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Research Article

DECISION MAKING METHOD TO OPTIMAL SELECTION OF AREA FOR BUILDING PROJECT BASED ON FUZZY PARAMETERIZED NEUTROSOPHIC SOFT EXPERT SET

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Abstract: Fuzzy parameterized neutrosophic soft expert sets is a useful extension of fuzzy parameterized intuitionistic fuzzy soft expert and neutrosophic soft sets. This set deals with attributes as well but with many definitive assessments, whereas the soft set only addresses one set of attributes. The development of this model addresses the limitations of existing soft set-based models, particularly their inability to handle multiple experts' opinions in a decision-making context. This paper's major goal is to introduce the theory of fuzzy parameterized neutrosophic soft expert set theory. It also discusses the notion's numerous related ideas and the basic operations on it, such as the complement, union, intersection, AND, and OR. The essential features of this idea are established, as are relevant rules such as De Morgan's laws. A generalised algorithm is then used to apply the suggested idea of fuzzy parameterized neutrosophic soft expert sets in a decision-making situation. Moreover, in order to check the validity of the proposed algorithm, an example related to the choice of the area of building project using the decision-making method is presented. The suggested method provides reliable, reliable decision-making outcomes, and its enhanced capacity and dependability are confirmed by a robust comparison with earlier models.

Keywords: Soft set, soft expert set, neutrosophic soft set, neutrosophic soft expert set, decision-making, optimization.

MSC: 03B52; 03E72; 52A01.

1. INTRODUCTION

Decision making in uncertain environments refers to the process of making choices or decisions when there is a lack of complete information or knowledge about potential outcomes. This can include situations where there is a high degree of risk or ambiguity, and decision makers must weigh the potential outcomes of different choices in order to make the best decision possible. This can be done using techniques such as decision analysis, game theory, or other decision-making models that take into account factors such as risk, probability, and uncertainty. We are aware that the real world is rife with uncertainty, inaccuracy, and ambiguity. In actuality, the majority of the issues we encountered are ambiguous rather than specific. Classical approaches are not always effective when dealing with unclear data since these difficulties contain a wide variety of uncertainty. It is well known that mathematical theories such as the theories of probabilities, fuzzy sets [1], rough sets [2], and others are useful tools for characterising uncertainty. Zare Ahmadabadi et al. [3] used hesitant fuzzy to study factors affecting the supply chain management. Rahadian et al. [4] used decision-making approach for the evaluation of different countries energy investment. The neutrosophic set(NS) was defined by Smarandache [5] as a generalisation of fuzzy sets, intuitionistic fuzzy sets, and classical sets. The NS finds wide-ranging applications in several fields such as databases, control theory, topology, and medical diagnosis. Membership in truth, indeterminacy, and falsehood are completely independent to one another in a NS, where indeterminacy is quantified explicitly. A scientific or technical description of the operators of the set-theoretic view and the NS is required. It will be difficult to implement in real applications if not. For this reason, before demonstrating its numerous properties and set theoretic operations, Wang et al. [6] created a single valued neutrosophic set (SVNS). Ghaforiyan et al. [7] used neutrosophic structure to describe the different applications. In order to overcome the shortcomings of decision-making techniques when dealing with ambiguous data, Nafei et al. [8, 9, 10] presented a unique framework for decision-making that combines the TOPSIS with neutrosophic triplets. The NS and their hybrid designs have advanced in theory and application [11, 12].

Molodtsov [13] pointed out that there were drawbacks to each of these theories, though. To circumvent these issues, Molodtsov presented a novel approach to describing uncertainty called a soft set. This theory has since been applied to many other domains, including probability theory, measurement theory, game theory, operations research, and smoothness of functions.

Cybernetics, machine learning, intelligent systems, information sciences, and other disciplines are similarly impacted. Most of these uses are previously illustrated in Molodtsov's book. Vellapandi et al. [14] presented an original method to choose an option for the soft set across all universes using the certainty of a paramter, and they established a new notion of certainty and coverage of a parameter of the soft set. Karatas et al. [15] gave the idea of an effective neutrosophic soft set with its valuable usage. Zhao et al. [16] used the neutrosophic with soft set-like structure in decision making problems. Pethaperumal et al. [17] used a soft set-like structure in the health system. Debnath [18] made an extension in the soft set-like structure. Smarandache et al. [19, 20, 21] gave some characteristics in soft set-like structures.

A few basic operations on soft sets were defined by Maji [22]. In 2009, Ali [23] introduced a number of innovative soft set operations. Furthermore, Maji et al. [24] defined a fuzzy soft set and gave an example of its application in a scenario involving decision-making [25]. Research on applications of soft set theory has increased as a result of these definitions. Çağman [26, 27] applied the soft set theory to decision-making.

Chen [28] discussed the parameterization reduction of soft sets and its applications. Simultaneously, Feng [29] introduced the use of soft sets in interval-valued fuzzy soft set-based decision-making. Jiang [30] introduced a novel technique for applying interval-valued intuitionistic fuzzy sets in decision-making. Later studies by Feng, Maji, and others have looked at the more general features and applications of soft set theory; see, for example, [31, 32]. It is observed that fuzzy soft sets have extended the theory of soft sets; these are cited in [33, 34]. Rough soft sets were recently added to semi-rings for the first time, and Zhan [35] documented several properties of rough soft semi-rings. Muniba et al. [36] introduced a new method for the prediction of climate change by using MCDM technique. Smarandache [37] gave the idea of a super hypersoft set with different applications. Many interesting and contemporary applications of soft set theory have involved the use of intuitionistic fuzzy sets and rough set theory. Using fuzzy soft sets, Jiang et al. [39] extended the customisable technique for decision-making. They explicitly offered an adjustable strategy to intuitionistic fuzzy soft set-based decision-making and provided various pertinent examples by using soft sets of intuitionistic fuzzy soft sets. Different convexity (concavity) structures on soft sets were presented by Rahman et al. [40, 41]. With some different results, they investigated the various concavity and convexity properties in soft set and fuzzy set environments. While soft set structures address the opinions of a single expert, some situations necessitate the opinions of multiple experts, such as in a questionnaire. To address this shortcoming, Alkhazaleh et al. [42] developed the notion of the soft expert set (SE-set) and offered applications in decision-making. Ihsan et al.[43] explored the special properties of the convexity on the FSE-set, which was gestated. Alkhazaleh et al. [44] employed a soft expert set in an uncertain environment to conceptualise fuzzy soft expert set (FSE-set). They discussed their potential application in making decisions. It was theorised that convexity-cum-concavity on SE-set, and Ihsan et al. [45] addressed various features. Intuitionistic fuzzy soft expert sets were developed by Broumi et al. [46, 47], who also presented their use in decision-making.

1.1. Research gap and novelty

Fuzzy parameterized soft set (FPS-set) was developed by *Çağman* et al. [48], who also applied a significant degree to parameters. A concept of operations on fuzzy parameterized fuzzy soft sets (FPSF-set) was presented by *Çağman* et al. [49]. Subsequently, the FPSF-set-aggregation operator was established in order to generate the FPSF-set-decision making method, which facilitates the development of more efficient decision processes. The t-norms and t-conorms products of fuzzy parameterized fuzzy soft sets were defined by Zhu and Zhan [50]. The AND-FPSF-set decision making method and the OR-FPSF-set decision making method are built using these products, respectively. Lastly, decision-making strategies are used to solve situations including uncertainties. Fuzzy parameterized fuzzy soft sets (FPFS-sets) are a useful tool for solving decision-making problems, as explored by Riaz and Hashmi [51]. They defined AND and OR operations for FPFS-sets. They examined the comparison tables, reduct, and aggregation operation for the FPFS-set. They illustrated our processes in the form of a few algorithms with the help of several examples. Sulukan et al.[52] proposed fuzzy parameterized intuitionistic fuzzy soft sets. Ihsan et al. [53] constructed the structure of a fuzzy parameterized single-valued neutrosophic soft set. They next used these sets to the performance-based value assignment (PVA) problem. The architectures of fuzzy parameterized and soft expert sets were combined, according to Bashir et al. [54], to generate hybrid fuzzy parameterized soft expert sets (FPSE-set) with applications in DMPs. As an extension of the fuzzy soft expert set, Hazaymeh et al. [55] introduced the concept of fuzzy parameterized fuzzy soft expert set by giving a membership value to each parameter in a collection of parameters. They also talked about the characteristics of its basic operations, such as complement, union intersection, "AND," and "OR." Lastly, they offered an application of this concept to decision-making issues. The notion of fuzzy parameterized intuitionistic fuzzy soft expert sets was put forth by Ganeshsree and Salleh [56]. They also specified a number of words related to this concept, as well as the fundamental operations on it, such as complement, union, intersection, AND, and OR. This idea was demonstrated by De Morgan's laws and other relevant fundamental characteristics and rules. A generalised technique is then used to apply the proposed concept of fuzzy parameterized neutrosophic soft expert sets to a decision-making problem. A novel mathematical framework, the fuzzy parameterized neutrosophic soft expert set, is developed in response to this literary demand. It is more versatile and flexible because it can handle the following scenarios as a whole:

1. The parameterization of fuzzy set-like structures in soft set environments with fuzzy set-like settings has been the subject of much research. It is clear from the literature assessment of the most pertinent models presented above that the models discussed above used conventional soft approximate mapping to examine the parameterized nature of the parametric domain and concentrated on a single set of attributes. They are unable to handle scenarios involving three-dimensional type data (membership, non-membership, and indeterminacy) when there are several deciding factors.
2. Above described soft sets structured for the necessity of involving the information of single expert opinions, but this literature demands to handle the situation of multiple experts' decisive opinions in a single structure which leads to the development of new models like this one.
3. The $\mathbb{FPNSE}$-set is developed to handle such scenarios since the incapability of soft expert set-like models with neutrosophic settings results in the necessity for new structure. This novel model stresses careful examination of three-dimensional data, making it a flexible and trustworthy model for a fair decision-support system.

1.2. Main contributions

The main contributions of the suggested work have been discussed below:

1. Using vivid numerical examples, the ideas and algebraic characteristics of the $\mathbb{FPNSE}$-set are explained.
2. By identifying their pertinent fuzzy parameterized grades, the ambiguous characteristics of the parameters and sub-parameters under examination are assessed.

3. Using set-based operations of $\mathbb{FPNSES}$-set, a novel robust MADM-based algorithm is provided to evaluate construction project area analyses. The suggested algorithms' processes are simple to comprehend and devoid of computational complexity.
4. In the end, a strong comparison of the suggested structure with the existing structures has been made.

We develop innovative structures of fuzzy parameterized neutrosophic soft expert sets and characterise them with the aid of algorithm-based decision-support systems, inspired by the aforementioned literature in general and [54, 55, 56] in particular. The remainder of the study is structured as follows: For a better understanding of the primary topic, Section 2 reviews some key definitions of elementary nature from the literature. In Sections 3 and 4, respectively, theories of $\mathbb{FPNSE}$-set are developed together with their decision-support system. In order to see the benefits of the proposed study, a comparison with some current relevant models is offered in Section 5, and the discussion of generalisation and the advantages of the proposed work is presented in Section 6. In the final section, the article is summarised along with suggested next steps.

2. PRELIMINARIES

This section provides definitions and explanations of some basic words from the literature that are connected to the primary topic.

Definition 1. *[5] The definition of neutrosophic set $\mathfrak{M}$ on $\hat{\aleph}$ is given by* $\mathfrak{M} = \{\langle \chi, (\mathrm{T}_{\mathfrak{M}}(\chi), l_{\mathfrak{M}}(\chi), \mathrm{F}_{\mathfrak{M}}(\chi))\rangle : \chi \in \mathbb{E}, \mathrm{T}_{\mathfrak{M}}, l_{\mathfrak{M}}, \mathrm{F}_{\mathfrak{M}} \in]^{-}0, 1^{+}[\}$ *having the condition* $0^{-} \leq \mathrm{T}_{\mathfrak{M}}(\chi), l_{\mathfrak{M}}(\chi), \mathrm{F}_{\mathfrak{M}}(\chi) \leq 3^{+}$, *where* $\mathrm{T}_{\mathfrak{M}}, l_{\mathfrak{M}}, \mathrm{F}_{\mathfrak{M}}$ *are constructed here for membership, indeterminacy, and non-membership functions.*

Definition 2. *[5] Consider two neutrosophic sets $\mathfrak{P}$ and $\mathfrak{T}$ such that*
$\mathfrak{P} = \{\langle \ltimes, (\mathrm{T}_{\mathfrak{P}}(\ltimes), l_{\mathfrak{P}}(\ltimes), \mathrm{F}_{\mathfrak{P}}(\ltimes))\rangle : \ltimes \in \mathbb{E}, \mathrm{T}_{\mathfrak{P}}, l_{\mathfrak{P}}, \mathrm{F}_{\mathfrak{P}} \in]^{-}0, 1^{+}[\}$,
$\mathfrak{T} = \{\langle \ltimes, (\mathrm{T}_{\mathfrak{T}}(\ltimes), l_{\mathfrak{T}}(\ltimes), \mathrm{F}_{\mathfrak{T}}(\ltimes))\rangle : \ltimes \in \mathbb{E}, \mathrm{T}_{\mathfrak{T}}, l_{\mathfrak{T}}, \mathrm{F}_{\mathfrak{T}} \in]^{-}0, 1^{+}[\}$, *then,*

1. *Neutrosophic set* $\mathfrak{M} \subseteq \mathfrak{N}$ *if*
   $\mathrm{T}_{\mathfrak{M}}(\bar{\wedge}) \leq \mathrm{T}_{\mathfrak{N}}(\bar{\wedge})$, $l_{\mathfrak{M}} \geq l_{\mathfrak{N}}$, $\mathrm{F}_{\mathfrak{M}}(\bar{\wedge}) \leq \mathrm{F}_{\mathfrak{P}}(\bar{\wedge})$.
2. *The compliment operation is*
   $\mathfrak{M}^{c} = \{\langle \bar{\wedge}, (\mathrm{T}_{\mathfrak{M}}(\bar{\wedge}), 1 - l_{\mathfrak{M}}(\bar{\wedge}), \mathrm{F}_{\mathfrak{M}}(\bar{\wedge}))\rangle : \bar{\wedge} \in \mathbb{E}, \mathrm{T}_{\mathfrak{M}}, l_{\mathfrak{M}}, \mathrm{F}_{\mathfrak{M}} \in]^{-}0, 1^{+}[\}$,
3. *The operation union is*
   $max(\mathrm{T}_{\mathfrak{M}}(\bar{\wedge}), \mathrm{T}_{\mathfrak{N}}(\bar{\wedge})$, $min(l_{\mathfrak{M}}(\bar{\wedge}), l_{\mathfrak{N}}(\bar{\wedge}))$, $min(\mathrm{F}_{\mathfrak{M}}(\bar{\wedge}), \mathrm{F}_{\mathfrak{N}}(\bar{\wedge}))$,
4. *The operation intersection is*
   $min(\mathrm{T}_{\mathfrak{M}}(\bar{\wedge}), \mathrm{T}_{\mathfrak{N}}(\bar{\wedge})$, $max(l_{\mathfrak{M}}(\bar{\wedge}), l_{\mathfrak{N}}(\bar{\wedge}))$, $max(\mathrm{F}_{\mathfrak{M}}(\bar{\wedge}), \mathrm{F}_{\mathfrak{N}}(\bar{\wedge}))$.

Definition 3. *[52] Let $\mathscr{H}$ is the cartesian product of $\heartsuit$, ϖ, $\diamondsuit$ and collection of experts and parameters are $\heartsuit$, ϖ respectively. While $\diamondsuit$ is the symbol for the opinions of experts i.e, $\diamondsuit$ = $\{0$ = disagree (dag), 1 = agree (ag) $\}$ and $\hat{\aleph}$ for the universe with $P(\hat{\aleph})$ as a power set and $\mathbb{I}$ =$[0,1]$. For the sake of simplicity, it is assumed that there are only two possible values for the collection of opinions in this study: agree and disagree. It is possible to incorporate more alternatives for the collection of opinions, such as more focused opinions. A pair (I_{ζ}, β) can be considered as a FPIFSE-set with g_{ζ} is $I_{\zeta} : \beta \to P(\hat{\aleph})$ such that β is being used as a subsets of ϖ.*

Definition 4. *[52] The property of being a subset of FPIFSE-set can be considered when certain conditions are fulfilled (i) when the set $\Re$ of FPIFSE-set $(I_\zeta, \Re)$ is a subset of χ of another FPIFSE-set (F_η, χ) and (ii) $I_\zeta(\xi)$ is being as an intuitionistic fuzzy subset of $F_\eta(\xi)$ for all $\xi \in I_\zeta$.*

Definition 5. *[52] Another subset property of FPIFSE-set is an agree FPIFSE-set which is $\subseteq$ FPIFSE-set $(g_\Gamma, \mp)^1 = \{g_\Gamma(\ddot{\varphi}) : \ddot{\varphi} \in \heartsuit \times \varpi \times 1\}$.*

Definition 6. *[52] Another subset property of FPIFSE-set is a dis-agree FPIFSE-set which is actually $\subseteq$ FPIFSE-set $(g_\Gamma, \mp)^0 = \{g_\Gamma(\ddot{\varphi}) : \ddot{\varphi} \in \heartsuit \times \varpi \times 0\}$.*

3. FUZZY PARAMETERIZED NEUTROSOPHIC SOFT EXPERT SET ($\mathbb{FPNSE}$-SET)

Some fundamental properties are described here. This section describes the development of the $\mathbb{FPNSE}$-set using the concept of the fuzzy parameterized intuitionistic fuzzy soft expert set.

Definition 7. ***Fuzzy Parameterized Neutrosophic Soft Expert Set***
A fuzzy parameterized neutrosophic soft expert set $\partial_{\mathscr{F}\mathscr{P}}$ over $\hat{\aleph}$ is defined as

$$\partial_{\mathscr{F}\mathscr{P}} = \left\{ \left(\left(\frac{\hat{\Lambda}}{\chi_{\mathscr{F}\mathscr{P}}(\hat{\Lambda})}, \hat{K}_i, \hat{Y}_i \right), \frac{\hat{\delta}}{\phi_{\mathscr{F}\mathscr{P}}(\hat{\delta})} \right); \forall \hat{\Lambda} \in \mathscr{H}, \hat{K}_i \in \check{\zeta}, \hat{Y}_i \in \diamondsuit, \hat{\delta} \in \hat{\aleph} \right\}$$

where $\chi_{\mathscr{F}\mathscr{P}} : \check{\zeta} \to FP(\hat{\aleph})$, and $\phi_{\mathscr{F}\mathscr{P}} : \check{\zeta} \to NP(\hat{\aleph})$ is called approximate function of $\mathbb{FPNSE}$-set, $\check{\zeta} \subseteq \mathscr{H} = \heartsuit \times \varpi \times \diamondsuit$.

Example 8. *Consider a scenario where a network of universities seeks a construction company to update the campus building in response to globalisation and requires specialised personnel (experts) to evaluate the facility's functionality. Let $\hat{\aleph} = \{\triangledown_1, \triangledown_2\}$ be a set of companies and distinct attributes set is $\{\Re_1 = cheap, \Re_2 = standard\}$. Let $\{\Re_1/0.2, \Re_2/0.3\}$ be the fuzzy subset of $I^\heartsuit$(set of fuzzy subsets of $\heartsuit$. Now $\mathscr{H} = \heartsuit \times \varpi \times \diamondsuit$.*

$$\mathscr{H} = \left\{ \begin{array}{l} (\Re_1/0.2, \Bbbk_1, 0), (\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_2, 0), (\Re_1/0.2, \Bbbk_2, 1) \\ , (\Re_1/0.2, \Bbbk_3, 0), (\Re_1/0.2, \Bbbk_3, 1), (\Re_2/0.3, \Bbbk_1, 0), (\Re_2/0.3, \Bbbk_1, 1), \\ (\Re_2/0.3, \Bbbk_2, 0), (\Re_2/0.3, \Bbbk_2, 1), (\Re_2/0.3, \Bbbk_3, 0), (\Re_2/0.3, \Bbbk_3, 1), \end{array} \right\}$$

and $\varpi = \{\Bbbk_1, \Bbbk_2, \Bbbk_3, \}$ be a set of specialists. We have:

$$\beth_1 = \beth(\Re_1/0.2, \Bbbk_1, 1) = \left\{ \frac{\triangledown_1}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{\triangledown_2}{\langle 0.7, 0.4, 0.5 \rangle} \right\},$$

$$\beth_2 = \beth(\Re_1/0.2, \Bbbk_2, 1) = \left\{ \frac{\triangledown_1}{\langle 0.4, 0.6, 0.3 \rangle}, \frac{\triangledown_2}{\langle 0.8, 0.4, 0.5 \rangle} \right\},$$

$$\beth_3 = \beth(\Re_1/0.2, \Bbbk_3, 1) = \left\{ \frac{\triangledown_1}{\langle 0.7, 0.4, 0.3 \rangle}, \frac{\triangledown_2}{\langle 0.5, 0.3, 0.6 \rangle}, \right\},$$

$$\beth_4 = \beth(\Re_2/0.3, \Bbbk_1, 1) = \left\{ \frac{\triangledown_1}{\langle 0.9, 0.6, 0.3 \rangle}, \frac{\triangledown_2}{\langle 0.4, 0.3, 0.4 \rangle}, \right\},$$

$$\gimel_5 = \gimel(\Re_2/0.3, \Bbbk_2, 1) = \left\{ \frac{\nabla_1}{\langle 0.4,0.7,0.6\rangle}, \frac{\nabla_2}{\langle 0.8,0.9,0.7\rangle}, \right\},$$

$$\gimel_6 = \gimel(\Re_2/0.3, \Bbbk_3, 1) = \left\{ \frac{\nabla_1}{\langle 0.5,0.3,0.7\rangle}, \frac{\nabla_2}{\langle 0.3,0.6,0.4\rangle}, \right\},$$

$$\gimel_7 = \gimel(\Re_1/0.2, \Bbbk_1, 0) = \left\{ \frac{\nabla_1}{\langle 0.3,0.2,0.6\rangle}, \frac{\nabla_2}{\langle 0.7,0.4,0.5\rangle}, \right\},$$

$$\gimel_8 = \gimel(\Re_1/0.2, \Bbbk_2, 0) = \left\{ \frac{\nabla_1}{\langle 0.3,0.8,0.4\rangle}, \frac{\nabla_2}{\langle 0.9,0.2,0.4\rangle}, \right\},$$

$$\gimel_9 = \gimel(\Re_1/0.2, \Bbbk_3, 0) = \left\{ \frac{\nabla_1}{\langle 0.3,0.7,0.5\rangle}, \frac{\nabla_2}{\langle 0.2,0.8,0.6\rangle}, \right\},$$

$$\gimel_{10} = \gimel(\Re_2/0.3, \Bbbk_1, 0) = \left\{ \frac{\nabla_1}{\langle 0.8,0.7,0.6\rangle}, \frac{\nabla_2}{\langle 0.3,0.6,0.7\rangle}, \right\},$$

$$\gimel_{11} = \gimel(\Re_2/0.3, \Bbbk_2, 0) = \left\{ \frac{\nabla_1}{\langle 0.7,0.3,0.5\rangle}, \frac{\nabla_2}{\langle 0.7,0.6,0.4\rangle} \right\},$$

$$\gimel_{12} = \gimel(\Re_2/0.3, \Bbbk_3, 0) = \left\{ \frac{\nabla_1}{\langle 0.6,0.8,0.5\rangle}, \frac{\nabla_2}{\langle 0.7,0.9,0.4\rangle} \right\}.$$

The $\mathbb{FPNSE}$*-set can be described as* $(\gimel, \mathbb{Q}) =$

$$\left\{ \begin{array}{l}
\left((\Re_1/0.2, \Bbbk_1, 1), \left\{ \frac{\nabla_1}{\langle 0.3,0.5,0.4\rangle}, \frac{\nabla_2}{\langle 0.7,0.4,0.5\rangle} \right\} \right), \\
\left((\Re_1/0.2, \Bbbk_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4,0.6,0.3\rangle}, \frac{\nabla_2}{\langle 0.8,0.4,0.5\rangle} \right\} \right), \\
\left((\Re_1/0.2, \Bbbk_3, 1), \left\{ \frac{\nabla_1}{\langle 0.7,0.4,0.3\rangle}, \frac{\nabla_2}{\langle 0.5,0.3,0.6\rangle} \right\} \right), \\
\left((\Re_2/0.3, \Bbbk_1, 1), \left\{ \frac{\nabla_1}{\langle 0.9,0.6,0.3\rangle}, \frac{\nabla_2}{\langle 0.4,0.3,0.4\rangle}, \right\} \right), \\
\left((\Re_2/0.3, \Bbbk_2, 1), \left\{ \frac{\nabla_1}{\langle 0.4,0.7,0.6\rangle}, \frac{\nabla_2}{\langle 0.8,0.9,0.7\rangle}, \right\} \right), \\
\left((\Re_2/0.3, \Bbbk_3, 1), \left\{ \frac{\nabla_1}{\langle 0.5,0.3,0.7\rangle}, \frac{\nabla_2}{\langle 0.3,0.6,0.4\rangle}, \right\} \right), \\
\left((\Re_1/0.2, \Bbbk_1, 0), \left\{ \frac{\nabla_1}{\langle 0.3,0.2,0.6\rangle}, \frac{\nabla_2}{\langle 0.7,0.4,0.5\rangle}, \right\} \right), \\
\left((\Re_1/0.2, \Bbbk_2, 0), \left\{ \frac{\nabla_1}{\langle 0.3,0.8,0.4\rangle}, \frac{\nabla_2}{\langle 0.9,0.2,0.4\rangle}, \right\} \right), \\
\left((\Re_1/0.2, \Bbbk_3, 0), \left\{ \frac{\nabla_1}{\langle 0.3,0.7,0.5\rangle}, \frac{\nabla_2}{\langle 0.2,0.8,0.6\rangle}, \right\} \right), \\
\left((\Re_2/0.3, \Bbbk_1, 0), \left\{ \frac{\nabla_1}{\langle 0.8,0.7,0.6\rangle}, \frac{\nabla_2}{\langle 0.3,0.6,0.7\rangle}, \right\} \right), \\
\left((\Re_2/0.3, \Bbbk_2, 0), \left\{ \frac{\nabla_1}{\langle 0.7,0.3,0.5\rangle}, \frac{\nabla_2}{\langle 0.7,0.6,0.4\rangle}, \right\} \right), \\
\left((\Re_2/0.3, \Bbbk_3, 0), \left\{ \frac{\nabla_1}{\langle 0.6,0.8,0.5\rangle}, \frac{\nabla_2}{\langle 0.7,0.9,0.4\rangle}, \right\} \right)
\end{array} \right\}.$$

Definition 9. *A subset property of A* $\mathbb{FPNSE}$*-set* $(\gimel_1, \mathbb{Q})$ *with another* $\mathbb{FPNSE}$*-set* $(\gimel_2, \mathbb{P})$ *can be seen as when set* $\mathbb{Q} \subseteq \mathbb{P}$*, and also fulfills the condition* $\forall\, \gamma \in \mathbb{Q}, \gimel_1(\gamma) \subseteq \gimel_2(\gamma)$ *and symbolized as* $(\gimel_1, \mathbb{Q}) \subseteq (\gimel_2, \mathbb{P})$.

Example 10. *Consider the Example 8, suppose*
$\mathbb{Q}_1 = \left\{ (\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_2, 1), , (\Re_1/0.2, \Bbbk_3, 0) \right\}$,
$\mathbb{Q}_2 = \left\{ (\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_2, 1), (\Re_1/0.2, \Bbbk_2, 0), (\Re_1/0.2, \Bbbk_3, 0), (\Re_1/0.2, \Bbbk_3, 1) \right\}$.
It is clear that $\mathbb{Q}_1 \subset \mathbb{Q}_2$.

Suppose $(\gimel_1,\mathbb{Q}_1)$ *and* $(\gimel_2,\mathbb{Q}_2)$ *be defined as following*

$$(\gimel_1,\mathbb{Q}_1)=\left\{\begin{array}{l}\left((\Re_1/0.2,\Bbbk_1,1),\frac{\triangledown_1}{\langle 0.3,0.5,0.4\rangle},\frac{\triangledown_2}{\langle 0.7,0.4,0.5\rangle}\right),\\ \left((\Re_1/0.2,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.3,0.4,0.5\rangle},\frac{\triangledown_2}{\langle 0.6,0.4,0.6\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_3,0),\left\{\frac{\triangledown_1}{\langle 0.1,0.6,0.3\rangle},\frac{\triangledown_2}{\langle 0.1,0.7,0.4\rangle}\right\}\right)\end{array}\right\},$$

$$(\gimel_2,\mathbb{Q}_2)=\left\{\begin{array}{l}\left((\Re_1/0.2,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.2,0.3,0.6\rangle},\frac{\triangledown_2}{\langle 0.7,0.4,0.7\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.3,0.4\rangle},\frac{\triangledown_2}{\langle 0.8,0.3,0.5\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_3,1),\left\{\frac{\triangledown_1}{\langle 0.7,0.2,0.4\rangle},\frac{\triangledown_2}{\langle 0.5,0.2,0.6\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_3,0),\left\{\frac{\triangledown_1}{\langle 0.2,0.5,0.1\rangle},\frac{\triangledown_2}{\langle 0.2,0.6,0.3\rangle}\right\}\right)\end{array}\right\}$$

which shows that $(\gimel_1,\mathbb{Q}_1)\subseteq(\gimel_2,\mathbb{Q}_2)$.

Definition 11. *Two* $\mathbb{FPNSE}$*-sets will be* $(\gimel_1,\mathbb{Q}_1)=(\gimel_2,\mathbb{Q}_2)$ *if* $(\gimel_1,\mathbb{Q}_1)\subseteq(\gimel_2,\mathbb{Q}_2)$ *and* $(\gimel_2,\mathbb{Q}_2)\subseteq(\gimel_1,\mathbb{Q}_1)$.

Definition 12. *The complement of a* $\mathbb{FPNSE}$*-set* $(\gimel,\mathbb{Q})$*, denoted by* $(\gimel,\mathbb{Q})^c$*, is defined by* $(\gimel,\mathbb{Q})^c=\tilde{c}(\gimel(\varsigma))\ \forall\ \varsigma\in\hat{\aleph}$ *while* $\tilde{c}$ *is a NF complement.*

Example 13. *Making use of Example 8 for complement, we have*

$$(\gimel,\mathbb{Q})^c=\left\{\begin{array}{l}\left\{(\Re_1/0.2,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.5,0.2\rangle},\frac{\triangledown_2}{\langle 0.5,0.8,0.7\rangle}\right\}\right\},\\ \left((\Re_1/0.2,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.3,0.8,0.6\rangle},\frac{\triangledown_2}{\langle 0.5,0.9,0.8\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_3,1),\left\{\frac{\triangledown_1}{\langle 0.3,0.8,0.7\rangle},\frac{\triangledown_2}{\langle 0.6,0.7,0.5\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.3,0.9,0.9\rangle},\frac{\triangledown_2}{\langle 0.4,0.5,0.4\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.6,0.5,0.4\rangle},\frac{\triangledown_2}{\langle 0.7,0.9,0.8\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_3,1),\left\{\frac{\triangledown_1}{\langle 0.7,0.6,0.5\rangle},\frac{\triangledown_2}{\langle 0.4,0.4,0.3\rangle}\right\}\right)\end{array}\right\}.$$

Definition 14. *An agree-*$\mathbb{FPNSE}$*-set* $(\gimel,\mathbb{Q})_{ag}$ *over* $\hat{\aleph}$*, is a* $\mathbb{FPNSE}$*-subset of* $(\gimel,\mathbb{Q})$ *and is characterized as* $(\gimel,\mathbb{Q})_{ag}=\{\gimel_{ag}(\varsigma):\varsigma\in G\times D\times\{1\}\}$.

Example 15. *Finding agree-*$\mathbb{FPNSE}$*-set determined in 8, we get* $(\gimel,\mathbb{Q})=$

$$\left\{\begin{array}{l}\left((\Re_1/0.2,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.3,0.5,0.4\rangle},\frac{\triangledown_2}{\langle 0.7,0.4,0.5\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.6,0.3\rangle},\frac{\triangledown_2}{\langle 0.8,0.4,0.5\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_3,1),\left\{\frac{\triangledown_1}{\langle 0.7,0.4,0.3\rangle},\frac{\triangledown_2}{\langle 0.5,0.3,0.6\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.9,0.6,0.3\rangle},\frac{\triangledown_2}{\langle 0.4,0.3,0.4\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.7,0.6\rangle},\frac{\triangledown_2}{\langle 0.8,0.9,0.7\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_3,1),\left\{\frac{\triangledown_1}{\langle 0.5,0.3,0.7\rangle},\frac{\triangledown_2}{\langle 0.3,0.6,0.4\rangle}\right\}\right)\end{array}\right\}.$$

Definition 16. *A disagree-*$\mathbb{FPNSE}$*-set* $(\beth,\mathbb{Q})_{dag}$ *over* $\hat{\aleph}$*, is a* $\mathbb{FPNSE}$*-subset of* $(\beth,\mathbb{Q})$ *and is characterized as* $(\beth,\mathbb{Q})_{dag} = \{\beth_{dag}(\varsigma) : \varsigma \in \heartsuit \times \varpi \times \{0\}\}$.

Example 17. *Getting disagree-*$\mathbb{FPNSE}$*-set determined in example 8,* $(\beth,\mathbb{Q}) =$

$$\left\{\begin{array}{l} \left((\Re_1/0.2,\Bbbk_1,0), \left\{ \frac{\nabla_1}{\langle 0.3,0.2,0.6\rangle}, \frac{\nabla_2}{\langle 0.7,0.4,0.5\rangle}, \right\} \right), \\ \left((\Re_1/0.2,\Bbbk_2,0), \left\{ \frac{\nabla_1}{\langle 0.3,0.8,0.4\rangle}, \frac{\nabla_2}{\langle 0.9,0.2,0.4\rangle}, \right\} \right), \\ \left((\Re_1/0.2,\Bbbk_3,0), \left\{ \frac{\nabla_1}{\langle 0.3,0.7,0.5\rangle}, \frac{\nabla_2}{\langle 0.2,0.8,0.6\rangle}, \right\} \right), \\ \left((\Re_2/0.3,\Bbbk_1,0), \left\{ \frac{\nabla_1}{\langle 0.8,0.7,0.6\rangle}, \frac{\nabla_2}{\langle 0.3,0.6,0.7\rangle}, \right\} \right), \\ \left((\Re_2/0.3,\Bbbk_2,0), \left\{ \frac{\nabla_1}{\langle 0.7,0.3,0.5\rangle}, \frac{\nabla_2}{\langle 0.7,0.6,0.4\rangle}, \right\} \right), \\ \left((\Re_2/0.3,\Bbbk_3,0), \left\{ \frac{\nabla_1}{\langle 0.6,0.8,0.5\rangle}, \frac{\nabla_2}{\langle 0.7,0.9,0.4\rangle}, \right\} \right) \end{array}\right\}.$$

Definition 18. *A relative null* $\mathbb{FPNSE}$*-set of a* $\mathbb{FPNSE}$*-set* $(\beth_1,\mathbb{Q}_1)$ *w.r.t* $\mathbb{Q}_1 \subset \mathbb{Q}$*, denoted by* $(\beth_1,\mathbb{Q}_1)_{RN}$*, if* $\beth_1(g) = \emptyset, \forall g \in \mathbb{Q}_1$.

Example 19. *Taking Example 8, if*

$$(\beth_1,\mathbb{Q}_1)_{RN} = \{((\Re_1/0.1,\Bbbk_1,1),\emptyset),((\Re_2/0.3,\Bbbk_2,1),\emptyset),((\Re_2/0.3,\Bbbk_3,1),\emptyset)\}.$$

Definition 20. *A* $\mathbb{FPNSE}$*-set* $(\beth_2,\mathbb{Q}_2)$ *is called a relative whole* $\mathbb{FPNSE}$*-set w.r.t* $\mathbb{Q}_2 \subset \mathbb{Q}$*, denoted by* $(\beth_2,\mathbb{Q}_2)_{\hat{\aleph}}$ *, if* $\beth_1(g) = \hat{\aleph}, \forall\, g \in \mathbb{Q}_2$.

Example 21. *Taking Example 8, if*

$$(\beth_2,\mathbb{Q}_2)_{\hat{\aleph}} = \{((\Re_1/0.2,\Bbbk_1,1),\hat{\aleph}),((\Re_2/0.3,\Bbbk_2,1),\hat{\aleph}),((\Re_2/0.3,\Bbbk_3,1),\hat{\aleph})\}$$

where $\mathbb{Q}_2 \subseteq \mathbb{Q}$.

Definition 22. *The absolute whole* $\mathbb{FPNSE}$*-set of a* $\mathbb{FPNSE}$*-set* $(\beth,\mathbb{Q})$ *denoted by* $(\beth,\mathbb{Q})_{AW}$*, if* $\beth(g) = \hat{\aleph}, \forall\, g \in \mathbb{Q}$.

Example 23. *Using Example 8, if*

$$(\beth,\mathbb{Q})_{\text{AW}} = \left\{\begin{array}{l} \left(\Re_1/0.2,\Bbbk_1,1),\hat{\aleph}\right), \left(\Re_1/0.2,\Bbbk_2,1),\hat{\aleph}\right), \left(\Re_1/0.2,\Bbbk_3,1),\hat{\aleph}\right), \\ \left(\Re_2/0.3,\Bbbk_1,1),\hat{\aleph}\right), \left(\Re_2/0.3,\Bbbk_2,1),\hat{\aleph}\right), \left(\Re_2/0.3,\Bbbk_3,1),\hat{\aleph}\right), \\ \left(\Re_2/0.3,\Bbbk_1,1),\hat{\aleph}\right), \left(\Re_2/0.3,\Bbbk_2,1),\hat{\aleph}\right) \end{array}\right\}.$$

4. SET THEORETIC OPERATIONS OF $\mathbb{FPNSE}$-SET

In this portion, some set theoretic operations are presented with detailed examples.

Definition 24. *The union of* $\mathbb{FPNSE}$*-sets* $(\beth_1,\top)$ *and* $(\beth_2,\bot)$ *over* $\hat{\aleph}$ *is* $(\beth_3,\mathbb{L})$ *with* $\mathbb{L} = \top \cup \bot$*, defined as*

$$\beth_3(\varsigma) = \begin{cases} \beth_1(\varsigma) & ; \varsigma \in \top \backslash \bot \\ \beth_2(\varsigma) & ; \varsigma \in \bot \backslash \top \\ \cup(\beth_1(\varsigma),\beth_2(\varsigma)) & ; \varsigma \in \top \cap \bot \end{cases}$$

where

$$\cup(\beth_1(\varsigma),\beth_2(\varsigma)) = \{\langle u, max(\beth_1(\varsigma),\beth_2(\varsigma)), min(\beth_1(\varsigma),(\beth_2(\varsigma)), min(\beth_1(\varsigma),(\beth_2(\varsigma))\}\rangle : u \in \hat{\aleph}\}.$$

Example 25. *Using Example 8, with two sets*

$$\mathbb{Q}_1 = \left\{(\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_2, 1), (\Re_2/0.3, \Bbbk_2, 1)\right\}$$

$$\mathbb{Q}_2 = \left\{(\Re_1/0.2, \Bbbk_1, 1), (\Re_2/0.3, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_2, 1), (\Re_2/0.3, \Bbbk_2, 1)\right\}.$$

Suppose $(\beth_1, \mathbb{Q}_1)$ *and* $(\beth_2, \mathbb{Q}_2)$ *over* $\hat{\aleph}$ *are two* $\mathbb{FPNSE}$*-sets such that*

$$(\beth_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} \left((\Re_1/0.2, \Bbbk_1, 1), \left\{\frac{\nabla_1}{\langle 0.3,0.5,0.4\rangle}, \frac{\nabla_2}{\langle 0.7,0.4,0.5\rangle}\right\}\right), \\ \left((\Re_1/0.2, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.6,0.3\rangle}, \frac{\nabla_2}{\langle 0.8,0.4,0.5\rangle}\right\}\right), \\ \left((\Re_2/0.3, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.7,0.6\rangle}, \frac{\nabla_2}{\langle 0.8,0.9,0.7\rangle}\right\}\right), \end{array} \right\},$$

$$(\beth_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} \left((\Re_1/0.2, \Bbbk_1, 1), \left\{\frac{\nabla_1}{\langle 0.2,0.3,0.4\rangle}, \frac{\nabla_2}{\langle 0.7,0.3,0.2\rangle}\right\}\right), \\ \left((\Re_1/0.2, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.3,0.8\rangle}, \frac{\nabla_2}{\langle 0.8,0.3,0.5\rangle}\right\}\right), \\ \left((\Re_2/0.3, \Bbbk_1, 1), \left\{\frac{\nabla_1}{\langle 0.1,0.3,0.6\rangle}, \frac{\nabla_2}{\langle 0.9,0.1,0.7\rangle}\right\}\right), \\ \left((\Re_2/0.3, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.2,0.3\rangle}, \frac{\nabla_2}{\langle 0.6,0.3,0.5\rangle}\right\}\right) \end{array} \right\}.$$

Then $(\beth_1, \mathbb{Q}_1) \cup (\beth_2, \mathbb{Q}_2) = (\beth_3, \mathbb{Q}_3)$

$$(\beth_3, \mathbb{Q}_3) = \left\{ \begin{array}{l} \left((\Re_1/0.2, \Bbbk_1, 1), \left\{\frac{\nabla_1}{\langle 0.3,0.3,0.4\rangle}, \frac{\nabla_2}{\langle 0.7,0.3,0.2\rangle}\right\}\right), \\ \left((\Re_1/0.2, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.3,0.3\rangle}, \frac{\nabla_2}{\langle 0.8,0.3,0.5\rangle}\right\}\right), \\ \left((\Re_2/0.3, \Bbbk_1, 1), \left\{\frac{\nabla_1}{\langle 0.1,0.30,0.6\rangle}, \frac{\nabla_2}{\langle 0.9,0.10,0.7\rangle}\right\}\right), \\ \left((\Re_2/0.3, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.2,0.3\rangle}, \frac{\nabla_2}{\langle 0.6,0.3,0.5\rangle}\right\}\right) \end{array} \right\}.$$

Definition 26. *Restricted Union of two* $\mathbb{FPNSE}$*-sets* $(\beth_1, \mathbb{Q}_1)$, $(\beth_2, \mathbb{Q}_2)$ *over* $\hat{\aleph}$ *is* $(\beth_3, \mathbb{L})$ *with* $\mathbb{L} = \mathbb{Q}_1 \cap \mathbb{Q}_2$, *defined as* $\beth_3(\varsigma) = \beth_1(\varsigma) \cup_{\mathbb{R}} \beth_2(\varsigma)$ *for* $\varsigma \in \mathbb{Q}_1 \cap \mathbb{Q}_2$.

Example 27. *Consider the above Example, we have*
Then $(\beth_1, \mathbb{Q}_1) \cup_{\mathbb{R}} (\beth_2, \mathbb{Q}_2) = (\beth_3, \mathbb{L})$

$$(\beth_3, \mathbb{L}) = \left\{ \begin{array}{l} \left((\Re_1/0.2, \Bbbk_1, 1), \left\{\frac{\nabla_1}{\langle 0.3,0.3,0.4\rangle}, \frac{\nabla_2}{\langle 0.7,0.3,0.2\rangle}\right\}\right), \\ \left((\Re_1/0.2, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.3,0.3\rangle}, \frac{\nabla_2}{\langle 0.8,0.3,0.5\rangle}\right\}\right), \\ \left((\Re_2/0.3, \Bbbk_2, 1), \left\{\frac{\nabla_1}{\langle 0.4,0.2,0.3\rangle}, \frac{\nabla_2}{\langle 0.6,0.3,0.5\rangle}\right\}\right) \end{array} \right\}.$$

Proposition 28. *If* $(\beth_1, \mathbb{Q}_1), (\beth_2, \mathbb{Q}_2)$ *and* $(\beth_3, \mathbb{Q}_3)$ *are three* $\mathbb{FPNSE}$*-sets over* $\hat{\aleph}$, *then*

1. $(\beth_1, \mathbb{Q}_1) \cup (\beth_2, \mathbb{Q}_2) = (\beth_2, \mathbb{Q}_2) \cup (\beth_1, \mathbb{Q}_1)$,
2. $((\beth_1, \mathbb{Q}_1) \cup (\beth_2, \mathbb{Q}_2)) \cup (\beth_3, \mathbb{Q}_3) = (\beth_1, \mathbb{Q}_1) \cup ((\beth_2, \mathbb{Q}_2) \cup (\beth_3, N_3))$,
3. $(\beth, \mathbb{Q}) \cup \Phi = (\beth, \mathbb{Q})$.

Definition 29. *The intersection of* $\mathbb{FPNSE}$*-sets* $(\beth_1,\top)$ *and* $(\beth_2,\bot)$ *over* $\hat{\aleph}$ *is* $(\beth_3,\mathbb{L})$ *with* $\mathbb{L}=\top\cap\bot$, *defined as*

$$\beth_3(\varsigma)=\begin{cases}\beth_1(\varsigma) & ;\varsigma\in\top-\bot\\ \beth_2(\varsigma) & ;\varsigma\in\bot-\top\\ \cap(\beth_1(\varsigma),\beth_2(\varsigma)) & ;\varsigma\in\top\cap\bot\end{cases}$$

$\cap(\beth_1(\varsigma),\beth_2(\varsigma))=\{\langle u,min(\beth_1(\varsigma),\beth_2(\varsigma)),max(\beth_1(\varsigma),(\beth_2(\varsigma)),max(\beth_1(\varsigma),(\beth_2(\varsigma))\}\rangle:u\in\hat{\aleph}\}$.

Example 30. *Using data of Example 25, we have*
Then $(\beth_1,\mathbb{Q}_1)\cap(\beth_2,\mathbb{Q}_2)=(\beth_3,\mathbb{Q}_3)$

$$(\beth_3,\mathbb{Q}_3)=\left\{\begin{array}{l}\left((\Re_1/0.2,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.2,0.5,0.4\rangle},\frac{\triangledown_2}{\langle 0.7,0.4,0.5\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.6,0.8\rangle},\frac{\triangledown_2}{\langle 0.8,0.4,0.5\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.7,0.6\rangle},\frac{\triangledown_2}{\langle 0.6,0.9,0.7\rangle}\right\}\right)\end{array}\right\}.$$

Definition 31. *Extended intersection of* $(\beth_1,\mathbb{S})$ *and* $(\beth_2,\mathbb{R})$ *over* $\hat{\aleph}$ *is* $(\beth_3,\mathbb{L})$ *with* $\mathbb{L}=\mathbb{S}\cup\mathbb{R}$, *defined as*

$$\beth_3(\varsigma)=\begin{cases}\beth_1(\varsigma) & ;\varsigma\in\mathbb{S}-\mathbb{R}\\ \beth_2(\varsigma) & ;\varsigma\in\mathbb{R}-\mathbb{S}\\ \beth_1(\varsigma)\cap\beth_2(\varsigma) & ;\varsigma\in\mathbb{S}\cap\mathbb{R}.\end{cases}$$

Example 32. *Reconsidering data from Example 25, we have*
Then $(\beth_1,\mathbb{Q}_1)\cap_{\mathbb{E}}(\beth_2,\mathbb{Q}_2)=(\beth_3,\mathbb{L})$

$$(\beth_3,\mathbb{L})=\left\{\begin{array}{l}\left((\Re_1/0.2,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.2,0.5,0.4\rangle},\frac{\triangledown_2}{\langle 0.7,0.4,0.5\rangle}\right\}\right),\\ \left((\Re_1/0.2,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.6,0.8\rangle},\frac{\triangledown_2}{\langle 0.8,0.4,0.5\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_2,1),\left\{\frac{\triangledown_1}{\langle 0.4,0.7,0.6\rangle},\frac{\triangledown_2}{\langle 0.6,0.9,0.7\rangle}\right\}\right),\\ \left((\Re_2/0.3,\Bbbk_1,1),\left\{\frac{\triangledown_1}{\langle 0.1,0.3,0.6\rangle},\frac{\triangledown_2}{\langle 0.9,0.1,0.7\rangle}\right\}\right)\end{array}\right\}.$$

Proposition 33. *If* $(\beth_1,\mathbb{Q}_1),(\beth_2,\mathbb{Q}_2)$ *and* $(\beth_3,\mathbb{Q}_3)$ *are three* $\mathbb{FPNSE}$*-sets over* $\hat{\aleph}$, *then*

1. $(\beth_1,\mathbb{Q}_1)\cap(\beth_2,\mathbb{Q}_2)=(\beth_2,\mathbb{Q}_2)\cap(\beth_1,\mathbb{Q}_1)$,
2. $((\beth_1,\mathbb{Q}_1)\cap(\beth_2,\mathbb{Q}_2))\cap(\beth_3,\mathbb{Q}_3)=(\beth_1,\mathbb{Q}_1)\cap((\beth_2,\mathbb{Q}_2)\cap(\beth_3,\mathbb{Q}_3))$,
3. $(\beth,\mathbb{Q})\cap\phi=\phi$.

Proposition 34. *If* $(\beth_1,\mathbb{Q}_1),(\beth_2,\mathbb{Q}_2)$ *and* $(\beth_3,\mathbb{Q}_3)$ *are three* $\mathbb{FPNSE}$*-sets over* $\hat{\aleph}$, *then*

1. $(\beth_1,\mathbb{Q}_1)\cup((\beth_2,\mathbb{Q}_2)\cap(\beth_3,\mathbb{Q}_3))=((\beth_1,\mathbb{Q}_1)\cup((\beth_2,\mathbb{Q}_2))\cap((\beth_1,\mathbb{Q}_1)\cup(\beth_3,\mathbb{Q}_3))$,
2. $(\beth_1,\mathbb{Q}_1)\cap((\beth_2,\mathbb{Q}_2)\cup(\beth_3,\mathbb{Q}_3))=((\beth_1,\mathbb{Q}_1)\cap((\beth_2,\mathbb{Q}_2))\cup((\beth_1,\mathbb{Q}_1)\cap(\beth_3,\mathbb{Q}_3))$.

Definition 35. *If* $(\beth_1,\mathbb{Q}_1)$ *and* $(\beth_2,\mathbb{Q}_2)$ *are two* $\mathbb{FPNSE}$*-sets over* $\hat{\aleph}$ *then* $(\beth_1,\mathbb{Q}_1)$ *AND* $(\beth_2,\mathbb{Q}_2)$ *denoted by* $(\beth_1,\mathbb{Q}_1)\wedge(\beth_2,\mathbb{Q}_2)$ *is defined by* $(\beth_1,\mathbb{Q}_1)\wedge(\beth_2,\mathbb{Q}_2)=(\beth_3,\mathbb{Q}_1\times\mathbb{Q}_2)$, *while* $\beth_3(\varsigma,\gamma)=\beth_1(\varsigma)\cap\beth_2(\gamma),\forall(\varsigma,\gamma)\in\mathbb{Q}_1\times\mathbb{Q}_2$.

Example 36. *Retaking Example 8,*
Suppose $(\daleth_1, \mathbb{Q}_1)$ *and* $(\daleth_2, \mathbb{Q}_2)$ *over* $\hat{\aleph}$ *are two* $\mathbb{FPNSE}$*-sets such that*

$$(\daleth_1, \mathbb{Q}_1) = \left\{ \begin{array}{l} \left((\Re_1/0.2, \Bbbk_1, 1), \left\{ \frac{\triangledown_1}{\langle 0.3,0.5,0.4 \rangle}, \frac{\triangledown_2}{\langle 0.7,0.4,0.5 \rangle} \right\} \right), \\ \left((\Re_1/0.2, \Bbbk_2, 1), \left\{ \frac{\triangledown_1}{\langle 0.4,0.6,0.3 \rangle}, \frac{\triangledown_2}{\langle 0.8,0.4,0.5 \rangle} \right\} \right) \end{array} \right\},$$

$$(\daleth_2, \mathbb{Q}_2) = \left\{ \begin{array}{l} \left((\Re_1/0.2, \Bbbk_1, 0), \left\{ \frac{\triangledown_1}{\langle 0.2,0.3,0.4 \rangle}, \frac{\triangledown_2}{\langle 0.7,0.3,0.2 \rangle} \right\} \right), \\ \left((\Re_1/0.2, \Bbbk_2, 0), \left\{ \frac{\triangledown_1}{\langle 0.4,0.3,0.8 \rangle}, \frac{\triangledown_2}{\langle 0.8,0.3,0.5 \rangle} \right\} \right) \end{array} \right\}.$$

Then $(\daleth_1, \mathbb{Q}_1) \wedge (\daleth_2, \mathbb{Q}_2) = (\daleth_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$

$$\left\{ \begin{array}{l} \left(((\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_1, 0)), \left\{ \frac{\triangledown_1}{\langle 0.2,0.5,0.4 \rangle}, \frac{\triangledown_2}{\langle 0.7,0.4,0.5 \rangle} \right\} \right), \\ \left(((\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_2, 0)), \left\{ \frac{\triangledown_1}{\langle 0.3,0.5,0.8 \rangle}, \frac{\triangledown_2}{\langle 0.7,0.4,0.5 \rangle} \right\} \right), \\ \left(((\Re_1/0.2, \Bbbk_2, 1), (\Re_1/0.2, \Bbbk_1, 0)), \left\{ \frac{\triangledown_1}{\langle 0.2,0.6,0.4 \rangle}, \frac{\triangledown_2}{\langle 0.7,0.4,0.5 \rangle} \right\} \right), \\ \left(((\Re_1/0.2, \Bbbk_2, 1), (\Re_1/0.2, \Bbbk_2, 0)), \left\{ \frac{\triangledown_1}{\langle 0.4,0.6,0.8 \rangle}, \frac{\triangledown_2}{\langle 0.8,0.4,0.5 \rangle} \right\} \right) \end{array} \right\}.$$

Definition 37. *If* $(\daleth_1, \mathbb{Q}_1)$ *and* $(\daleth_2, \mathbb{Q}_2)$ *are two* $\mathbb{FPNSE}$*-sets over* $\hat{\aleph}$*, then* $(\daleth_1, \mathbb{Q}_1)$ *OR* $(\daleth_2, \mathbb{Q}_2)$ *denoted by* $(\daleth_1, \mathbb{Q}_1) \vee (\daleth_2, \mathbb{Q}_2)$ *is defined by* $(\daleth_1, \mathbb{Q}_1) \vee (\daleth_2, \mathbb{Q}_2) = (\daleth_3, \mathbb{Q}_1 \times \mathbb{Q}_2)$*, while* $\daleth_3(\delta, \gamma) = \daleth_1(\delta) \cup \daleth_2(\gamma), \forall (\delta, \gamma) \in \mathbb{Q}_1 \times \mathbb{Q}_2$.

Example 38. *Reconsidering Example 36,*
Then $(\daleth_3, \mathbb{Q}_3) \vee (\daleth_2, \mathbb{Q}_2) = (\daleth_3, \mathbb{Q}_1 \times \mathbb{Q}_2)=$

$$\left\{ \begin{array}{l} \left(((\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_1, 0)), \left\{ \frac{\triangledown_1}{\langle 0.3,0.3,0.4 \rangle}, \frac{\triangledown_2}{\langle 0.7,0.3,0.2 \rangle} \right\} \right), \\ \left(((\Re_1/0.2, \Bbbk_1, 1), (\Re_1/0.2, \Bbbk_2, 0)), \left\{ \frac{\triangledown_1}{\langle 0.4,0.3,0.4 \rangle}, \frac{\triangledown_2}{\langle 0.8,0.3,0.5 \rangle} \right\} \right), \\ \left(((\Re_1/0.2, \Bbbk_2, 1), (\Re_1/0.2, \Bbbk_1, 0)), \left\{ \frac{\triangledown_1}{\langle 0.4,0.3,0.3 \rangle}, \frac{\triangledown_2}{\langle 0.8,0.3,0.2 \rangle} \right\} \right), \\ \left(((\Re_1/0.2, \Bbbk_2, 1), (\Re_1/0.2, \Bbbk_2, 0)), \left\{ \frac{\triangledown_1}{\langle 0.4,0.3,0.3 \rangle}, \frac{\triangledown_2}{\langle 0.8,0.3,0.5 \rangle} \right\} \right) \end{array} \right\}.$$

Proposition 39. *If* $(\daleth_1, \mathbb{Q}_1), (\daleth_2, \mathbb{Q}_2)$ *and* $(\daleth_3, \mathbb{Q}_3)$ *are three* $\mathbb{FPNSE}$*-sets over* $\hat{\aleph}$*, then*

1. $((\daleth_1, \mathbb{Q}_1) \wedge (\daleth_2, \mathbb{Q}_2))^c = ((\daleth_1, \mathbb{Q}_1))^c \vee ((\daleth_2, \mathbb{Q}_2))^c$
2. $((\daleth_1, \mathbb{Q}_1) \vee (\daleth_2, \mathbb{Q}_2))^c = ((\daleth_1, \mathbb{Q}_1))^c \wedge ((\daleth_2, \mathbb{Q}_2))^c$.

Proof. (1) $((\daleth_1, \mathbb{Q}_1) \wedge (\daleth_2, \mathbb{Q}_2))^c = (\daleth_1(\phi) \cap (\daleth_2(\psi))^c$
$= (\daleth_1^c(\phi) \cup (\daleth_2^c(\psi))$
$= c(\daleth_1(\sim \phi) \cup c(\daleth_2(\sim \psi))$
$= ((\daleth_1, \mathbb{Q}_1))^c \vee ((\daleth_2, \mathbb{Q}_2))^c$.
(2) This is similar to the first. □

Proposition 40. *If* $(\daleth_1, \mathbb{Q}_1), (\daleth_2, \mathbb{Q}_2)$ *and* $(\daleth_3, \mathbb{Q}_3)$ *are three* $\mathbb{FPNSE}$*-sets over* $\hat{\aleph}$*, then*

1. $((\daleth_1, \mathbb{Q}_1) \wedge (\daleth_2, \mathbb{Q}_2)) \wedge (\daleth_3, \mathbb{Q}_3) = (\daleth_1, \mathbb{Q}_1) \wedge ((\daleth_2, \mathbb{Q}_2) \wedge (\daleth_3, \mathbb{Q}_3))$
2. $((\daleth_1, \mathbb{Q}_1) \vee (\daleth_2, \mathbb{Q}_2)) \vee (\daleth_3, \mathbb{Q}_3) = (\daleth_1, \mathbb{Q}_1) \vee ((\daleth_2, \mathbb{Q}_2) \vee (\daleth_3, \mathbb{Q}_3))$
3. $(\daleth_1, \mathbb{Q}_1) \vee ((\daleth_2, \mathbb{Q}_2) \wedge (\daleth_3, \mathbb{Q}_3) = ((\daleth_1, \mathbb{Q}_1) \vee ((\daleth_2, \mathbb{Q}_2)) \wedge ((\daleth_1, \mathbb{Q}_1) \vee (\daleth_3, \mathbb{Q}_3))$
4. $(\daleth_1, \mathbb{Q}_1) \wedge ((\daleth_2, \mathbb{Q}_2) \vee (\daleth_3, \mathbb{Q}_3)) = ((\daleth_1, \mathbb{Q}_1) \wedge ((\daleth_2, \mathbb{Q}_2)) \vee ((\daleth_1, \mathbb{Q}_1) \wedge (\daleth_3, \mathbb{Q}_3))$.

Proof. All propositions can be proved using proposition 4.16. □

5. AN APPLICATION TO FUZZY PARAMETERIZED NEUTROSOPHIC SOFT EXPERT SETS

This section presents an algorithmically suggested use of $\mathbb{FPNSE}$-set theory to a decision-making problem.

5.1. Statement of the problem

In this part, we demonstrate how fuzzy parameterized neutrosophic soft expert set theory is applied to a problem of decision-making.

Example 41. *Let's say a restaurant chain intends to invest in the construction of a new location in a city suburb. They asked some professionals to conduct an economic feasibility analysis on the place they had chosen for them. The expert's four choices are as follows:* $\Lambda = \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4\}$ *based on the criteria they utilised to arrive at their conclusions. Three parameters* Ψ_1= *Market research,* Ψ_2= *Location,*Ψ_3= *Menu,* Ψ_4= *Legal considerations,*Ψ_5= *Financing, are used to assess the restaurant site with the important degree 0.2,0.4,0.5,0.7,0.9 respectively.*

==

Proposed Algorithm : Selection of area for a newly Restaurant

==

▷ **Start**:
▷ **Input**:
———1. Consider set of alternatives
———2. Consider set of parameters
———3. Set of experts
———4. Set opinions of experts
▷ **Construction**:
———5. Construct $\mathbb{FPNSE}$-set (ξ, K)
▷ **Computation**:
———6. Determine agree-$\mathbb{FPNSE}$-set and disagree-$\mathbb{FPNSE}$-set.
———7. Calculation of values of $\mid \top(0_i) - \iota(0_i) - \bot(0_i) \mid$ for each $0_i \in \hat{\aleph}$.
———8. Calculate the the highest numerical grade for agree and disagree-$\mathbb{FPNSE}$-sets.
———9. Determine the score of each element $0_i \in \hat{\aleph}$ for agree and disagree-$\mathbb{FPNSE}$-sets.
———-10. Determine the score difference for each element $\Upsilon_i \in \hat{\aleph}$.
▷ **Output**:
———11. Compute n, for which M= max j_i to decide the best solution of the problem.
▷ **End**:

==

5.2. Role of parameters

This section includes the role and explanation of some selected parameters.

1. **Market research**: It is important to understand the local market and competition to ensure that there is demand for your restaurant and to determine the best location.

2. **Location**: The location of your restaurant will have a major impact on its success. Consider factors such as foot traffic, parking availability, and proximity to other restaurants and attractions.
3. **Menu**: Determine what type of food you will serve and how it will be priced. Consider the local market and competition when developing your menu. Developing a menu for a new restaurant is an important part of the planning process. Your menu should be carefully thought out and should reflect the concept and theme of your restaurant, as well as the preferences and tastes of your target market. Here are some things to consider when developing a menu for a new restaurant: Cuisine type: Determine the type of cuisine you will serve, and consider the local market and competition when making this decision. Pricing: Determine the price point for your menu items, keeping in mind the cost of ingredients and the local market. Portion size: Decide on the portion size for each menu item, considering the price point and the desired profit margin. Specials and seasonal items: Consider offering specials or rotating menu items based on the season or local availability of ingredients. Dietary restrictions: Consider offering options for customers with dietary restrictions, such as vegetarian, vegan, or gluten-free options. Plating and presentation: The presentation of your dishes can be just as important as the taste, so consider the plating and presentation of each menu item. Menu design: The design of menu should be visually appealing and easy to read. Consider using photos of your dishes, and be sure to include the name, description, and price of each menu item.
4. **Legal considerations**: Determine how you will finance the start-up costs of your restaurant, including the cost of purchasing or leasing a location, purchasing equipment, and hiring staff.
5. **Financing**: Obtain any necessary licenses and permits, and consider any local zoning laws or regulations that may impact your restaurant.

Step-1

Let Θ_1,Θ_2,Θ_3 be the three experts. We can determine the best place for the new restaurant to open in the network based on those findings. The committee creates the following fuzzy parameterized neutrosophic soft expert set after a thoughtful deliberation. Let $\Psi_1/0.2$, $\Psi_2/0.4$,$\Psi_3/0.5$, $\Psi_4/0.7$, $\Psi_5/0.9$ be fuzzy subset of $I^{\heartsuit}$(set of fuzzy subsets of $\heartsuit$. Then we can form the fuzzy parameterized neutrosophic soft expert set as

$$
(\xi,\mathrm{K})_1=\left\{\begin{array}{l}
\left((\Psi_1/0.2,\Theta_1,1),\left\{\frac{\Upsilon_1}{\langle 0.8,0.4,0.6\rangle},\frac{\Upsilon_2}{\langle 0.5,0.4,0.5\rangle},\frac{\Upsilon_3}{\langle 0.6,0.3,0.4\rangle},\frac{\Upsilon_4}{\langle 0.7,0.6,0.2\rangle}\right\}\right),\\
\left((\Psi_1/0.2,\Theta_2,1),\left\{\frac{\Upsilon_1}{\langle 0.7,0.3,0.4\rangle},\frac{\Upsilon_2}{\langle 0.8,0.4,0.3\rangle},\frac{\Upsilon_3}{\langle 0.6,0.5,0.3\rangle},\frac{\Upsilon_4}{\langle 0.9,0.2,0.5\rangle}\right\}\right),\\
\left((\Psi_1/0.2,\Theta_3,1),\left\{\frac{\Upsilon_1}{\langle 0.7,0.5,0.4\rangle},\frac{\Upsilon_2}{\langle 0.4,0.6,0.2\rangle},\frac{\Upsilon_3}{\langle 0.6,0.3,0.4\rangle},\frac{\Upsilon_4}{\langle 0.8,0.6,0.4\rangle}\right\}\right),\\
\left((\Psi_2/0.4,\Theta_1,1),\left\{\frac{\Upsilon_1}{\langle 0.8,0.4,0.2\rangle},\frac{\Upsilon_2}{\langle 0.4,0.2,0.1\rangle},\frac{\Upsilon_3}{\langle 0.7,0.1,0.5\rangle},\frac{\Upsilon_4}{\langle 0.5,0.2,0.1\rangle}\right\}\right),\\
\left((\Psi_2/0.4,\Theta_2,1),\left\{\frac{\Upsilon_1}{\langle 0.5,0.1,0.3\rangle},\frac{\Upsilon_2}{\langle 0.9,0.4,0.4\rangle},\frac{\Upsilon_3}{\langle 0.7,0.4,0.1\rangle},\frac{\Upsilon_4}{\langle 0.9,0.2,0.3\rangle}\right\}\right),\\
\left((\Psi_2/0.4,\Theta_3,1),\left\{\frac{\Upsilon_1}{\langle 0.7,0.3,0.2\rangle},\frac{\Upsilon_2}{\langle 0.8,0.2,0.4\rangle},\frac{\Upsilon_3}{\langle 0.9,0.2,0.5\rangle},\frac{\Upsilon_4}{\langle 0.8,0.2,0.5\rangle}\right\}\right),\\
\left((\Psi_3/0.5,\Theta_1,1),\left\{\frac{\Upsilon_1}{\langle 0.9,0.4,0.2\rangle},\frac{\Upsilon_2}{\langle 0.7,0.2,0.3\rangle},\frac{\Upsilon_3}{\langle 0.8,0.3,0.5\rangle},\frac{\Upsilon_4}{\langle 0.7,0.6,0.4\rangle}\right\}\right),\\
\left((\Psi_3/0.5,\Theta_2,1),\left\{\frac{\Upsilon_1}{\langle 0.8,0.3,0.4\rangle},\frac{\Upsilon_2}{\langle 0.9,0.2,0.5\rangle},\frac{\Upsilon_3}{\langle 0.6,0.7,0.3\rangle},\frac{\Upsilon_4}{\langle 0.9,0.5,0.3\rangle}\right\}\right),\\
\left((\Psi_3/0.5,\Theta_3,1),\left\{\frac{\Upsilon_1}{\langle 0.7,0.4,0.8\rangle},\frac{\Upsilon_2}{\langle 0.6,0.3,0.7\rangle},\frac{\Upsilon_3}{\langle 0.8,0.9,0.3\rangle},\frac{\Upsilon_4}{\langle 0.7,0.5,0.4\rangle}\right\}\right),\\
\left((\Psi_4/0.7,\Theta_1,1),\left\{\frac{\Upsilon_1}{\langle 0.9,0.1,0.7\rangle},\frac{\Upsilon_2}{\langle 0.8,0.3,0.5\rangle},\frac{\Upsilon_3}{\langle 0.7,0.6,0.4\rangle},\frac{\Upsilon_4}{\langle 0.6,0.3,0.2\rangle}\right\}\right),\\
\left((\Psi_4/0.7,\Theta_2,1),\left\{\frac{\Upsilon_1}{\langle 0.8,0.1,0.4\rangle},\frac{\Upsilon_2}{\langle 0.4,0.3,0.1\rangle},\frac{\Upsilon_3}{\langle 0.8,0.3,0.1\rangle},\frac{\Upsilon_4}{\langle 0.7,0.2,0.8\rangle}\right\}\right),\\
\left((\Psi_4/0.7,\Theta_3,1),\left\{\frac{\Upsilon_1}{\langle 0.6,0.2,0.4\rangle},\frac{\Upsilon_2}{\langle 0.9,0.3,0.2\rangle},\frac{\Upsilon_3}{\langle 0.9,0.5,0.3\rangle},\frac{\Upsilon_4}{\langle 0.6,0.1,0.3\rangle}\right\}\right),\\
\left((\Psi_5/0.9,\Theta_1,1),\left\{\frac{\Upsilon_1}{\langle 0.6,0.7,0.2\rangle},\frac{\Upsilon_2}{\langle 0.8,0.2,0.4\rangle},\frac{\Upsilon_3}{\langle 0.3,0.2,0.9\rangle},\frac{\Upsilon_4}{\langle 0.7,0.6,0.8\rangle}\right\}\right),\\
\left((\Psi_5/0.9,\Theta_2,1),\left\{\frac{\Upsilon_1}{\langle 0.5,0.3,0.7\rangle},\frac{\Upsilon_2}{\langle 0.6,0.2,0.3\rangle},\frac{\Upsilon_3}{\langle 0.4,0.3,0.6\rangle},\frac{\Upsilon_4}{\langle 0.6,0.8,0.2\rangle}\right\}\right),\\
\left((\Psi_5/0.9,\Theta_3,1),\left\{\frac{\Upsilon_1}{\langle 0.8,0.3,0.4\rangle},\frac{\Upsilon_2}{\langle 0.6,0.7,0.9\rangle},\frac{\Upsilon_3}{\langle 0.3,0.6,0.2\rangle},\frac{\Upsilon_4}{\langle 0.9,0.8,0.5\rangle}\right\}\right)
\end{array}\right\}.
$$

and

$$
(\xi,\mathrm{K})_0=\left\{\begin{array}{l}
\left((\Psi_1/0.2,\Theta_1,0),\left\{\frac{\Upsilon_1}{\langle 0.6,0.2,0.8\rangle},\frac{\Upsilon_2}{\langle 0.9,0.3,0.2\rangle},\frac{\Upsilon_3}{\langle 0.8,0.4,0.3\rangle},\frac{\Upsilon_4}{\langle 0.6,0.5,0.4\rangle}\right\}\right),\\
\left((\Psi_1/0.2,\Theta_2,0)\left\{\frac{\Upsilon_1}{\langle 0.7,0.8,0.4\rangle},\frac{\Upsilon_2}{\langle 0.6,0.4,0.5\rangle},\frac{\Upsilon_3}{\langle 0.7,0.2,0.5\rangle},\frac{\Upsilon_4}{\langle 0.8,0.7,0.9\rangle}\right\}\right),\\
\left((\Psi_1/0.2,\Theta_3,0),\left\{\frac{\Upsilon_1}{\langle 0.8,0.6,0.4\rangle},\frac{\Upsilon_2}{\langle 0.6,0.2,0.6\rangle},\frac{\Upsilon_3}{\langle 0.7,0.4,0.9\rangle},\frac{\Upsilon_4}{\langle 0.9,0.7,0.5\rangle}\right\}\right),\\
\left((\Psi_2/0.4,\Theta_1,0),\left\{\frac{\Upsilon_1}{\langle 0.9,0.4,0.5\rangle},\frac{\Upsilon_2}{\langle 0.9,0.7,0.4\rangle},\frac{\Upsilon_3}{\langle 0.8,0.6,0.7\rangle},\frac{\Upsilon_4}{\langle 0.8,0.5,0.8\rangle}\right\}\right),\\
\left((\Psi_2/0.4,\Theta_2,0),\left\{\frac{\Upsilon_1}{\langle 0.9,0.6,0.7\rangle},\frac{\Upsilon_2}{\langle 0.7,0.8,0.3\rangle},\frac{\Upsilon_3}{\langle 0.6,0.5,0.9\rangle},\frac{\Upsilon_4}{\langle 0.5,0.3,0.8\rangle}\right\}\right),\\
\left((\Psi_2/0.4,\Theta_3,0),\left\{\frac{\Upsilon_1}{\langle 0.7,0.8,0.9\rangle},\frac{\Upsilon_2}{\langle 0.8,0.5,0.2\rangle},\frac{\Upsilon_3}{\langle 0.7,0.8,0.6\rangle},\frac{\Upsilon_4}{\langle 0.8,0.5,0.4\rangle}\right\}\right),\\
\left((\Psi_3/0.5,\Theta_1,0),\left\{\frac{\Upsilon_1}{\langle 0.9,0.7,0.6\rangle},\frac{\Upsilon_2}{\langle 0.7,0.1,0.9\rangle},\frac{\Upsilon_3}{\langle 0.9,0.4,0.5\rangle},\frac{\Upsilon_4}{\langle 0.8,0.2,0.4\rangle}\right\}\right),\\
\left((\Psi_3/0.5,\Theta_2,0),\left\{\frac{\Upsilon_1}{\langle 0.8,0.2,0.5\rangle},\frac{\Upsilon_2}{\langle 0.9,0.2,0.8\rangle},\frac{\Upsilon_3}{\langle 0.9,0.6,0.4\rangle},\frac{\Upsilon_4}{\langle 0.9,0.6,0.7\rangle}\right\}\right),\\
\left((\Psi_3/0.5,\Theta_3,0),\left\{\frac{\Upsilon_1}{\langle 0.6,0.7,0.4\rangle},\frac{\Upsilon_2}{\langle 0.6,0.4,0.3\rangle},\frac{\Upsilon_3}{\langle 0.9,0.4,0.3\rangle},\frac{\Upsilon_4}{\langle 0.7,0.4,0.9\rangle}\right\}\right),\\
\left((\Psi_4/0.7,\Theta_1,0),\left\{\frac{\Upsilon_1}{\langle 0.6,0.3,0.2\rangle},\frac{\Upsilon_2}{\langle 0.7,0.5,0.9\rangle},\frac{\Upsilon_3}{\langle 0.8,0.2,0.9\rangle},\frac{\Upsilon_4}{\langle 0.9,0.4,0.6\rangle}\right\}\right),\\
\left((\Psi_4/0.7,\Theta_2,0),\left\{\frac{\Upsilon_1}{\langle 0.5,0.2,0.4\rangle},\frac{\Upsilon_2}{\langle 0.5,0.2,0.1\rangle},\frac{\Upsilon_3}{\langle 0.3,0.7,0.5\rangle},\frac{\Upsilon_4}{\langle 0.6,0.2,0.3\rangle}\right\}\right),\\
\left((\Psi_4/0.7,\Theta_3,0),\left\{\frac{\Upsilon_1}{\langle 0.7,0.9,0.3\rangle},\frac{\Upsilon_2}{\langle 0.7,0.2,0.9\rangle},\frac{\Upsilon_3}{\langle 0.6,0.7,0.4\rangle},\frac{\Upsilon_4}{\langle 0.9,0.2,0.6\rangle}\right\}\right),\\
\left((\Psi_5/0.9,\Theta_1,0),\left\{\frac{\Upsilon_1}{\langle 0.9,0.5,0.4\rangle},\frac{\Upsilon_2}{\langle 0.6,0.2,0.1\rangle},\frac{\Upsilon_3}{\langle 0.9,0.8,0.3\rangle},\frac{\Upsilon_4}{\langle 0.9,0.2,0.5\rangle}\right\}\right),\\
\left((\Psi_5/0.9,\Theta_2,0),\left\{\frac{\Upsilon_1}{\langle 0.8,0.6,0.7\rangle},\frac{\Upsilon_2}{\langle 0.9,0.2,0.1\rangle},\frac{\Upsilon_3}{\langle 0.8,0.5,0.8\rangle},\frac{\Upsilon_4}{\langle 0.7,0.3,0.5\rangle}\right\}\right),\\
\left((\Psi_5/0.9,\Theta_3,0),\left\{\frac{\Upsilon_1}{\langle 0.8,0.5,0.3\rangle},\frac{\Upsilon_2}{\langle 0.5,0.2,0.4\rangle},\frac{\Upsilon_3}{\langle 0.8,0.6,0.7\rangle},\frac{\Upsilon_4}{\langle 0.6,0.8,0.4\rangle}\right\}\right),
\end{array}\right\}
$$

Table 1: Values $\aleph = \mid \top(0) - \imath(0) - \bot(0) \mid$ for each $0 \in \hat{\aleph}$ for agree-$\mathbb{FPNSE}$-set

Pairs	ℓ_1	ℓ_2	ℓ_3	ℓ_4
$(\Psi_1/0.2, \Theta_1, 1)$	0.2	0.4	0.1	0.1
$(\Psi_1/0.2, \Theta_2, 1)$	0.0	0.1	0.2	0.3
$(\Psi_1/0.2, \Theta_3, 1)$	0.2	0.4	0.1	0.2
$(\Psi_2/0.4, \Theta_1, 1)$	0.3	0.1	0.1	0.2
$(\Psi_2/0.4, \Theta_2, 1)$	0.1	0.1	0.2	0.4
$(\Psi_2/0.4, \Theta_3, 1)$	0.2	0.2	0.4	0.1
$(\Psi_3/0.5, \Theta_1, 1)$	0.4	0.2	0.0	0.3
$(\Psi_3/0.5, \Theta_2, 1)$	0.1	0.2	0.5	0.1
$(\Psi_3/0.5, \Theta_3, 1)$	0.5	0.4	0.2	0.2
$(\Psi_4/0.7, \Theta_1, 1)$	0.1	0.0	0.3	0.1
$(\Psi_4/0.7, \Theta_2, 1)$	0.3	0.0	0.4	0.2
$(\Psi_4/0.7, \Theta_3, 1)$	0.0	0.4	0.1	0.2
$(\Psi_5/0.9, \Theta_1, 1)$	0.3	0.2	0.8	0.7
$(\Psi_5/0.9, \Theta_2, 1)$	0.6	0.1	0.5	0.4
$(\Psi_5/0.9, \Theta_3, 1)$	0.1	1.0	0.5	0.4

Table 2: Values $\pounds = \mid \top(\emptyset) - \imath(\emptyset) - \bot(\emptyset) \mid$ for each $\emptyset \in \Omega$ for disagree-$\mathbb{FPNSE}$-set

Pairs	ℓ_1	ℓ_2	ℓ_3	ℓ_4
$(\Psi_1/0.2, \Theta_1, 0)$	0.0	0.4	0.1	0.3
$(\Psi_1/0.2, \Theta_2, 0)$	0.4	0.2	0.0	0.8
$(\Psi_1/0.2, \Theta_3, 0)$	0.2	0.2	1.2	0.3
$(\Psi_2/0.4, \Theta_1, 0)$	0.0	0.2	0.9	0.5
$(\Psi_2/0.4, \Theta_2, 0)$	1.0	0.4	1.0	0.0
$(\Psi_2/0.4, \Theta_3, 0)$	1.0	0.1	0.9	0.1
$(\Psi_3/0.5, \Theta_1, 0)$	0.4	0.3	0.0	0.2
$(\Psi_3/0.5, \Theta_2, 0)$	0.1	0.1	0.1	1.0
$(\Psi_3/0.5, \Theta_3, 0)$	0.0	0.2	0.4	0.1
$(\Psi_4/0.7, \Theta_1, 0)$	0.5	0.1	0.2	1.2
$(\Psi_4/0.7, \Theta_2, 0)$	0.1	0.2	0.5	0.1
$(\Psi_4/0.7, \Theta_3, 0)$	0.5	0.4	0.5	0.1
$(\Psi_5/0.9, \Theta_1, 0)$	0.0	0.3	0.2	0.2
$(\Psi_5/0.9, \Theta_2, 0)$	0.5	0.6	0.5	0.1
$(\Psi_5/0.9, \Theta_3, 0)$	0.0	0.1	0.5	0.6

are $\mathbb{FPNSE}$-sets.

Step-2 To calculate the values of $\top(0)$-$\imath(0)$-$\bot(0)$, Tables 1 and 2 are constructed. For the example in table 1, the value 0.2 with respect to the $< 0.8, 0.4, 0.6 >$ is calculated by using the formula $\mid \top(0)$-$\imath(0)$-$\bot(0) \mid$, as $\mid 0.8 - 0.4 - 0.6 \mid = 0.2$.

Step-3
Tables 3 and 4 have been constructed for the grade values of agree and disagree-$\mathbb{FPNSE}$-sets respectively.

Table 3: Numerical Grades of agree-$\mathbb{FPNSE}$-set

Pairs	ℓ_i	Grade
$(\Psi_1/0.2,\Theta_1,1)$	ℓ_2	0.4
$(\Psi_1/0.2,\Theta_2,1)$	ℓ_4	0.3
$(\Psi_1/0.2,\Theta_3,1)$	ℓ_2	0.4
$(\Psi_2/0.4,\Theta_1,1)$	ℓ_1	0.3
$(\Psi_2/0.4,\Theta_2,1)$	ℓ_4	0.4
$(\Psi_2/0.4,\Theta_3,1)$	ℓ_3	0.4
$(\Psi_3/0.5,\Theta_1,1)$	ℓ_1	0.4
$(\Psi_3/0.5,\Theta_2,1)$	ℓ_3	0.5
$(\Psi_3/0.5,\Theta_3,1)$	ℓ_1	0.5
$(\Psi_4/0.7,\Theta_1,1)$	ℓ_3	0.3
$(\Psi_4/0.7,\Theta_2,1)$	ℓ_3	0.4
$(\Psi_4/0.7,\Theta_3,1)$	ℓ_2	0.4
$(\Psi_5/0.9,\Theta_1,1)$	ℓ_3	0.8
$(\Psi_5/0.9,\Theta_2,1)$	ℓ_1	0.6
$(\Psi_5/0.9,\Theta_3,1)$	ℓ_3	0.5

Table 4: Numerical Grades of disagree $\mathbb{FPNSE}$-set

Pairs	ℓ_i	Grade
$(\Psi_1/0.2,\Theta_1,0)$	ℓ_2	0.4
$(\Psi_1/0.2,\Theta_2,0)$	ℓ_4	0.8
$(\Psi_1/0.2,\Theta_3,0)$	ℓ_3	1.2
$(\Psi_2/0.4,\Theta_1,0)$	ℓ_3	0.9
$(\Psi_2/0.4,\Theta_2,0)$	ℓ_2	0.4
$(\Psi_2/0.4,\Theta_3,0)$	ℓ_3	0.9
$(\Psi_3/0.5,\Theta_1,0)$	ℓ_1	0.3
$(\Psi_3/0.5,\Theta_2,0)$	ℓ_4	1.0
$(\Psi_3/0.5,\Theta_3,0)$	ℓ_3	0.4
$(\Psi_4/0.7,\Theta_1,0)$	ℓ_1	0.5
$(\Psi_4/0.7,\Theta_2,0)$	ℓ_3	0.5
$(\Psi_4/0.7,\Theta_3,0)$	ℓ_1	0.5
$(\Psi_4/0.9,\Theta_1,0)$	ℓ_2	0.3
$(\Psi_4/0.9,\Theta_2,0)$	ℓ_2	0.6
$(\Psi_4/0.9,\Theta_3,0)$	ℓ_4	0.6

Step-(4-5)

The purpose of Table 5 is to find the difference of scores of agree and disagree-$\mathbb{FPNSE}$-sets. The scores of agree-$\mathbb{FPNSE}$-set are :

$S(\ell_1) = 0.6$, $S(\ell_2) = 1.3$, $S(\ell_3) = 0.6$ and $S(\ell_4) = 2.0$

and scores for disagree-$\mathbb{FPNSE}$-set are:

$S(\ell_1) = 1.3$, $S(\ell_2) = 1.6$, $S(\ell_3) = 1.8$ and $S(\ell_4) = 2.2$.

Step-6; Decision

Table 5: Numerical values of $\S_i = |\dagger_i - \ddagger_i|$

$\dagger_i$	$\ddagger_i$	$\S_i = \lvert\dagger_i - \ddagger_i\rvert$
$S(\ell_1) = 1.8$	$S(\ell_1) = 1.3$	0.6
$S(\ell_2) = 1.2$	$S(\ell_2) = 1.7$	0.5
$S(\ell_3) = 2.9$	$S(\ell_3) = 3.9$	1.0
$S(\ell_4) = 0.7$	$S(\ell_4) = 2.4$	1.7

Above analysis shows that, ℓ_4 is best and it is represented in Figure 1.

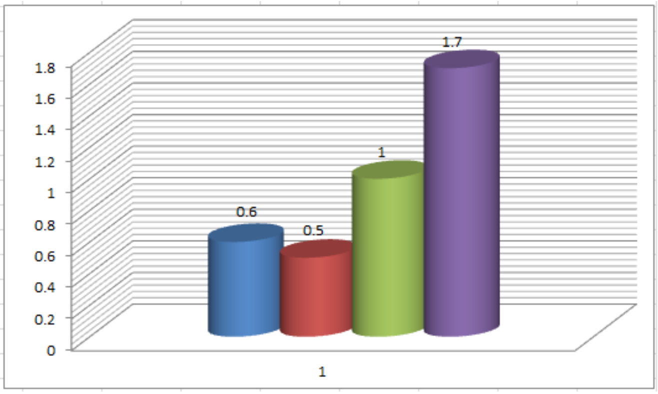

Figure 1: Ranking of alternatives for algorithm

6. DISCUSSION AND COMPARISON

To illustrate the advantages of our proposed method using $\mathbb{FPNSE}$-set as compared to fuzzy parameterised intuitionistic fuzzy soft expert set as proposed by Selvachandran and Salleh [56] which is a generalisation of fuzzy parameterised fuzzy soft expert. The $\mathbb{FPNSE}$-set, as demonstrated in the example above, can provide a more detailed explanation of the universal with three membership functions, particularly when numerous parameters are involved. In contrast, fuzzy parameterised intuitionistic fuzzy soft expert set [56] can only provide a limited amount of information about the universal. The $\mathbb{FPNSE}$-set is more accurate and realistic than fuzzy parameterized intuitionistic fuzzy soft expert set because it can handle problems involving imprecise, indeterminate, and inconsistent data, whereas it can only handle incomplete information when taking into account both the truth-membership and falsity-membership values [56]. The $\mathbb{FPNSE}$-set gives the best resemblance, accuracy, and acceptability when compared to the already accessible soft set-like models. This will be shown by contrasting $\mathbb{FPNSE}$-set to other models. This proposed model is more advantageous than others due to the inclusion of the multi-argument approximate function, which is particularly helpful in decision-making scenarios. In comparison to earlier models, including FPS-set, FPFS-set, FPIFS-set, FPSES-set, FPFSE-set, FPNS-set, and FPIFS-set, the results of the $\mathbb{FPNSE}$-set study show that it is superior. In contrast to these previous models, FPNSE-sets handle ambiguity and conflicting facts in complex decision-making more effectively by incorporating truth, falsity, and indeterminacy across different experts and attributes. By offering a more reliable framework for multi-expert evaluations, FPNSE-set improve decision accuracy in contrast to models such as FPFS-set and FPIFS-set, which do not fully address indeterminacy. By generating dependable and consistent results, skillfully handling a range of expert viewpoints, and taking into account subtle uncertainties, this study demonstrates that FPNSE-set perform better than these models in real-world situations. The comparison analysis is shown in Table 6.

Table 6: Comparison Analysis

Models	Exp. Opinions	Mship	Non-Mship	Indeterminacy
FPS-set [48]	No	No	No	No
FPSE-set [54]	Yes	No	No	No
FPFS-set [49]	Yes	Yes	No	No
FPFSE-set [55]	Yes	Yes	No	No
FPIFS-set [52]	No	Yes	Yes	No
FPIFSE-set [46]	Yes	Yes	Yes	No
$\mathbb{FPNSE}$-Set	Yes	Yes	Yes	Yes

7. CONCLUSION

Together with several generalizations of the theoretic operations union, intersection, complement, AND, and OR, this paper establishes the foundations of the fuzzy parameterized neutrosophic soft expert set. Some basic ideas, such as union, and intersection, and laws, such as idempotent, absorption, domination, identity, associative, and distributive, are addressed with reference to particular cases. Ultimately, an algorithm for making decisions is developed to characterize the procedure. The some limitations of the study are (a) the process can be difficult to comprehend and apply, especially for people who are not familiar with fuzzy logic, neutrosophic sets, or decision theory (b) accurate and thorough data are necessary for the method, but they might be challenging to find. Inaccurate or partial data can have a detrimental effect on the decision's quality. Though, it has certain advantages over different structures such as (a) with its strong foundation for integrating fuzzy and neutrosophic factors, it successfully addresses the innate ambiguity and uncertainty in decision-making processes (b) the decision-making process is more accurate and reliable when expert information and opinions are integrated using this strategy (c) it gives a more thorough examination of possible building places by taking into account a variety of criteria and features, resulting in more informed and balanced conclusions. This new study inspires further advancements of related research and practical applications while providing an excellent expansion to current theories for handling indeterminacy, falsehood, and truthiness.

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