 are defined as follows:

$$H_i^1 = \left(\prod_{k=1}^{3} S_i^k\right)^{\frac{1}{2}} \left[1 - \prod_{k=1}^{3}\left(1 - S_i^k\right)\right]^{\frac{1}{2}}, \quad \forall i \in \{1,\dots,n\}, \tag{8}$$

This H_i^1 value is achieved with a t-norm and t-conorm combination based on the product function.

$$H_i^2 = \sum_{k=1}^{3} \frac{S_i^k - \underline{S}^k}{\bar{S}^k - \underline{S}^k}, \quad \forall i \in \{1,\dots,n\}, \tag{9}$$

where,

$$\bar{S}^k = \max_{i \in \{1,\dots,n\}} (S_i^k) \quad \text{and} \quad \underline{S}^k = \min_{i \in \{1,\dots,n\}} (S_i^k), \qquad \forall k \in \{1,\dots,3\}. \tag{10}$$

This H_i^2 value is the sum of the relative scores of the distance compared with the best distance.

Step (4): Calculate the overall score H_i for each alternative and rank the alternatives in descending order according to their overall score. H_i is hybrid aggregation based on the generalized average using the exponential function and its inverse the logarithm function. H_i is given by the following formula:

$$H_i = \ln\left(\frac{H_i^1 \exp(H_i^1) + H_i^2 \exp(H_i^2)}{H_i^1 + H_i^2}\right), \quad \forall i \in \{1,\dots,n\}. \tag{11}$$

3. EXPERIMENTAL RESULTS

3.1. Logistic Provider Selection Problem (LPSP)

This problem was introduced by Yazdani et al. [2]. The French Association of Supply Chain and Logistics (ASLOG) aims to provide forward-looking visions, to generate standards and qualifications, to measure and evaluate logistics performance, and finally to produce a dissemination of research in partnership with the academic sector and a benchmark of best practices. Therefore, it encouraged companies to engage logistics and supply chain departments at the highest level of management decision-making. In this problem, seven logistics companies (i.e., alternatives) are chosen to be selected: Mathez (A1), Bansard (A2), GEFCO (A3), Schneider Transport (A4), LDI Dimotrans (A5), SAGA (A6) and GETMA (A7). These companies specialise in logistics coordination and international transport (air, sea and road). Their main services include air freight management, sea freight, road transport, storage, packaging, supply and distribution management and optimisation, port agent, assistance with customs operations, cargo and cruise ship management, logistics and industrial projects, etc. The selection criteria considered in this problem are: storage capacity (C1), price offered by logistics providers (C2), volume of batches or deliveries (C3), degree of flexibility (C4) and use of technology (C5). The price offered by logistics providers (C2) is a cost criterion and the others are benefit criteria. The data of this problem and the criteria weights are summarized in Table 1.

Table 1: Summary information of the logistics provider selection problem

	Criteria				
Alternatives	C1 (max)	C2 (min)	C3 (max)	C4 (max)	C5 (max)
A_1	60	0.4	2540	500	990
A_2	6.35	0.15	1016	3000	1041
A_3	6.8	0.1	1727.2	1500	1676
A_4	10	0.2	1000	2000	965
A_5	2.5	0.1	560	500	915
A_6	4.5	0.08	1016	350	508
A_7	3	0.1	1778	1000	920
Weights	0.036	0.192	0.326	0.326	0.12

Table 2 shows the results obtained after step (1) (i.e., calculation of normalized decision-making matrix using equations 2, 3, 4).

Table 2: The normalized decision-making matrix (step(1), equations 2, 3, 4)

Alternatives	C1	C2	C3	C4	C5
A_1	0.9861	0	0.7291	0.0442	0.3095
A_2	0.0660	0.3628	0.1679	0.7803	0.3423
A_3	0.0737	0.4353	0.4298	0.3386	0.7501
A_4	0.1286	0.2902	0.1620	0.4858	0.2935
A_5	0	0.4353	0	0.0442	0.2614
A_6	0.0343	0.4644	0.1679	0	0
A_7	0.0086	0.4353	0.4485	0.1914	0.2646

After the calculation of the normalized decision-making matrix, the next step consists of calculating the normalized sum of the power weight of the weighted sequence (S_i^1), the power sum of the weighted power products (S_i^2) and the weighted average comparability S_i^3. The value of ε is fixed at 10^{-8}. This value was adjusted experimentally. The obtained results are shown in Tables 3, 4 and 5.

Table 3: The normalized sum of the power weight of the weighted sequence (S^1, step(2), eq. 5)

Alternatives	C1	C2	C3	C4	C5	S^1
A_1	0.8868	0	0.6260	0.2510	0.6736	0.1351
A_2	0.8045	0.5996	0.3879	0.6400	0.6818	0.1725
A_3	0.8077	0.6209	0.5269	0.4875	0.7491	0.1769
A_4	0.8241	0.5744	0.3834	0.5484	0.6693	0.1662
A_5	0	0.6209	0	0.2510	0.6600	0.0849
A_6	0.7858	0.6287	0.3879	0	0	0.0999
A_7	0.7475	0.6209	0.5343	0.4048	0.6610	0.1645

Table 4: The power sum of the weighted power products (S^2, step(2), equation 6)

Alternatives	C1	C2	C3	C4	C5	Products	S^2
A1	$(r_{11})^{w_1} = 0.9995$ $(1-r_{11})^{w_1} = 0.8573$	0 1.0000	0.9021 0.6533	0.3617 0.9854	0.8687 0.9565	$\prod_{j=1}^{5}(r_{1j})^{w_j} = 0$ $\prod_{j=1}^{5}(1-r_{1j})^{w_j} = 0.5279$	1.0000
A2	0.9068 0.9975	0.8231 0.9171	0.5589 0.9418	0.9223 0.6102	0.8793 0.9510	0.3383 0.5000	1.3383
A3	0.9104 0.9975	0.8524 0.8961	0.7593 0.8327	0.7026 0.8739	0.9661 0.8467	0.4000 0.5506	1.4000
A4	0.9288 0.9951	0.7886 0.9363	0.5525 0.9440	0.7903 0.8051	0.8632 0.9592	0.2761 0.6791	1.2761
A5	0 1.0000	0.8524 0.8961	0 1.0000	0.3617 0.9854	0.8513 0.9643	0 0.8515	1.0000
A6	0.8857 0.9987	0.8630 0.8870	0.5589 0.9418	0 1.0000	0 1.0000	0 0.8344	1.0000
A7	0.8426 0.9997	0.8524 0.8961	0.7700 0.8237	0.5833 0.9331	0.8525 0.9638	0.2750 0.6635	1.2750

Table 5: The weighted average comparability (S^3, step(2), equation 7)

Alternatives	C1	C2	C3	C4	C5	S^3
A_1	1.0361	1.0000	1.2683	1.0145	1.0378	1.6784
A_2	1.0024	1.0721	1.0563	1.2896	1.0419	1.6979
A_3	1.0027	1.0872	1.1504	1.1167	1.0942	1.6958
A_4	1.0046	1.0573	1.0542	1.1716	1.0358	1.6722
A_5	1.0000	1.0872	1.0000	1.0145	1.0319	1.6358
A_6	1.0012	1.0933	1.0563	1.0000	1.0000	1.6391
A_7	1.0003	1.0872	1.1574	1.0644	1.0323	1.6755

The next step consists of calculating the scores using the results obtained in the previous step, and finally, the alternatives are ranked according to their overall scores. The obtained results are summarized in Table 6.

Table 6: Alternatives scores and HybSO ranking of alternatives (step(3) and step(4), eqs. 8, 9, 11)

	H^1	Ranks	H^2	Ranks	H	Final ranks
A_1	0.4761	5	1.2310	5	1.0710	5
A_2	0.5617	2	2.7986	2	2.6369	2
A_3	0.5690	1	2.9669	1	2.8087	1
A_4	0.5475	3	2.1599	4	1.9832	4
A_5	0.3726	7	0.0000	7	0.3726	6
A_6	0.4046	6	0.2168	6	0.3430	7
A_7	0.5449	4	2.1927	3	2.0174	3

The results revealed that the best logistics companie is GEFCO (A3) and the next best is Bansard (A2), the last alternative is A6 (SAGA). Therefore, the final ranking of the logistics companies is as follows:

$$A_6 \prec A_5 \prec A_1 \prec A_4 \prec A_7 \prec A_2 \prec A_3.$$

3.1.1. Comparative analysis using the logistic provider selection problem

As mentioned earlier, to study the validity of the proposed algorithm (HybSo), we used the logistic provider selection problem. This is, by comparing the ranking results obtained by HybSo with the ones obtained using eight state-of-the-art methods: TOPSIS, VIKOR, COPRAS, WASPAS, MOORA, CODAS, ARAS and CoCoSo. In this comparison, we have calculated the Spearman correlation coefficients (CCs) between the HybSo ranking and the rankings of the competing methods mentioned above. If the CCs of two rankings is higher than 0.8 (80%), it means that these two methods are highly similar. The rankings obtained by the different methods, and the correlation coefficients between HybSo algorithm and the others are summarized in Table 7. The comparative results show that there is a high similarity between the results obtained using the proposed algorithm (HybSo) and the results of other competing algorithms. Therefore, HybSo is at least comparable with state-of-the-art algorithms and can be considered as a possible alternative method for solving MCDM problems. In other terms, the experimental results illustrates the validity of HybSo's decision outcomes.

Table 7: The ranking comparison of the MCDM methods and the correlation coefficients

	TOPSIS	VIKOR	COPRAS	WASPAS	MOORA	CODAS	ARAS	CoCoSo	HybSo
A_1	5	5	3	5	5	3	3	5	5
A_2	1	2	1	2	1	1	1	2	2
A_3	2	1	2	1	2	2	2	1	1
A_4	3	4	5	4	4	5	5	4	4
A_5	7	7	7	7	7	7	7	7	6
A_6	6	6	6	6	6	6	6	6	7
A_7	4	3	4	3	3	4	4	3	3
CC	0,893	0,964	0,821	0,964	0,929	0,821	0,821	0,964	1,000

3.1.2. Sensitivity analysis using the logistic provider selection problem

Sensitivity analysis is another technique used by decision makers and MCDM method developers to study the stability and the validity of their methods. In this analysis, we have made some modifications in the initial model of the used problem (i.e., the logistic provider selection problem), we swaped the criteria weights (we permuted the weights between the criteria and taken all possible cases). After deleting the repetitions, we have obtained 60 possible cases. We have organized these cases as 60 different tests of weight sensitivity analysis. These tests are summarized in Table 8. Table 9 shows the ranking results of the alternatives for each case of the sensitivity analysis. The results of the ranking show that alternative A3 ranked first in 47 tests (in 78.33% of the tests) and second in 12 tests, alternative A5 is ranked last in 32 tests and A6 in 28 tests. Figure 1 shows the variation of alternative rankings according to the sensitivity analysis tests (i.e., it shows the weight perturbation's influence on the ranking of alternatives). We can conclude that the best alternative is A3 and the worst one is either A5 or A6. We have calculated Spearman's correlation coefficients (CCs) between the initial ranking of the proposed algorithm (HybSo) and the rankings of the sensitivity analysis tests, the results are summarized in Figure 2. 55 tests (i.e., 92% of the tests) show a high similarity with the initial ranking (i.e., over 80 per cent), which proves the high stability of the proposed algorithm.

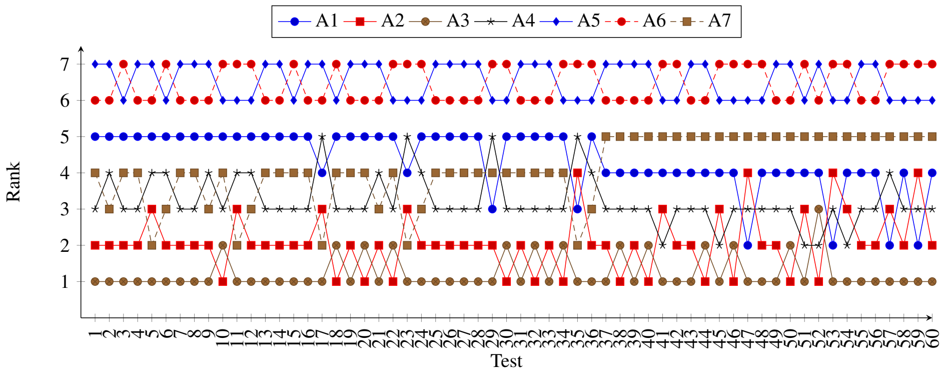

Figure 1: The variation of alternative rankings according to the sensitivity analysis tests for LPSP

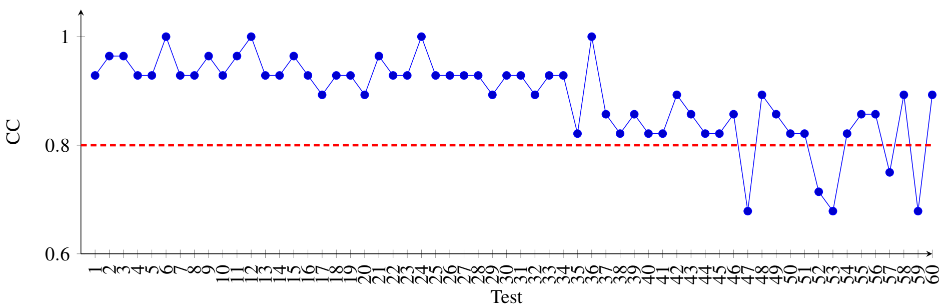

Figure 2: Spearman's correlation coefficients between the initial ranking of HybSo and the rankings of the sensitivity analysis tests for LPSP

We also performed the same sensitivity analysis for the other MCDM methods: TOPSIS, VIKOR, COPRAS, WASPAS, MOORA, CODAS, ARAS and CoCoSo, i.e., for each method, we calculated the Spearman correlation coefficients between the initial ranking and the ranking obtained in the sensitivity tests. Figure 3 shows the Spearman correlation coefficients (CCs) between the initial ranking and each ranking of the sensitivity analysis tests obtained using the nine methods. Figure 3 also shows the 0.8 threshold to see the CCs that are greater than 0.8. Within 60 tests for each method, 14 tests (23% of the tests) have a high similarity for the TOPSIS method, 30 tests (50% of the tests) for VIKOR, 40 tests (66% of the tests) for COPRAS, 25 tests (42% of the tests) for WASPAS, 23 tests (38% of the tests) for MOORA, 22 tests (37% of the tests) for CODAS, 37 tests (62% of the tests) for ARAS and 39 tests (65% of the tests) for CoCoSo. In conclusion, the results obtained show that the HybSo method has the highest number of tests that have a Spearman's correlation coefficients (CCs) over 0.8 with a proportion of 92% (the highest proportion compared with the other methods), therefore HybSo can be considered the most stable method for this problem(i.e., the logistic provider selection problem). Finally, according to the results found in the sensitivity analysis, we can confirm the high similarity with the rankings obtained by HybSo and the high stability of HybSo results. Therefore, we can affirm the validity and stability of the proposed method and that it can be used to solve MCDM problems.

Table 8: The different tests (weights of the criteria) used in the sensitivity analysis

	C1	C2	C3	C4	C5		C1	C2	C3	C4	C5
Test 1	0,036	0,12	0,192	0,326	0,326	Test 31	0,192	0,326	0,036	0,12	0,326
Test 2	0,036	0,12	0,326	0,192	0,326	Test 32	0,192	0,326	0,036	0,326	0,12
Test 3	0,036	0,12	0,326	0,326	0,192	Test 33	0,192	0,326	0,12	0,036	0,326
Test 4	0,036	0,192	0,12	0,326	0,326	Test 34	0,192	0,326	0,12	0,326	0,036
Test 5	0,036	0,192	0,326	0,12	0,326	Test 35	0,192	0,326	0,326	0,036	0,12
Test 6	0,036	0,192	0,326	0,326	0,12	Test 36	0,192	0,326	0,326	0,12	0,036
Test 7	0,036	0,326	0,12	0,192	0,326	Test 37	0,326	0,036	0,12	0,192	0,326
Test 8	0,036	0,326	0,12	0,326	0,192	Test 38	0,326	0,036	0,12	0,326	0,192
Test 9	0,036	0,326	0,192	0,12	0,326	Test 39	0,326	0,036	0,192	0,12	0,326
Test 10	0,036	0,326	0,192	0,326	0,12	Test 40	0,326	0,036	0,192	0,326	0,12
Test 11	0,036	0,326	0,326	0,12	0,192	Test 41	0,326	0,036	0,326	0,12	0,192
Test 12	0,036	0,326	0,326	0,192	0,12	Test 42	0,326	0,036	0,326	0,192	0,12
Test 13	0,12	0,036	0,192	0,326	0,326	Test 43	0,326	0,12	0,036	0,192	0,326
Test 14	0,12	0,036	0,326	0,192	0,326	Test 44	0,326	0,12	0,036	0,326	0,192
Test 15	0,12	0,036	0,326	0,326	0,192	Test 45	0,326	0,12	0,192	0,036	0,326
Test 16	0,12	0,192	0,036	0,326	0,326	Test 46	0,326	0,12	0,192	0,326	0,036
Test 17	0,12	0,192	0,326	0,036	0,326	Test 47	0,326	0,12	0,326	0,036	0,192
Test 18	0,12	0,192	0,326	0,326	0,036	Test 48	0,326	0,12	0,326	0,192	0,036
Test 19	0,12	0,326	0,036	0,192	0,326	Test 49	0,326	0,192	0,036	0,12	0,326
Test 20	0,12	0,326	0,036	0,326	0,192	Test 50	0,326	0,192	0,036	0,326	0,12
Test 21	0,12	0,326	0,192	0,036	0,326	Test 51	0,326	0,192	0,12	0,036	0,326
Test 22	0,12	0,326	0,192	0,326	0,036	Test 52	0,326	0,192	0,12	0,326	0,036
Test 23	0,12	0,326	0,326	0,036	0,192	Test 53	0,326	0,192	0,326	0,036	0,12
Test 24	0,12	0,326	0,326	0,192	0,036	Test 54	0,326	0,192	0,326	0,12	0,036
Test 25	0,192	0,036	0,12	0,326	0,326	Test 55	0,326	0,326	0,036	0,12	0,192
Test 26	0,192	0,036	0,326	0,12	0,326	Test 56	0,326	0,326	0,036	0,192	0,12
Test 27	0,192	0,036	0,326	0,326	0,12	Test 57	0,326	0,326	0,12	0,036	0,192
Test 28	0,192	0,12	0,036	0,326	0,326	Test 58	0,326	0,326	0,12	0,192	0,036
Test 29	0,192	0,12	0,326	0,036	0,326	Test 59	0,326	0,326	0,192	0,036	0,12
Test 30	0,192	0,12	0,326	0,326	0,036	Test 60	0,326	0,326	0,192	0,12	0,036

Table 9: The ranking results of the sensitivity analysis

	A1	A2	A3	A4	A5	A6	A7		A1	A2	A3	A4	A5	A6	A7
T1	5	2	1	3	7	6	4	T31	5	2	1	3	7	6	4
T2	5	2	1	4	7	6	3	T32	5	1	2	3	7	6	4
T3	5	2	1	3	6	7	4	T33	5	2	1	3	7	6	4
T4	5	2	1	3	7	6	4	T34	5	1	2	3	6	7	4
T5	5	3	1	4	7	6	2	T35	3	4	1	5	6	7	2
T6	5	2	1	4	6	7	3	T36	5	2	1	4	6	7	3
T7	5	2	1	3	7	6	4	T37	4	2	1	3	7	6	5
T8	5	2	1	3	7	6	4	T38	4	1	2	3	7	6	5
T9	5	2	1	4	7	6	3	T39	4	2	1	3	7	6	5
T10	5	1	2	3	6	7	4	T40	4	1	2	3	7	6	5
T11	5	3	1	4	6	7	2	T41	4	3	1	2	6	7	5
T12	5	2	1	4	6	7	3	T42	4	2	1	3	6	7	5
T13	5	2	1	3	7	6	4	T43	4	2	1	3	7	6	5
T14	5	2	1	3	7	6	4	T44	4	1	2	3	7	6	5
T15	5	2	1	3	6	7	4	T45	4	3	1	2	6	7	5
T16	5	2	1	3	7	6	4	T46	4	1	2	3	6	7	5
T17	4	3	1	5	7	6	2	T47	2	4	1	3	6	7	5
T18	5	1	2	3	6	7	4	T48	4	2	1	3	6	7	5
T19	5	2	1	3	7	6	4	T49	4	2	1	3	7	6	5
T20	5	1	2	3	7	6	4	T50	4	1	2	3	7	6	5
T21	5	2	1	4	7	6	3	T51	4	3	1	2	6	7	5
T22	5	1	2	3	6	7	4	T52	4	1	3	2	7	6	5
T23	4	3	1	5	6	7	2	T53	2	4	1	3	6	7	5
T24	5	2	1	4	6	7	3	T54	4	3	1	2	6	7	5
T25	5	2	1	3	7	6	4	T55	4	2	1	3	7	6	5
T26	5	2	1	3	7	6	4	T56	4	2	1	3	7	6	5
T27	5	2	1	3	7	6	4	T57	2	3	1	4	6	7	5
T28	5	2	1	3	7	6	4	T58	4	2	1	3	6	7	5
T29	3	2	1	5	6	7	4	T59	2	4	1	3	6	7	5
T30	5	1	2	3	6	7	4	T60	4	2	1	3	6	7	5

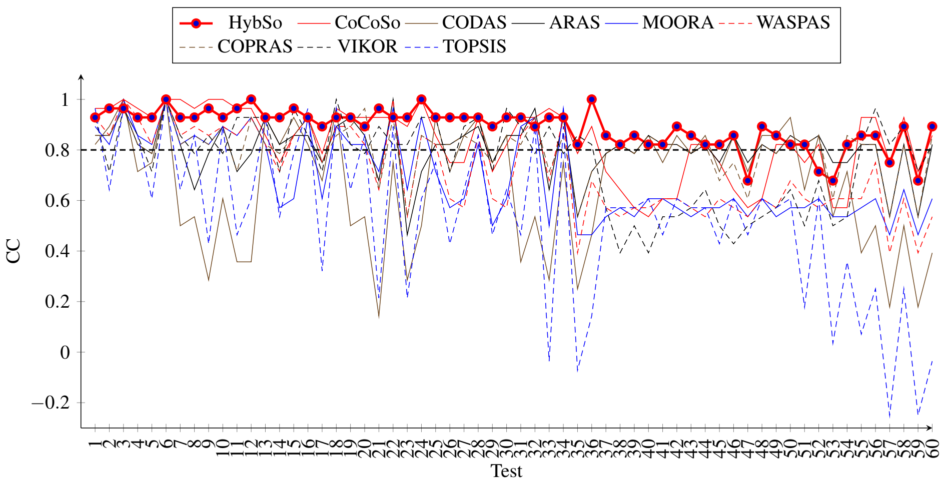

Figure 3: Spearman's correlation coefficients between the initial ranking and the rankings of the sensitivity analysis tests obtained using the nine competing algorithms for LPSP

3.2. Evaluation of Microclimate in Office Rooms (EMOR)

The problem of evaluating the microclimate in an office room was presented by Zavadskas and Turskis in 2010 [5] to test the ARAS method. The aim of this problem is to determine the climate inside the premises, in which people work, and the identification of measures to be taken in order to improve their environment [5]. Accordingly, we used this problem to assess the performance of HybSo method. In this problem, six criteria are chosen to rank 14 rooms:

1. C1: The amount of air per head in m^3/h,
2. C2: Relative air humidity in %,
3. C3: Air temperature in $^\circ C$,
4. C4: Illumination during work hours (8h to 17h) in lx,
5. C5: Rate of air flow in m/s,
6. C6: Dew point in $^\circ C$.

The rate of air flow and the dew point are cost criteria and the others are benefit criteria. The data of this study are summarized in Table 10.

Table 10: Evaluation of the microclimate in office rooms data

	Criteria					
Alternatives	C1 (max)	C2 (max)	C3 (max)	C4 (max)	C5 (min)	C6 (min)
A_1	7.6	46	18	390	0.1	11
A_2	5.5	32	21	360	0.05	11
A_3	5.3	32	21	290	0.05	11
A_4	5.7	37	19	270	0.05	9
A_5	4.2	38	19	240	0.1	8
A_6	4.4	38	19	260	0.1	8
A_7	3.9	42	16	270	0.1	5
A_8	7.9	44	20	400	0.05	6
A_9	8.1	44	20	380	0.05	6
A_{10}	4.5	46	18	320	0.1	7
A_{11}	5.7	48	20	320	0.05	11
A_{12}	5.2	48	20	310	0.05	11
A_{13}	7.1	49	19	280	0.1	12
A_{14}	6.9	50	16	250	0.05	10
Weights	0.21	0.16	0.26	0.17	0.12	0.08

Tables 11-15 summarize the results found in the different steps of HybSo algorithm. Table 11 shows the normalized decision-making matrix. The normalized sum of the power weight of the weighted sequence (S_i^1), the power sum of the weighted power products (S_i^2) and the weighted average comparability S_i^3 are shown in Tables 12, 13 and 14 respectively. As it is mentioned above, the value of ε is fixed experimentally at 10^{-8}. Table 15 shows the scores of each alternative and HybSo's ranking of the alternatives.

Table 11: The normalized decision-making matrix (step(1), equations 2, 3, 4)

Alternatives	C1	C2	C3	C4	C5	C6
A_1	0.4138	0.3129	0.1591	0.4603	0	0.0714
A_2	0.1789	0	0.3978	0.3682	0.3536	0.0714
A_3	0.1566	0	0.3978	0.1534	0.3536	0.0714
A_4	0.2013	0.1117	0.2387	0.0921	0.3536	0.2143
A_5	0.0335	0.1341	0.2387	0	0	0.2857
A_6	0.0559	0.1341	0.2387	0.0614	0	0.2857
A_7	0	0.2235	0	0.0921	0	0.5000
A_8	0.4473	0.2682	0.3182	0.4910	0.3536	0.4286
A_9	0.4697	0.2682	0.3182	0.4296	0.3536	0.4286
A_{10}	0.0671	0.3129	0.1591	0.2455	0	0.3571
A_{11}	0.2013	0.3576	0.3182	0.2455	0.3536	0.0714
A_{12}	0.1454	0.3576	0.3182	0.2148	0.3536	0.0714
A_{13}	0.3579	0.3799	0.2387	0.1227	0	0
A_{14}	0.3355	0.4023	0	0.0307	0.3536	0.1429

Table 12: The normalized sum of the power weight of the weighted sequence (S^1, step(2), equation 5)

Alternatives	C1	C2	C3	C4	C5	C6	S^1
A_1	0.5987	0.6193	0.4369	0.6485	0	0.6615	0.0709
A_2	0.5020	0	0.5544	0.6243	0.6844	0.6615	0.0724
A_3	0.4881	0	0.5544	0.5380	0.6844	0.6615	0.0700
A_4	0.5146	0.5253	0.4854	0.4932	0.6844	0.7223	0.0820
A_5	0.3532	0.5408	0.4854	0	0	0.7391	0.0507
A_6	0.3932	0.5408	0.4854	0.4604	0	0.7391	0.0627
A_7	0	0.5869	0	0.4932	0	0.7730	0.0443
A_8	0.6086	0.6042	0.5231	0.6556	0.6844	0.7635	0.0919
A_9	0.6148	0.6042	0.5231	0.6409	0.6844	0.7635	0.0917
A_{10}	0.4086	0.6193	0.4369	0.5827	0	0.7524	0.0670
A_{11}	0.5146	0.6327	0.5231	0.5827	0.6844	0.6615	0.0861
A_{12}	0.4806	0.6327	0.5231	0.5697	0.6844	0.6615	0.0850
A_{13}	0.5807	0.6389	0.4854	0.5180	0	0	0.0532
A_{14}	0.5729	0.6447	0	0.4092	0.6844	0.6993	0.0720

Table 13: The power sum of the weighted power products (S^2, step(3), equation 6)

Alternatives	C1	C2	C3	C4	C5	C6	Products	S^2
A1	$(r_{11})^{w_1} = 0.8308$	0.8304	0.6201	0.8764	0	0.8097	$\prod_{j=1}^{6}(r_{1j})^{w_j} = 0$	1.0000
	$(1-r_{11})^{w_1} = 0.8939$	0.9417	0.9559	0.9005	1.0000	0.9941	$\prod_{j=1}^{6}(1-r_{1j})^{w_j} = 0.7203$	
A2	0.6967	0	0.7869	0.8438	0.8827	0.8097	0	1.0000
	0.9594	1.0000	0.8765	0.9249	0.9490	0.9941	0.7337	
A3	0.6775	0	0.7869	0.7271	0.8827	0.8097	0	1.0000
	0.9649	1.0000	0.8765	0.9721	0.9490	0.9941	0.7755	
A4	0.7142	0.7042	0.6890	0.6666	0.8827	0.8841	0.1803	1.1803
	0.9539	0.9812	0.9316	0.9837	0.9490	0.9809	0.7984	
A5	0.4902	0.7251	0.6890	0	0	0.9046	0	1.0000
	0.9929	0.9772	0.9316	1.0000	1.0000	0.9734	0.8798	
A6	0.5457	0.7251	0.6890	0.6222	0	0.9046	0	1.0000
	0.9880	0.9772	0.9316	0.9893	1.0000	0.9734	0.8661	
A7	0	0.7868	0	0.6666	0	0.9461	0	1.0000
	1.0000	0.9603	1.0000	0.9837	1.0000	0.9461	0.8937	
A8	0.8446	0.8101	0.7425	0.8861	0.8827	0.9345	0.3713	1.3713
	0.8829	0.9513	0.9052	0.8916	0.9490	0.9562	0.6151	
A9	0.8533	0.8101	0.7425	0.8662	0.8827	0.9345	0.3667	1.3667
	0.8753	0.9513	0.9052	0.9090	0.9490	0.9562	0.6217	
A10	0.5670	0.8304	0.6201	0.7876	0	0.9209	0	1.0000
	0.9855	0.9417	0.9559	0.9532	1.0000	0.9653	0.8163	
A11	0.7142	0.8483	0.7425	0.7876	0.8827	0.8097	0.2532	1.2532
	0.9539	0.9316	0.9052	0.9532	0.9490	0.9941	0.7234	
A12	0.6670	0.8483	0.7425	0.7699	0.8827	0.8097	0.2312	1.2312
	0.9675	0.9316	0.9052	0.9597	0.9490	0.9941	0.7388	
A13	0.8059	0.8566	0.6890	0.7001	0	0	0	1.0000
	0.9112	0.9264	0.9316	0.9780	1.0000	1.0000	0.7690	
A14	0.7950	0.8644	0	0.5531	0.8827	0.8558	0	1.0000
	0.9178	0.9210	1.0000	0.9947	0.9490	0.9877	0.7881	

Table 14: The weighted average comparability (S^3, step(2), equation 7)

Alternatives	C1	C2	C3	C4	C5	C6	S^3
A_1	1.0908	1.0513	1.0422	1.0814	1.0000	1.0057	1.8360
A_2	1.0383	1.0000	1.1090	1.0646	1.0433	1.0057	1.8343
A_3	1.0334	1.0000	1.1090	1.0264	1.0433	1.0057	1.8274
A_4	1.0432	1.0180	1.0640	1.0158	1.0433	1.0173	1.8248
A_5	1.0071	1.0217	1.0640	1.0000	1.0000	1.0231	1.8109
A_6	1.0118	1.0217	1.0640	1.0105	1.0000	1.0231	1.8134
A_7	1.0000	1.0364	1.0000	1.0158	1.0000	1.0408	1.8071
A_8	1.0985	1.0438	1.0863	1.0870	1.0433	1.0349	1.8553
A_9	1.1037	1.0438	1.0863	1.0758	1.0433	1.0349	1.8544
A_{10}	1.0142	1.0513	1.0422	1.0426	1.0000	1.0290	1.8212
A_{11}	1.0432	1.0589	1.0863	1.0426	1.0433	1.0057	1.8374
A_{12}	1.0310	1.0589	1.0863	1.0372	1.0433	1.0057	1.8346
A_{13}	1.0780	1.0627	1.0640	1.0211	1.0000	1.0000	1.8287
A_{14}	1.0730	1.0665	1.0000	1.0052	1.0433	1.0115	1.8245

Table 15: Alternative scores and HybSO ranking of alternatives (step(3) and step(4), equations 8, 9, 11)

Alternatives	H^1	Ranks	H^2	Ranks	H	Final ranks
A_1	0.3609	8	1.1587	6	1.0188	6
A_2	0.3645	6	1.1549	7	1.0144	7
A_3	0.3577	9	0.9613	8	0.8302	8
A_4	0.3904	5	1.6437	5	1.4962	5
A_5	0.3030	13	0.2115	13	0.2664	14
A_6	0.3371	11	0.5150	12	0.4484	12
A_7	0.2831	14	0.0000	14	0.2831	13
A_8	0.4078	1	3.0000	1	2.8827	1
A_9	0.4077	2	2.9635	2	2.8453	2
A_{10}	0.3493	10	0.7687	10	0.6555	10
A_{11}	0.3999	3	2.1882	3	2.0504	3
A_{12}	0.3976	4	2.0469	4	1.9060	4
A_{13}	0.3119	12	0.6337	11	0.5385	11
A_{14}	0.3625	7	0.9424	9	0.8120	9

The final alternatives' ranking obtained by HybSo is as follows:

$$A_5 \prec A_7 \prec A_6 \prec A_{13} \prec A_{10} \prec A_{14} \prec A_3 \prec A_2 \prec A_1 \prec A_4 \prec A_{12} \prec A_{11} \prec A_9 \prec A_8.$$

According to the result found by HybSo, the best alternatives is A8 and the worst one is A5. In other terms, the room with the best microclimate is room number 8 and the worst is in room number 5.

We performed a similarity study between the proposed algorithm (HybSo) and other state-of-the-art methods (i.e., TOPSIS, VIKOR, COPRAS, WASPAS, MOORA, CODAS, ARAS and CoCoSo) on the evaluation of microclimate in office rooms problem. In this study, we have measured the Spearman correlation coefficients (CCs) between the ranking of HybSo and the rankings of other methods. The results of this study are summarized in Table 16. The results show that there is a high similarity between the ranking of HybSo and the rankings of the other methods. Therefore, HybSo agrees significantly with the other methods. Hence, we can say that HybSo's decision results are valid.

Table 16: Summary of the similarity study results

	TOPSIS	VIKOR	COPRAS	WASPAS	MOORA	CODAS	ARAS	CoCoSo	HybSo
A_1	3	7	3	4	4	3	4	6	6
A_2	7	6	7	6	6	6	6	5	7
A_3	10	9	10	10	10	9	10	8	8
A_4	9	8	9	9	8	10	9	7	5
A_5	14	12	13	14	14	14	14	13	14
A_6	13	11	12	12	12	13	13	12	12
A_7	12	14	14	13	13	12	12	14	13
A_8	1	1	1	1	1	1	2	1	1
A_9	2	2	2	2	2	2	1	2	2
A_{10}	11	10	11	11	11	11	11	11	10
A_{11}	4	3	4	3	3	4	3	3	3
A_{12}	8	4	5	5	5	7	5	4	4
A_{13}	6	5	8	8	9	8	8	10	11
A_{14}	5	13	6	7	7	5	7	9	9
CC	0,802	0,846	0,886	0,912	0,938	0,837	0,903	0,974	1

Following the same experimental evaluation using LPSP (see section 3.1.2), we performed a sensitivity analysis using the evaluation of microclimate in office rooms problem. In the same way as in the previous section, we have made some modifications to the initial model of the EMOR. That is, by generating all possible permutations between the criteria' weights. After removing the replicates, we obtained 720 possible cases, we organized these cases as 720 different tests of the weight sensitivity analysis. The ranking results show that A_8 ranked first in 696 tests (in 96.67% of tests) and second in 24 tests, so we can conclude that the best alternative is A_8. Figure 4 shows the variation of alternative rankings according to the sensitivity analysis tests (i.e., it shows the weight perturbation's influence on the ranking of alternatives).

Next, we calculated Spearman's correlation coefficients (CCs) between the initial ranking of HybSo and the sensitivity analysis test rankings, the results are summarized in Figure 5. 720 tests (100% of the tests) show a high similarity with the initial ranking (>80 per cent), which proves the high stability of the proposed algorithm. We also performed the same sensitivity analysis for the other MCDM methods (i.e., TOPSIS, VIKOR, COPRAS, WASPAS, MOORA, CODAS, ARAS and CoCoSo). 346 tests (48% of the tests) have a high similarity for the TOPSIS method, 334 tests (46% of the tests) for VIKOR, 635 tests (88% of the tests) for COPRAS, 596 tests (83% of the tests) for WASPAS, 644 tests (89% of the tests) for MOORA, 515 tests (72% of the tests) for CODAS, 543 tests (75% of the tests) for ARAS, and 720 tests (100% of the tests) for CoCoSo. The details of this comparison are illustrated in Figure 6. The results of this sensitivity analysis show that HybSo and CoCoSo are the most stable methods for this problem (The HybSo and CoCoSo methods have the highest number of tests that have a Spearman's correlation coefficients over 0.8 with a proportion of 100%). Finally, we are able to confirm the high similarity of HybSo rankings and the high stability of HybSo results. Therefore, we can affirm the validity and stability of HybSo results and that HybSo can be used to solve MCDM problems.

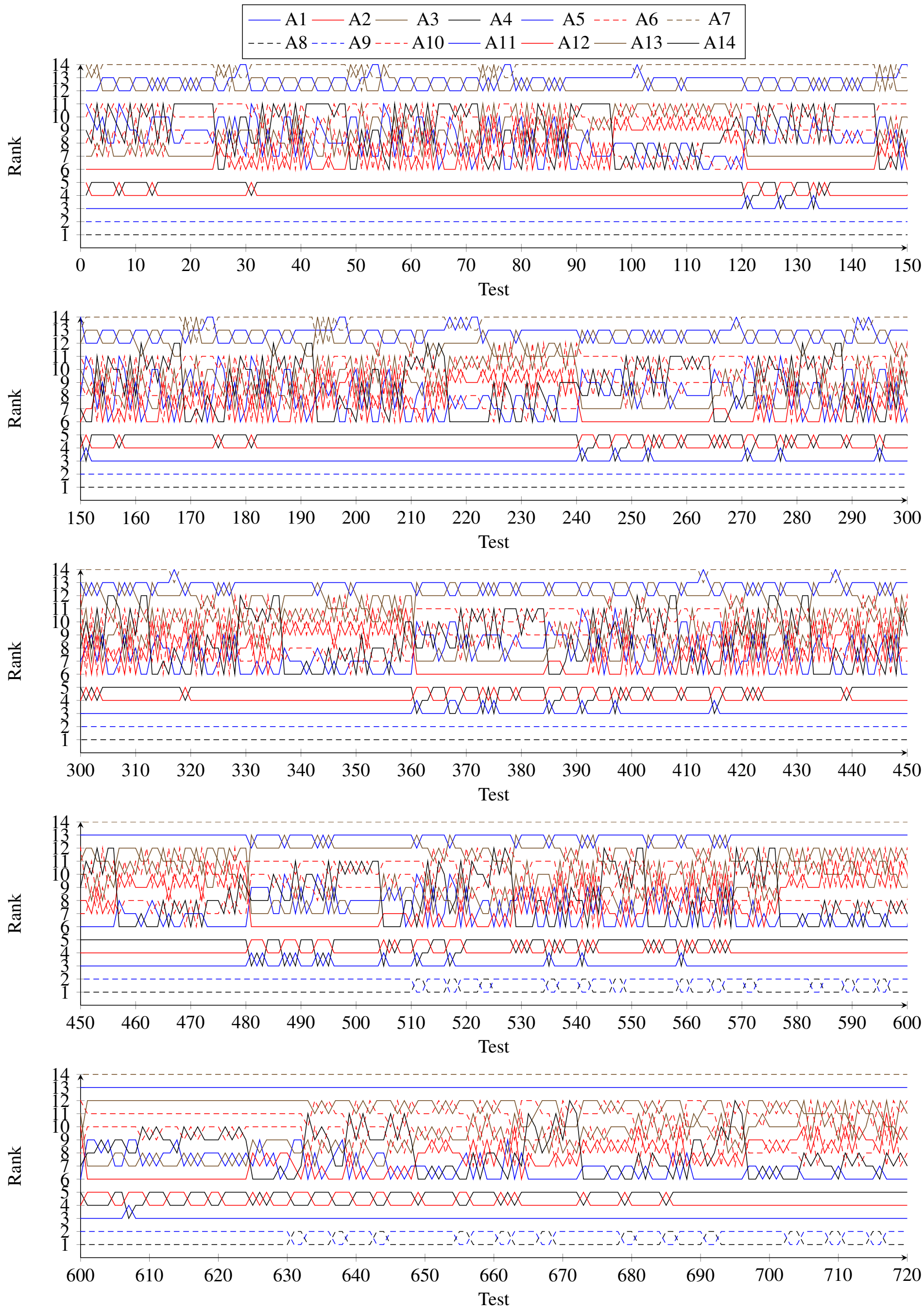

Figure 4: The variation of alternative rankings according to the sensitivity analysis tests for EMOR

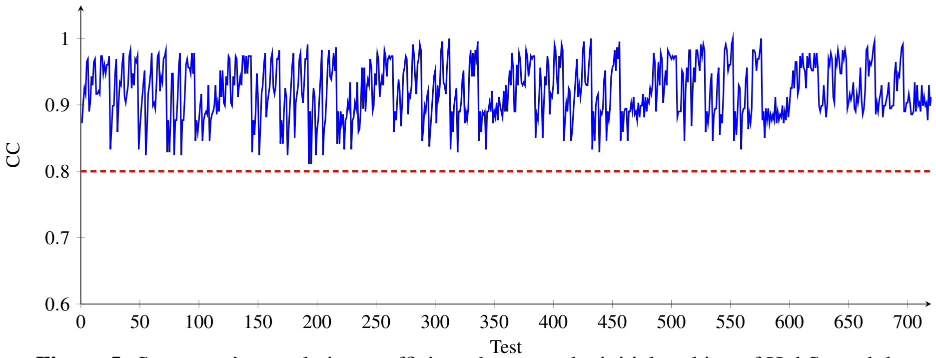

Figure 5: Spearman's correlation coefficients between the initial ranking of HybSo and the rankings of the sensitivity analysis tests for EMOR

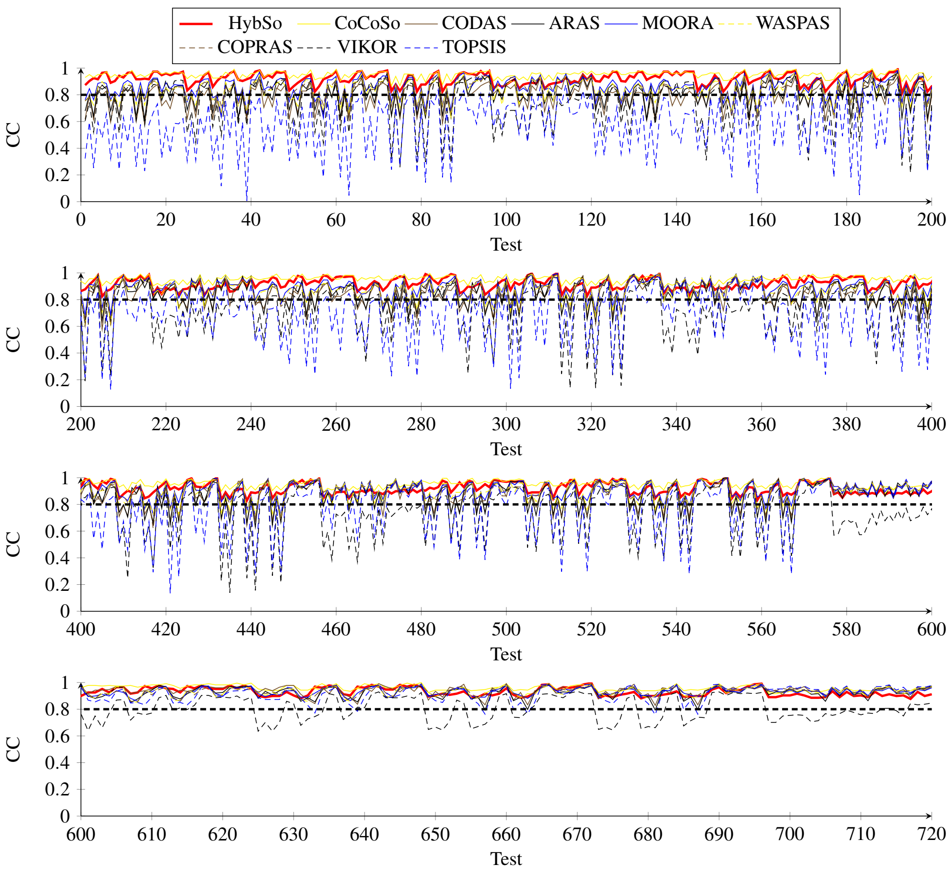

Figure 6: Spearman's correlation coefficients between the initial ranking and the rankings of the sensitivity analysis tests obtained using the nine competing algorithms for EMOR

4. CONCLUSION

Given the importance of multi-criteria decision making in human life and the development of multi-criteria decision making methods, in this paper, we addressed solving the multi-criteria decision making problems. Therefore, we have presented HybSo a new Hybrid Solution for the multi-criteria decision-making problems. The proposed method is based on a new hybrid normalization between the distance measure and the ratio system. As to improve the stability of the proposed method and the flexibility of the results, we introduced two hybrid equations to compute the weighted performance of alternatives. Finally, we also introduced a hybrid aggregation rule based on exponential and logarithmic functions to give the final ranking of the alternatives.

Furthermore, to assess the suggested algorithm, we have used the logistic provider selection (LPSP) problem and the evaluation of microclimate in an office (EMOR) problem. We presented an experimental study, where, we have used eight state-of-the-art MCDM methods (TOPSIS, VIKOR, COPRAS, WASPAS, MOORA, CODAS, ARAS and CoCoSo) for comparison. In this comparison, we calculated the Spearman correlation coefficients (CCs) between the HybSo ranking and other methods' rankings, all the CCs found are greater than 0.8 which means that there is a high similarity between the rankings of the proposed method (HybSo) and the rankings of other methods. We also performed a sensitivity analysis of the proposed method (HybSo) and the other methods using both the (LPSP) and (EMOR) problems. In this analysis, we have modified the initial model of each problem by swapping the weights of the criteria, after removing the repetitions, we have obtained 60 different tests of the sensitivity analysis for LPSP and 720 for EMOR. We calculated CCs between the initial ranking of each method and these rankings from the sensitivity analysis tests, 92% of the LPSP tests and 100% of the EMOR tests showed a high similarity with the initial ranking and this is the best result of all the methods. Therefore, the results obtained show that the proposed method (HybSo) is more stable than the state-of-the-art methods (the poposed method performs well in sensitivity analysis compared with the best state-of-the-art methods), and confirm the validity and stability of HybSo for solving MCDM problems. In future work, we expect to investigate the application of the proposed HybSo method in more real-world applications, including the ranking of climatic regions for wind turbine installation (wind power) and adding new criteria weight calculation techniques.

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