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Research Article

A PRIMAL NONLINEAR FRACTIONAL PROGRAMMING APPROACH FOR POSYNOMIAL GEOMETRIC PROGRAMMING

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Abstract: Geometric programming (GP) is a well-established optimization framework widely used in engineering design and related areas for solving nonlinear optimization problems. Classical Classi approaches for solving GP problems typically rely on dual formulations, which may become restrictive when the degree of difficulty is high. In this paper, we propose a direct primal approach for solving constrained posynomial geometric programming problems by reformulating them as nonlinear fractional programming problems, without resorting to duality. The proposed method transforms the original GP into a ratio optimization problem and applies a parametrization technique based on the Dinkelbach method to obtain the optimal solution. This approach allows GP problems with nonzero degrees of difficulty to be handled in a systematic and computationally efficient manner. Theoretical results supporting the proposed formulation are presented, with several lemmas developed within the paper and standard results clearly distinguished from those recalled from existing literature. Numerical examples are provided to demonstrate the effectiveness and accuracy of the proposed approach. In addition, potential

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challenges associated with large-scale geometric programming problems, such as computational complexity and convergence issues, are briefly discussed.

Keywords: Fractional programming, geometric programming, nonlinear programming, posynomial.

MSC: 90C30, 90C32.

1. INTRODUCTION

To model an engineering design problem with specific types of nonlinear optimization problems with flexible variables in powers are known as geometric programming. Geometric programming model was first introduced by Duffin, Peterson and Zener in 1964 and was developed by themselves in a pioneering book [1]. They proposed an excellent idea to solve engineering problems by developing basic theories of geometric programs. Geometric programming provides a powerful tool for solving a class of problems in the field of engineering optimization. Their structure is built in terms of a class of functions which we call positive polynomials, or posynomials for short.

In the last few decades, we have seen a rapid development in geometric programming used in a variety of optimization problems involving digital circuit design [2, 3] and the problem of temperature-aware floor planning in which the parameters of the problem is often undetermined [4].

Many applications of geometric programming have been considered in the engineering design problems such as posynomial geometric programming with parametric uncertainty [5], posynomial parametric geometric programming with interval valued coefficient [6] and on generalized geometric programming problems with non-positive variables [7], solving linear multi-objective geometric programming problems via reference point approach [8], solving a posynomial geometric programming problem with fully fuzzy approach [9] have been published as well.

Ojha and Das [10] developed a solution procedure using geometric programming technique by splitting the coefficients and exponents with the help of binary numbers. Multi-objective geometric programming problem is worked out by Ojha et al. [11], in which they have proposed ε -constraint method that has been applied to find the non-inferior solution. In view of Rajgopal et al. [12].

A lot of research works have been developed in the area of risk management, inventory management and planning [13, 14]. Mahapatra et al. got optimal solution of the objective function directly without solving the equivalent transformed problem. They also considered problem of fuzzy decision making on reliability systems, via fuzzy geometric programming [15].

An essential book about fuzzy geometric programming is written by Cao in [16]. A very pioneering work in fuzzy geometric programming is due to Yang and Cao [17]. Global optimization of signomial geometric programming problems is investigated by Xu [18], and a new global optimization algorithm for signomial geometric programming via Lagrangian relaxation by Qu [19].

Some exponential-based decomposition methods have been developed for solving global optimization of generalized geometric programming (GGP) problems. However, the use of logarithmic exponential transformations restricts these methods to strictly positive variables. Moreover, the fractional programming problems arises from the summation of minimization of several quotient terms, which are composed of terms appearing in the objective function subject to the given constraints.

Borza et al. [20] applied the Charnes and Cooper's technique to solve the LFP problem with interval coefficients in the objective function. Pandey and Punnen [21] applied some methods based on the simplex method for solving the LFP problem. Hasan and Acharjee [22] investigated another idea to solve the LFP problem. Tantawy [23] solved the LFP problem by proposing methods based on the conjugate gradient projection. Odior [24] solved the LFP problem by an algebraic approach, which depends on the duality concept and partial fractions.

Many researchers have recently worked on the higher-order symmetric duality in non-differential multi-objective fractional programming problem with duality for minimax fractional as well duality relation for a class of multi-objective linear fractional programming problem [25, 26, 27, 28].

In the literature the main goal in geometric programming is to minimize the costs of a design. Since the clear characteristic of geometric programming problems is the existence of exponential expressions; Therefor the main idea of this paper is to convert geometric programming to nonlinear fractional programming form by elimination of negative exponential in the problem and then converting the nonlinear fractional programming in to a nonlinear programming via Dinkelbach method [29], and finally on solving the problem by existing method in the literature such as Lagrange multipliers or any appropriate optimization technique.

2. MATHEMATICAL FORMULATION

The general form of a posynomial geometric programming problem is as follows

$$\min_X \ \varphi(X) = \sum_{t=1}^{s_0} c_{0t} \prod_{j=1}^{n} x_j^{a_{0tj}}$$

$$s.t$$

$$\sum_{t=1}^{s_i} c_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \leq b_i \quad i = 1, \ldots, m \tag{1}$$

$$x_j > 0 \quad j = 1, \ldots, n$$

In the model (1), the objective function contains s_0 terms, while inequality constraints contain s_i terms; $i = 1, \cdots, m$.

By definition of posynomial all b_i ; $i = 1, \cdots, m$ are positive real numbers and the exponents a_{itj} ; $i = 0, 1, \cdots, m$ are arbitrary constants and all the coefficients c_{it} ; $i = 0, 1, \cdots, m$ are positive. Since exponents a_{itj} ; $i = 0, 1, \cdots, m$ are arbitrary constants, therefore we define index sets A and B as follows

$$A = \left\{ j | \ a_{itj} > 0 \quad \forall i, t \right\}$$

$$B = \left\{ j | \ a_{itj} < 0 \quad \forall i, t \right\}$$

If $\forall j; a_{itj} = 0$, it's obvious that $x_j^{a_{itj}} = 1$.

Thus according to A and B index sets, model (1) can be transformed into the following problem

$$\begin{aligned}
&\min_X \ \ \varphi(X) = \sum_{t=1}^{s_0} c_{0t} \prod_{j\in A} x_j^{a_{0tj}} \prod_{j\in B} x_j^{a_{0tj}} \\
&s.t \\
&\quad \sum_{t=1}^{s_i} c_{it} \prod_{j\in A} x_j^{a_{itj}} \prod_{j\in B} x_j^{a_{itj}} \le b_i \quad i = 1,\dots,m \\
&\qquad x_j > 0 \quad\ j = 1,\dots,n
\end{aligned} \tag{2}$$

Since $\forall j$; $x_j > 0$, we convert the product of variables that are in set B to the variables with positive exponents, therefore the model (2) can be reformulated as follows

$$\begin{aligned}
&\min_X \ \ \varphi(X) = \sum_{t=1}^{s_0} c_{0t} \prod_{j\in A} x_j^{a_{0tj}} \prod_{j\in B} \frac{1}{x_j^{-a_{0tj}}} \\
&s.t \\
&\sum_{t=1}^{s_i} c_{it} \prod_{j\in A} x_j^{a_{itj}} \prod_{j\in B} \frac{1}{x_j^{-a_{itj}}} \le b_i \quad i = 1,\dots,m \\
&\qquad x_j > 0 \quad\ j = 1,\dots,n
\end{aligned} \tag{3}$$

3. CONVERTING GEOMETRIC PROGRAMMING TO NONLINEAR FRACTIONAL PROGRAMMING

In the model (3), regarding that all of the exponents in the problem are positive and there is no variable having negative exponent, so without loss of generality, we can assume that

$$\begin{aligned}
c_{it} \prod_{j\in A} x_j^{a_{itj}} &= f_{it}(X); \quad i = 0,...,m;\ t = 1,...,s_i \\
\prod_{j\in B} x_j^{-a_{itj}} &= g_{it}(X); \quad i = 0,...,m;\ t = 1,...,s_i
\end{aligned}$$

By substitution the above relations, model (3) can be written as the following nonlinear fractional programming

$$\begin{aligned}
&\min_X \ \ \varphi(X) = \sum_{t=1}^{s_0} \frac{f_{0t}(X)}{g_{0t}(X)} \\
&s.t \\
&\sum_{t=1}^{s_i} \frac{f_{it}(X)}{g_{it}(X)} \le b_i \quad i = 1,...,m \\
&\qquad X > 0
\end{aligned} \tag{4}$$

4. SOLVING NONLINEAR FRACTIONAL PROGRAMMING

To simplify the model (4), we assume that

$$\prod_{t=1}^{s_i} g_{it}(X) = G_i(X)\,; \quad i = 0, \ldots, m \tag{5}$$

$$\sum_{k=1}^{s_i} f_{ik}(X) \prod_{\substack{t=1 \\ t\neq k}}^{s_i} g_{it}(X) = F_i(X); \quad i = 0, \ldots, m \tag{6}$$

Therefore the model (4) can be rewritten as follows

$$\begin{aligned} &\min_X \ \varphi(X) = \frac{F_0(X)}{G_0(X)} \\ &\quad s.t \\ &\quad \frac{F_i(X)}{G_i(X)} \leq b_i \quad i = 1, \ldots, m \\ &\quad\quad X > 0 \end{aligned} \tag{7}$$

5. HOW TO CONVERT A NLFP TO NONLINEAR PARAMETRIC PROGRAMMING

Let S be a compact and connected subset of $\mathbb{R}$. Let $F_0(X)$ and $G_0(X)$ be continuous functions of $X \in S$. Furthermore, according to the structure of geometric programming problem is also made $G_0(X) > 0$ for all $X \in S$. Now we are interested in solving the following two problems (8) and (9)

$$\min\{\xi = \frac{F_0(X)}{G_0(X)} \,|\, X \in S\} \tag{8}$$

The problem in (8) can be equivalently written as follow:

$$Z(\xi) = \min\{\, F_0(X) - \xi G_0(X) \mid ,\, X \in S\} \tag{9}$$

Now the following lemmas is discussed to get the solution for $Z(\xi)$.

Lemma 1. *$Z(\xi)$ is concave over $\mathbb{R}$.*

Proof. Let $\xi', \xi'' \in \mathbb{R}$, $X_f \in S$ is mimimum of $\{F_0(X) - (\lambda\xi' + (1-\lambda)\xi'')G_0(X) \mid X \in S\}$ and more over $\xi' \neq \xi''$ and $\lambda \in [0\,,\,1]$.

$$\begin{aligned} Z(\lambda\xi' + (1-\lambda)\xi'') &= \min\{F_0(X) - (\lambda\xi' + (1-\lambda)\xi'')G_0(X)\ \ X \in S\} \\ &= F_0(X_f) - (\lambda\xi' + (1-\lambda)\xi'')G_0(X_f) \\ &= F_0(X_f) - (\lambda\xi' + (1-\lambda)\xi'')G_0(X_f) + \lambda F_0(X_f) - \lambda F_0(X_f) \\ &= \lambda\left[F_0(X_f) - \xi' G_0(X_f)\right] + (1-\lambda)\left[F_0(X_f) - \xi'' G_0(X_f)\right] \\ &\geq \lambda \min\{F_0(X) - \xi' G_0(X) \,\Big|\, X \in S\} \\ &\quad + (1-\lambda)\min\{F_0(X) - \xi'' G_0(X)\ \ X \in S\} \\ &= \lambda Z(\xi') + (1-\lambda)Z(\xi'') \\ &\Longrightarrow\ Z(\lambda\xi' + (1-\lambda)\xi'') \geq \lambda Z(\xi') + (1-\lambda)Z(\xi''). \end{aligned}$$

Thus $Z(\xi)$ is concave over $\mathbb{R}$. $\square$

Lemma 2. *Show that $X_f \in S$ and $\xi_f = \frac{F_0(X_f)}{G_0(X_f)}$ then $Z(\xi_f) \leq 0$.*

Proof. It is obvious that if we let $Z(\xi_f) = \min\{F_0(X) - \xi_f G_0(X) \mid X \in S\}$ then

$$\min\{F_0(X) - \xi_f G_0(X) \mid X \in S\} \leq F_0(X_f) - \xi_f G_0(X_f) = 0 \Longrightarrow Z(\xi_f) \leq 0$$

□

Lemma 3. *$Z(\xi)$is continuous for $\xi \in \mathbb{R}$.*

Proof. see Courant [30] □

Lemma 4. *$Z(\xi) = \min\{\, F_0(X) - \xi G_0(X) \mid X \in S\}$ is strictly monotonic decreasing.*

Proof. let $\xi' < \xi''$ and $\xi', \xi'' \in \mathbb{R}$ then

$$Z(\xi') = \min\{\, F_0(X) - \xi' G_0(X) \mid X \in S\} = F_0(X') - \xi' G_0(X')$$

It is to be noted that X' is a feasible solution to the problem in such a way that $Z(\xi') = F_0(X') - \xi' G_0(X')$ Since $\xi' < \xi''$ therefore

$$\begin{aligned} F_0(X') - \xi' G_0(X') > F_0(X') - \xi'' G_0(X') \geq \min\{\, F_0(X) - \xi'' G_0(X) \mid X \in S\} = Z(\xi'') \\ \Rightarrow Z(\xi') > Z(\xi'') \end{aligned}$$

□

Lemma 5. *$Z(\xi) = 0$ has an unique solution.*

Proof. This assertion results from lemma 3 and lemma 4 and the following fact

$$\lim_{\xi \to -\infty} Z(\xi) = +\infty \quad \textit{and} \quad \lim_{\xi \to +\infty} Z(\xi) = -\infty$$

□

For any $\xi = \xi^*$ let the $\min\{\, F_0(X) - \xi^* G_0(X) \mid X \in S\}$ to be obtained at X^*. that may be indicated by $Z(\xi^*, x^*)$.

Theorem 6. *$\xi_f = \frac{F_0(X_f)}{G_0(X_f)} = \min\{\xi = \frac{F_0(X)}{G_0(X)} | \, X \in S\}$ if and only if*

$$Z(\xi_f) = Z(\xi_f, X_f) = \min\{\, F_0(X) - \xi_f G_0(X) \mid X \in S\} = 0$$

Proof. See Dinkelbach [29]. □

According to the above assumptions and to solve the problem (7), we denote the optimal solution of (9) by starting from the feasible solution X_0 which yields the problem (10) as follows.

$$\begin{aligned} &\min \quad F_0(X) - \xi G_0(X) \\ &s.t \\ &\frac{F_i(X)}{G_i(X)} \le b_i \;\Rightarrow\; b_i G_i(X) - F_i(X) \ge 0;\; i = 1,...,m \\ &\quad X > 0 \end{aligned} \tag{10}$$

since $Z(\xi)$ is continuous, we find X_n and $\xi_n = \frac{F_0(X_n)}{G_0(X_n)}$ such that $|Z(\xi_n)| < \varepsilon$ for $\varepsilon > 0$.

Forthermore, we assume that $Z(0) = \min\{\, F_0(X) |\, X \in S\} \ge 0$ hence the algorithm can be started with $\xi > 0$ or by any feasible point $X = X_0 \in S$ with $\xi \ge 0$.

Now we follow the following algorithm due to Dinkelbach.
(I) Here we can start with $\xi_0 = 0$ or any feasible point. (our results in this paper state that it is recommended to use any feasible point X_f rather than $\xi_0 = 0$.)
(II) $Z(\xi_k) = \min\{\, F_0(X) - \xi_k G_0(X) \mid X \in S\}$ is a nonlinear programming that can be solved by any appropriate optimization technique. Let to denote its solution point by X_k.
If $|Z(\xi_k)| < \varepsilon$ stop.
(If $Z(\xi_k) = 0$, then $X_k = X^*$ is uniqe optimum solution.)
If $Z(\xi_k) \ge \varepsilon$; Evaluate $\xi_{k+1} = \frac{F_0(X_k)}{G_0(X_k)}$ and then replacing ξ_k by ξ_{k+1} in (II).

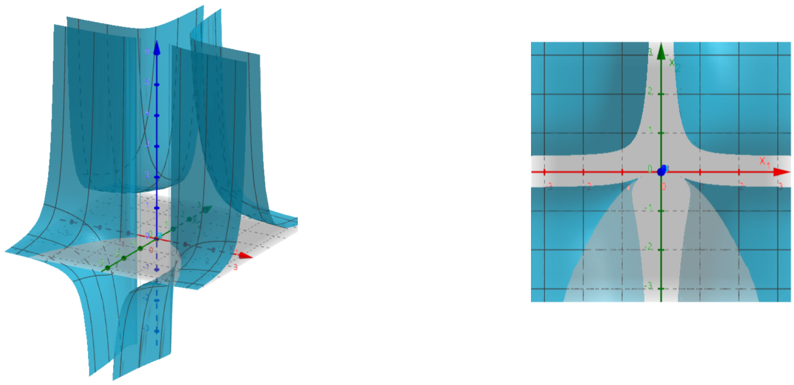
As we see the most important issue in this article is that we could directly obtain the objective function value and the primal optimum solution without considering the dual of the geometric programming model. More over we are also not in need to consider the degree of difficulty; i.e degree of difficulty is not an important issue in our approach.

6. NUMERICAL EXAMPLE

In the first example we will geometrically describe the behaviour of objective function and constraints to clarify our approach.

Example 7. *Consider the following optimization problem with two positive decision variables x_1 and x_2.*

$$\begin{aligned} &\min_X \quad \varphi(X) = 2x_1^{-2}x_2^{-1} + x_2^{-2} \\ &s.t \\ &4x_1x_2 + x_1x_2^{-2} \le 8 \\ &x_1^{-2}x_2 \le 2 \\ &\quad x_1, x_2 > 0 \end{aligned} \tag{11}$$



(a) figure of orginal objective function $\varphi(X) = 2x_1^{-2}x_2^{-1} + x_2^{-2}$

(b) The zero mapping of objective function in $x_1 - x_2$ palne

Figure 1: Geometrical representation of objective function

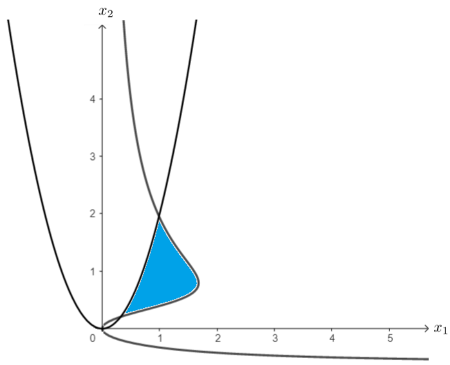


Figure 2: Geometrical representation of feasible region

We firstly convert the geometric programming in (11) *to a fractional programming as follows*

$$
\begin{aligned}
&\min_{X} \quad \varphi(X) = \frac{2x_2 + x_1^2}{x_1^2\, x_2^2} \\
&s.t \\
&\quad 8x_2^2 - 4x_1x_2^3 - x_1 \geq 0 \\
&\quad 2x_1^2 - x_2 \geq 0 \\
&\qquad x_1, x_2 > 0
\end{aligned}
\tag{12}
$$

Let

$$\xi_k = \frac{2x_2 + x_1^2}{x_1^2 x_2^2} \quad (k \in \mathbb{N})$$

Which can be furthere reduced to the following nonfractional as below

$$Z(\xi_k) = \min\{2x_2 + x_1^2 - \xi_k x_1^2\, x_2^2 |\, 8x_2^2 - 4x_1x_2^3 - x_1 \geq 0;\ 2x_1^2 - x_2 \geq 0;\ \ x_1, x_2 > 0\} \quad (13)$$

Now to solve (13) *we follow our algorithm.*
Step 1. Initially we begin with a feasible point $X_0 = (1, \frac{1}{2})$ *and set* $\xi_0 = \frac{2x_2+x_1^2}{x_1^2x_2^2}$ *for* X_0. *This feasible point help us to obtain* $\xi_0 = 8$ *and proceed with solving the following programming for* $Z(\xi_0)$.

$$Z(\xi_0) = \min\{2x_2 + x_1^2 - \xi_0 x_1^2\, x_2^2 |\, 8x_2^2 - 4x_1x_2^3 - x_1 \geq 0;\ 2x_1^2 - x_2 \geq 0;\ \ x_1, x_2 > 0\}$$

Equallently

$$Z(8) = \min\{2x_2 + x_1^2 - 8x_1^2\, x_2^2 |\, 8x_2^2 - 4x_1x_2^3 - x_1 \geq 0;\ 2x_1^2 - x_2 \geq 0;\ \ x_1, x_2 > 0\} \quad (14)$$

The solution for (14) *is obtained as* $Z(8) = -25.07$, $X_1 = (0.98, 1.95)$. *Figure 3 illustrate the geometrical representation of above problem* (14).

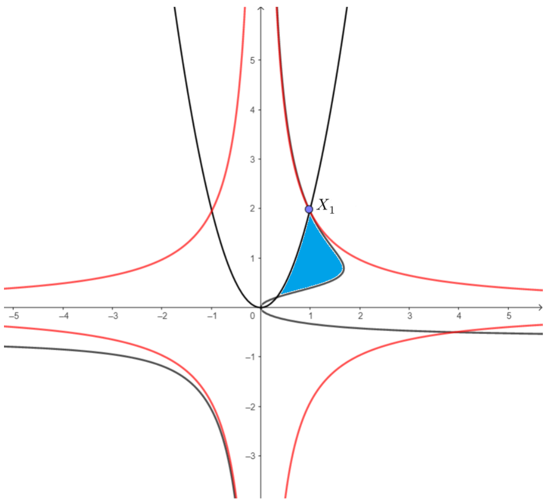


Figure 3: Geometrical representation of model (14)

Since $|Z(\xi_0)| > \varepsilon$ *for* ε*-tolerancy* $(\varepsilon = 0.1)$, *we therefore move to next step.*
Step 2. We set $\xi_1 = \frac{2x_2+x_1^2}{x_1^2x_2^2}$ *for* $X_1 = (0.98, 1.95)$ *and obtain* $\xi_1 = 1.331$.

$$Z(1.331) = \min\{2x_2 + x_1^2 - 1.331x_1^2\, x_2^2 |\, 8x_2^2 - 4x_1x_2^3 - x_1 \geq 0;\ 2x_1^2 - x_2 \geq 0;\ \ x_1, x_2 > 0\} \quad (15)$$

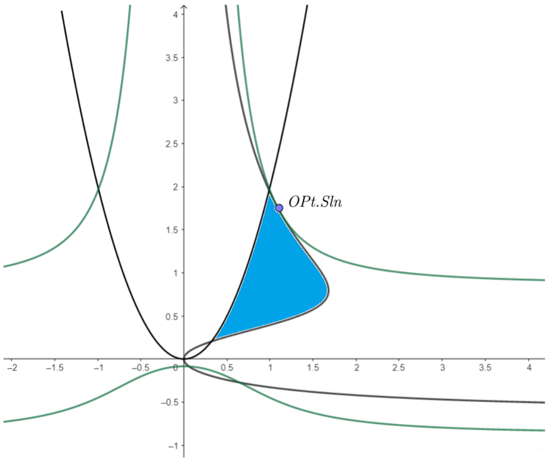


Figure 4: Geometrical representation of model (15)

The solution for (15) *is obtained as* $Z(1.331) = -0.169$, $X_2 = (1.11, 1.72)$. *Figure 4 illustrate the geometrical representation of above problem* (15).

Since $|Z(\xi_2| > 0.1$, *we therefore move to next step.*
Step 3. We obtain $\xi_2 = 1.299$ *for* $X_2 = (1.11, 1.72)$.

$$Z(1.331) = \min\{2x_2 + x_1^2 - 1.299x_1^2\, x_2^2 |\; 8x_2^2 - 4x_1x_2^3 - x_1 \geq 0;\; 2x_1^2 - x_2 \geq 0;\; x_1, x_2 > 0\} \tag{16}$$

The solution for (16) *is obtained as* $Z(1.299) = -0.052$, $X_3 = (1.11, 1.72)$.
Since $|Z(\xi_2| < 0.1$, *we therefore stop.*
Therefore let us take $(x_1^*, x_2^*) = (1.11, 1.72)$ *as an approximate optimal solution and the objective value is* $\varphi(X^*) = 1.299$. *This solution is optimum within the* $\varepsilon = 0.1$.

Example 8. *Consider the following geometric programming problem with three positive decision variables.*

$$\begin{aligned} &\min_X \quad \varphi(X) = 40x_1^{-1}x_2^{-1}x_3^{-1} + 40x_1x_3 \\ &s.t \\ &\quad 2x_1x_2 + x_2x_3 \leq 4 \\ &\qquad x_1, x_2, x_3 > 0 \end{aligned}$$

According to the models (3) *and* (4), *we rewrite the above problem as follows*

$$\begin{aligned} &\min_X \quad \varphi(X) = \frac{40 + 40x_1^2x_2x_3}{x_1\, x_2\, x_3} \\ &s.t \\ &\quad 4 - 2x_1x_2 - x_2x_3 \geq 0 \\ &\qquad x_1, x_2, x_3 > 0 \end{aligned}$$

Let

$$\xi_k = \frac{40+40x_1^2x_2x_3}{x_1\ x_2\ x_3} \quad (k \in \mathbb{N}) \tag{17}$$

we set the ε*-tolerance as* $\varepsilon = 0.01$ *and define* $Z(\xi_k)$ *as follows*

$$Z(\xi_k) = \min\{40+40x_1^2x_2x_3 - \xi_k x_1\ x_2\ x_3 |\ 4-2x_1x_2-x_2x_3 \geq 0;\ x_1,x_2,x_3 > 0\} \tag{18}$$

Now to solve (18) *we follow our algorithm.*
step1. Initially we begin with a feasible point $X_0 = (1,1,1)$*. This feasible point help us to obtain* $\xi_0 = 80$ *and proceed with solving the following programming for* $Z(80)$*.*

$$Z(80) = \min\{40+40x_1^2x_2x_3 - 80x_1\ x_2\ x_3 |\ 4-2x_1x_2-x_2x_3 \geq 0;\ x_1,x_2,x_3 > 0\} \tag{19}$$

The solution for (19) *with* $X_1 = (0.58,1.73,1.15)$ *is obtained as* $Z(80) = -21.58$*.*
Since $|Z(80)| > 0.01$*, we therefore move to next step.*
Step2. We obtain $\xi_1 = 61.34$ *for* $X_1 = (0.58,1.73,1.15)$*. Now we solve model* (20)

$$Z(61.34) = \min\{40+40x_1^2x_2x_3 - 61.34x_1\ x_2\ x_3 |\ 4-2x_1x_2-x_2x_3 \geq 0;\ x_1,x_2,x_3 > 0\} \tag{20}$$

The solution for (20) *is achieved as* $Z(61.34) = -1.34$ *with* $X_2 = (0.5,1.98,1.01)$*.*
Again since $|Z(61.34)| > 0.01$*, we therefore continue to next step.*
Step3. By substituting $X_2 = (0.58,1.73,1.15)$ *in* (17) *we get* $\xi_2 = 60.2$*. Now we solve model* (21)*.*

$$Z(60.2) = \min\{40+40x_1^2x_2x_3 - 60.2x_1\ x_2\ x_3 |\ 4-2x_1x_2-x_2x_3 \geq 0;\ x_1,x_2,x_3 > 0\} \tag{21}$$

The point $X_3 = (0.5,2,1)$ *will demonstrate* $Z(60.2) = -0.2$*.*
Now we verify whether this value is satisfied with the given tolerance or not. We see that still $|Z(60.2)| > 0.01$ *which indicates. That we have to move forether.*
Step4. As the previous steps we obtain $\xi_3 = 60$ *for* $X_3 = (0.5,2,1)$ *in which*

$$Z(60) = \min\{40+40x_1^2x_2x_3 - 60x_1\ x_2\ x_3 |\ 4-2x_1x_2-x_2x_3 \geq 0;\ x_1,x_2,x_3 > 0\} \tag{22}$$

The solution for (22) *is finally obtained as* $Z(60) = 0$*, we therefore stop.*
Thus let us take $X^* = (0.5,2,1)$ *as a optimal solution and the objective value is* $\varphi(X^*) = 60$*. since* $Z(\xi_3) = 0$*, this solution is uniqu optimum.*

7. CONCLUSION

Generalized geometric programming (GGP) problems occur frequently in engineering design and management. As we know geometric programming problems are solved via their duals by help of normality and orthogonally conditions. The advantage of the present approach is to directly obtain the objective function value and the primal optimum solution without considering the dual of the geometric programming model just by converting the geometric programming into a fractional programming and then solve the fractional programming problem by converting it to a nonlinear programming model. Numerical examples are given to ensure the efficiency of our approach.

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